The Continuous Fourier Transform

Image Processing

CSE 166

Lecture 5

Announcements

- Assignment 2 is due Apr 13, 11:59 PM
- Assignment 3 will be released Apr 18
  - Due Apr 25, 11:59 PM
- Reading
  - Chapter 4: Filtering in the Frequency Domain
    - Sections 4.1 and 4.2

Overview: Image processing in the frequency domain

Image in spatial domain $f(x,y)$ → Fourier transform → Image in frequency domain $F(u,v)$

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Frequency domain processing

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Image in frequency domain $G(u,v)$ → Inverse Fourier transform → Image in spatial domain $g(x,y)$

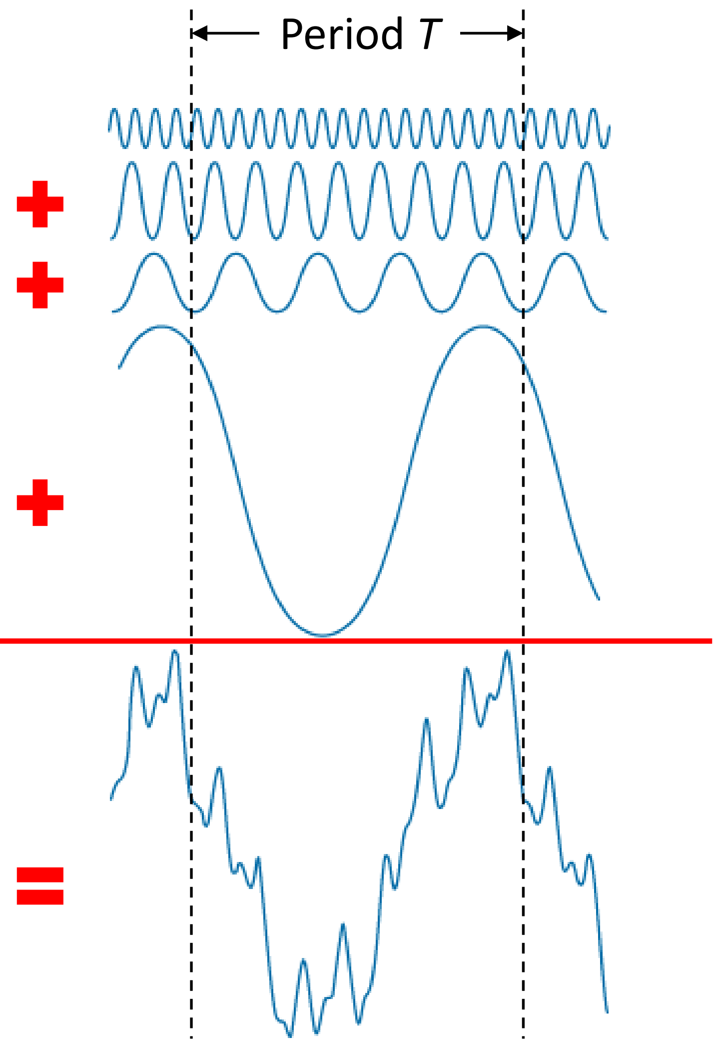
Jean-Baptiste Joseph Fourier
1768-1830

Review

- Complex numbers $C = R + jI$
- Euler's formula $e^{j\theta} = \cos\theta + j\sin\theta$
- Complex functions

