

University of California, San Diego
Department of Electrical and Computer Engineering



ECE 109: Problem Set #1

1. An ice cream store manufactures five basic flavors (vanilla, chocolate, mint, strawberry, and fudge) of ice cream, and then creates its specialty flavors by mixing two or more basic flavors in equal proportions.
 - (a) How many specialty flavors can be obtained by mixing three basic flavors? How many of these contain chocolate? How many of these contain either chocolate or fudge or both? How many of these contain either chocolate or fudge but not both?
 - (b) How many distinct flavors (including basic and specialty flavors) of ice cream can the store offer? *Optional:* What's the name of the store?
2. Express each of the following events in terms of the events A , B , and C , and the operations of complementation, union, and intersection:
 - (a) at least one of the events A, B, C occurs;
 - (b) at most one of the events A, B, C occurs;
 - (c) none of the events A, B, C occurs;
 - (d) all three events A, B, C occur;
 - (e) exactly one of the events A, B, C occurs;
 - (f) events A and B occur, but not C ;
 - (g) either event A occurs, or if not then B also does not occur.

In each case draw the corresponding Karnaugh maps.

3. Suppose that A and B are two events defined over the same sample space, with probabilities $P(A) = 3/4$ and $P(B) = 3/8$.
 - (a) Show that $P(A \cup B) \geq 3/4$.
 - (b) Show that $1/8 \leq P(AB) \leq 3/8$.
 - (c) Give inequalities analogous to (a) and (b) for $P(A) = 2/3$ and $P(B) = 1/2$.

4. Suppose an experiment consists of picking a student from the set of all students registered on the UCSD campus this quarter. It is *not* necessary to assume that all students are equally likely to be picked, but you may make this assumption if it makes you feel happier and more confident.

(a) Consider the two events:

- A — the student has had four years of high school science (FYS),
 B — the student has had calculus in high school.

If the probability that he/she has had at least one of FYS and calculus is 0.7, and the probability that he/she has missed at least one of the two is 0.8, what is the probability that he/she has had *exactly* one of the two?

(b) Let C denote the event that the student is registered in ECE109 this quarter, and let events A and B and their probabilities be as in part (a). If students who had had at most one of FYS and calculus did not register in ECE109 this quarter,

- What is $P(A^c \cap B^c \cap C^c)$?
- What is the probability that the student picked is not registered in ECE109 **and** has had exactly one of FYS or calculus in high school?

(c) Using the data given in parts (a) and (b), which of the following probabilities:

$$P(ABC), P(C), P(A^c B C^c), P(A), P(A^c B^c C), P(A \cup B \cup C^c)$$

can you compute? It is not necessary to actually compute each probability.

5. Let $\Omega = \{1, 2, 3, \dots\}$ be the countably infinite sample space whose elements (outcomes) are the positive integers. For each positive integer n , define the event

$$A_n = \{ k : k \text{ is a multiple of } n \}$$

(a) Find n and m such that $A_n = A_3 \cap A_4$ and $A_m = A_6 \cap A_9$.

(b) If $P(\{k\}) = \frac{1}{3} \cdot \left(\frac{2}{3}\right)^{k-1}$ find the probability of the event A_3 .

Note: an exact answer is required here; if you write a program to obtain answers of the form 0.210526... you will receive no credit.