The Continuous Fourier Transform

Image Processing
CSE 166
Lecture 5
Announcements

• Assignment 2 is due Apr 13, 11:59 PM
• Assignment 3 will be released Apr 18
  – Due Apr 25, 11:59 PM
• Reading
  – Chapter 4: Filtering in the Frequency Domain
    • Sections 4.1 and 4.2
Overview: Image processing in the frequency domain

Image in spatial domain $f(x,y)$ → Fourier transform → Image in frequency domain $F(u,v)$ → Frequency domain processing → Image in frequency domain $G(u,v)$ → Inverse Fourier transform → Image in spatial domain $g(x,y)$

Jean-Baptiste Joseph Fourier 1768-1830
Review

• Complex numbers \[ C = R + jI \]
• Euler’s formula \[ e^{j\theta} = \cos \theta + j \sin \theta \]
• Complex functions
1D Fourier series

Sines and cosines

Periodic function

Weighted by magnitude

Shifted by phase

Period $T$
1D Fourier transform

Sines and cosines

Periodic function

Period $T$  
Frequency $\mu$

$\mu_1$  
$\mu_2$  
$\mu_3$  
$\mu_4$

Magnitude

Frequency

CSE 166, Spring 2022
1D continuous Fourier transform

- (Forward) Fourier transform

\[ \mathcal{F} \{ f(t) \} = F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi \mu t} \, dt \]

- Inverse Fourier transform

\[ \mathcal{F}^{-1} \{ F(\mu) \} = f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi \mu t} \, d\mu \]
1D continuous Fourier transform

- Example: box function

![Diagram of a box function, its Fourier transform, and its spectrum. All functions extend to infinity in both directions. Note the inverse relationship between the width, W, of the function and the zeros of the transform.]
1D convolution theorem

- Fourier transform of 1D continuous convolution
  \[ \mathcal{F} \{ f(t) \star h(t) \} = H(\mu)F(\mu) \]

- 1D convolution theorem
  \[ f(t) \star h(t) \iff H(\mu)F(\mu) \]
  \[ f(t)h(t) \iff H(\mu) \star F(\mu) \]
Next Lecture

• Sampling and aliasing, and the discrete Fourier transform

• Reading
  – Chapter 4: Filtering in the Frequency Domain
    • Sections 4.2, 4.3, and 4.4