Spatial Filtering

Image Processing

CSE 166

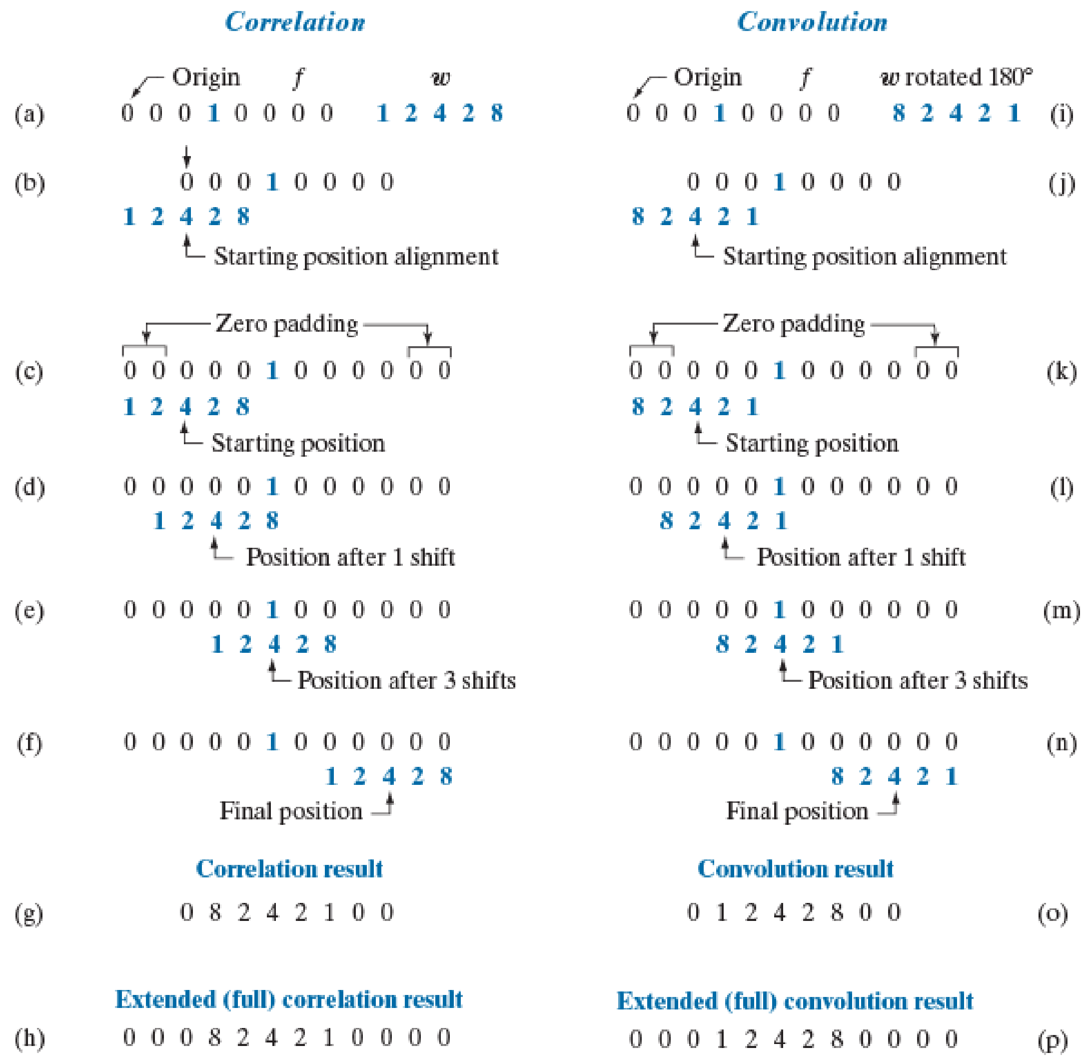
Lecture 4

Announcements

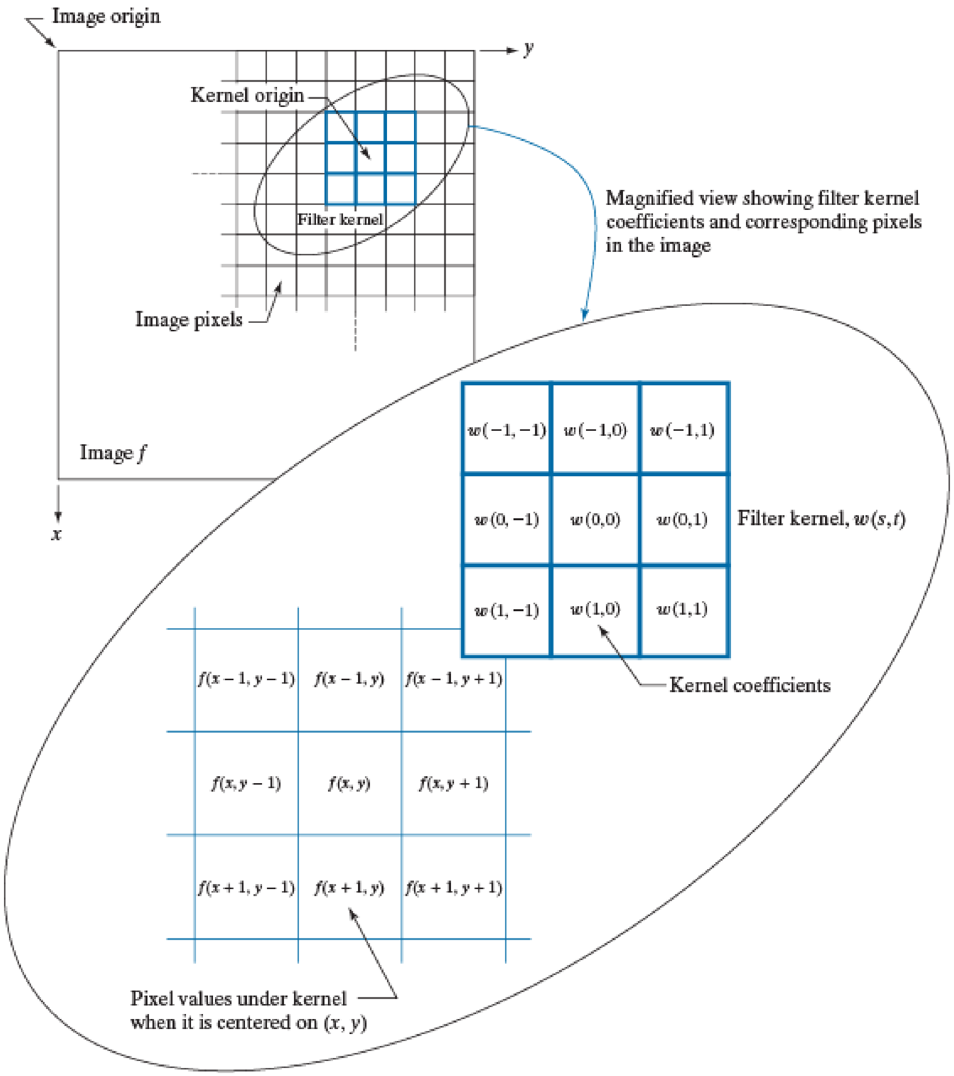
- Assignment 1 is due today, 11:59 PM
- Assignment 2 will be released today
  - Due Apr 13, 11:59 PM
- Reading
  - Chapter 3: Intensity Transformations and Spatial Filtering
    - Sections 3.4, 3.5, 3.6, and 3.8

Correlation and convolution (1D)

FIGURE 3.35 Illustration of 1-D correlation and convolution of a kernel, *w*, with a function *f* consisting of a discrete unit impulse. Note that correlation and convolution are functions of the variable *x*, which acts to *displace* one function with respect to the other. For the extended correlation and convolution results, the starting configuration places the right-most element of the kernel to be coincident with the origin of *f*. Additional padding must be used.