1D Fourier series

Sines and cosines



Periodic function

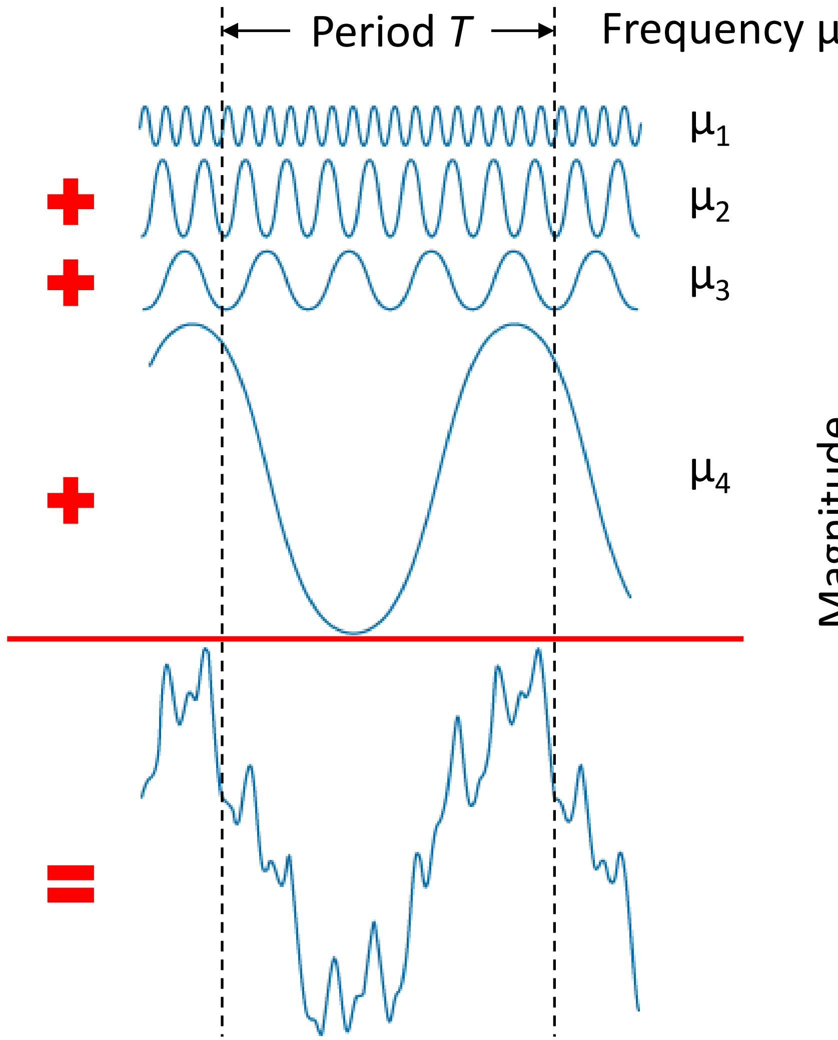
Weighted by magnitude

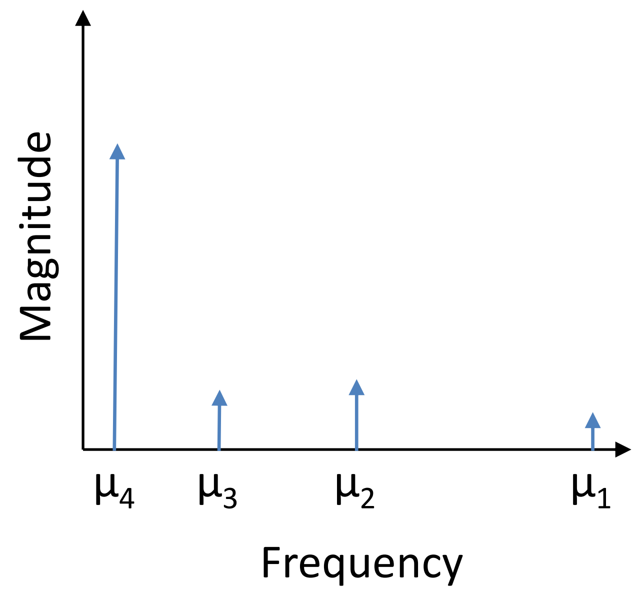
Shifted by phase

1D Fourier transform

Sines and cosines

Periodic function





1D continuous Fourier transform

- (Forward) Fourier transform

$$\mathfrak{F}\left\{f(t)\right\} = F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t}\,\mathrm{d}t$$

- Inverse Fourier transform

$$\mathfrak{F}^{-1}\left\{F(\mu)\right\} = f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t}\,\mathrm{d}\mu$$

1D continuous Fourier transform

- Example: box function

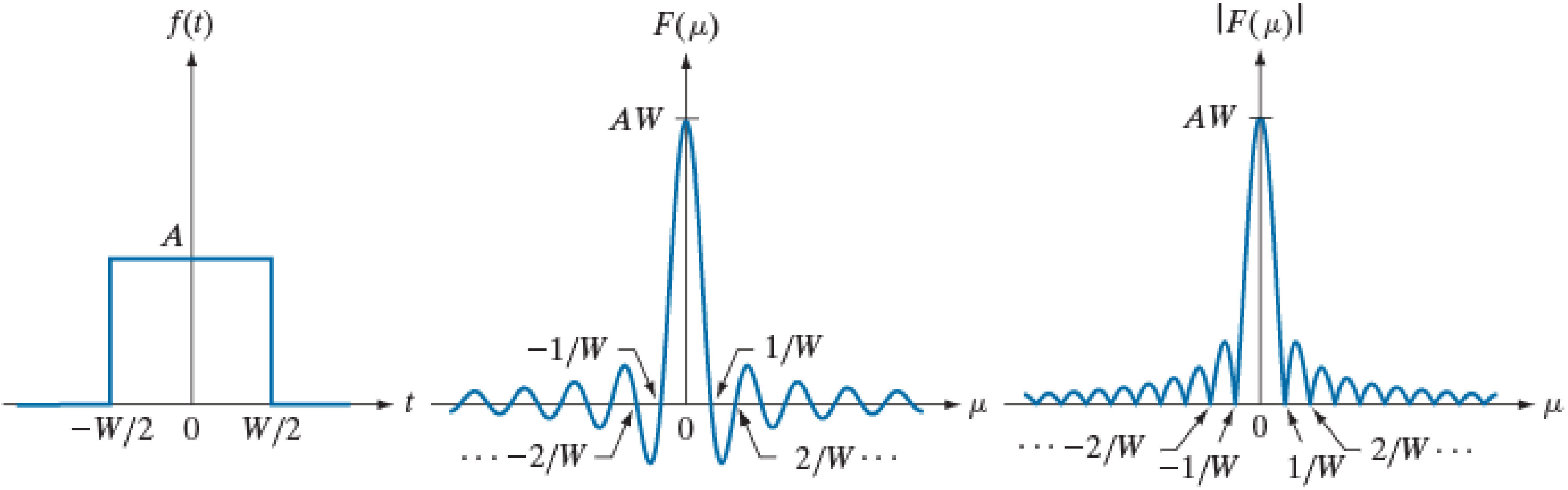


FIGURE 4.4 (a) A box function, (b) its Fourier transform, and (c) its spectrum. All functions extend to infinity in both directions. Note the inverse relationship between the width, W, of the function and the zeros of the transform.

1D convolution theorem

- Fourier transform of 1D continuous convolution

$$\mathfrak{F}\{f(t)\bigstar h(t)\} = H(\mu)F(\mu)$$

- 1D convolution theorem

$$f(t)\bigstar h(t) \Longleftrightarrow H(\mu)F(\mu)$$

$$f(t)h(t) \Longleftrightarrow H(\mu)\bigstar F(\mu)$$

Next Lecture

- Sampling and aliasing, and the discrete Fourier transform
- Reading
  - – Chapter 4: Filtering in the Frequency Domain
    - Sections 4.2, 4.3, and 4.4