Homework 2

CSE 105, Spring 2025
Due: Monday April 14, 11:59pm

Instructions:

Upload a single file to Gradescope for each group. All group members' names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning, where applicable. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Read each question carefully. Each question is worth 3 points (total 15 points).

Problem 1:

Let L be a finite language (that is, a language with a finite number of strings). Give a detailed proof showing that L is regular.

Problem 2:

The languages in this problem are over the alphabet {0,1}.
A is the language consisting of strings whose second symbol is 0.
B is the language consisting of strings whose last two symbols are 11.

(a) Construct an NFA recognizing $C = A \cap B$.
(b) Construct an NFA recognizing $D = A \circ B$.

Briefly explain your constructions.

Problem 3:

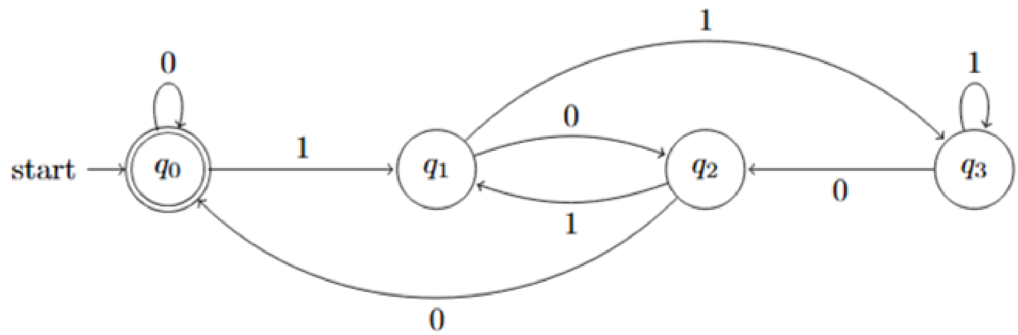
Construct an NFA for the language L over $\Sigma = \{0,1\}$ given by
$L = \{w \in \Sigma^* \mid w \text{ begins or ends with 00 or 11}\}$.

Exercise (Optional): In class, we discussed how to construct a DFA for the union of two languages as well as how to convert an NFA into a DFA, both of which will yield valid DFAs for the given question albeit with many states. Can you also construct a DFA with a small number of states, perhaps 8 or fewer?

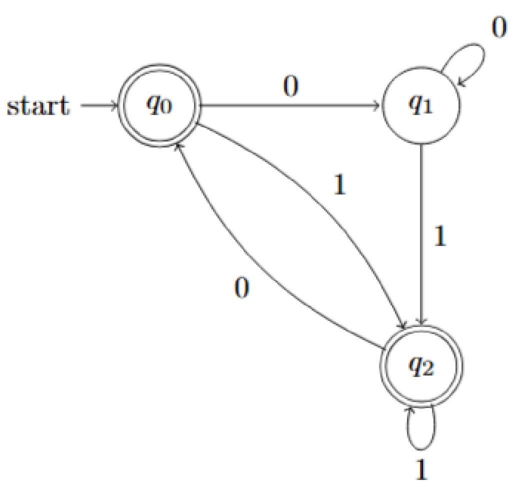
Problem 4:

Let L1 be the language recognized by the DFA M1 and L2 be the language recognized by M2 which are shown below. Note that the input strings w can be interpreted as binary numbers and you can also assume that w is provided to the automata in Big-Endian format. In other words, the automata processes w from the most significant bit to the least significant bit.

M1:



M2:



(a) Identify the languages L1 and L2. Briefly justify your answers. (You can provide an informal description of the languages in English)

(b) What are the languages L1 ∪ L2 and L1 ∩ L2? What would the DFAs for these languages look like?
For full credit, your descriptions of languages and DFAs should be as simple as possible. For both L1 ∪ L2 and L1 ∩ L2, there is a DFA with at most two states.

Problem 5:

Let L be any regular language over the alphabet Σ. Define $T(L) = \{yx \mid xy \in L, x \in \Sigma^*, y \in \Sigma^*\}$. Prove that T(L) is regular. (You may use without proof the fact that if a language can be computed by an NFA, then it can also be computed by a DFA.)

Problem 6 (Extra credit):

We use the following definition in this problem: a language $A \subset \{0,1\}^*$ is called <u>cofinite</u> if its complement is finite.

(a) Let k be any positive integer. Let $A_1, A_2, \ldots, A_k$ be cofinite subsets of $\{0, 1\}^*$. Prove that their intersection $A_1 \cap A_2 \cap \ldots \cap A_k$ is regular.

(b) Does your proof also work when there are infinitely many cofinite sets A_i? If not, explain what fails in your proof.