Problem 1 (3 points) Consider the query on a directed graph returning all nodes that are not reachable from a directed cycle. Write the query in stratified Datalog\(^{-}\).

Problem 2 (2 points) Recall that inclusion of Datalog programs is undecidable. Show that it is decidable, for a conjunctive query \(\varphi\) and a Datalog program \(P\), whether \(\varphi \subseteq P\) (i.e., \(\varphi(I) \subseteq P(I)\) for every input \(I\)).

Problem 3 (5 points) Consider the movie database consisting of two relations \(\text{Schedule}[\text{theater}, \text{title}]\) and \(\text{Movie}[\text{title}, \text{director}, \text{actor}]\). We do not make the unique director assumption, so one title may have multiple directors. Consider the query “List the theaters showing only movies by Hitchcock”.

(i) (2 points) Express the query in Datalog\(^{-}\) with inflationary fixpoint semantics.

(ii) (3 points) Suppose that in addition to the above relations, the database provides for each attribute \(A \in \{\text{theater, title, actor}\}\) a relation \(\text{succ}_A\) providing a successor relation on the set of values of attribute \(A\) in the database. In addition, assume \(\min_A\) and \(\max_A\) are constants providing the minimum and maximum values of \(A\). Express the same query using a semi-positive Datalog program.

Problem 4 (6 points) Let \(G\) be a directed graph. The length of a path in the graph is the number of edges along the path. The distance between two nodes \(a\) and \(b\) is the length of the shortest path from \(a\) to \(b\) (infinity if there is no path from \(a\) to \(b\)). The diameter of a directed graph is the maximum finite distance between any two nodes in the graph. Write a \(\text{while}^+\) program defining directed graphs whose diameter is even.

Problem 5 (4 points) Consider a database consisting of a unary relation \(R\) (i.e., \(R\) has just one attribute). Prove that every property of \(R\) definable by
a while\(^+\) program with no constants is also definable in CALC. **Hint:** there is a 3-line proof.