Spatial filtering (2D)



2D correlation

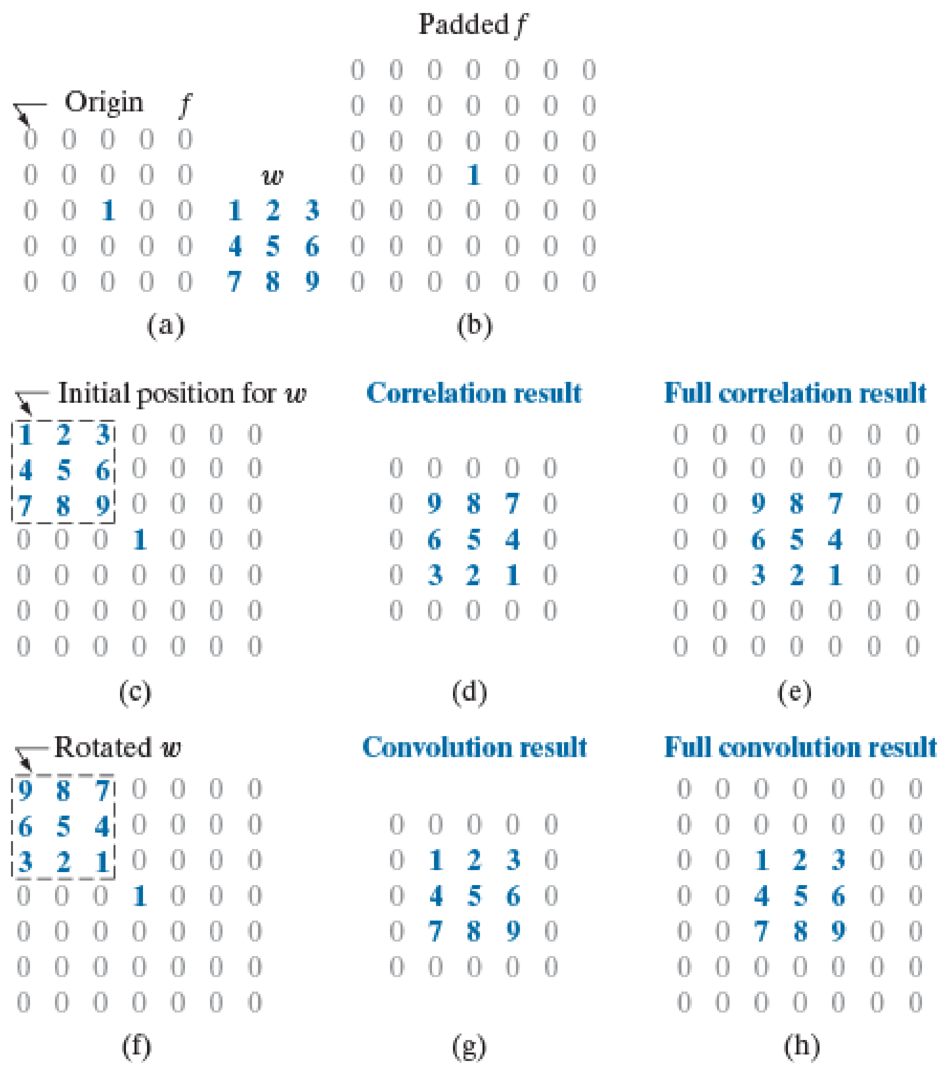
$$w(x,y) \star f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

2D convolution

$$w(x,y) \bigstar f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Correlation and convolution (2D)

FIGURE 3.36 Correlation (middle row) and convolution (last row) of a 2-D kernel with an image consisting of a discrete unit impulse. The 0's are shown in gray to simplify visual analysis. Note that correlation and convolution are functions of x and y. As these variable change, they *displace* one function with respect to the other. See the discussion of Eqs. (3-45) and (3-46) regarding full correlation and convolution.



Correlation and convolution

- Convolution is commutative and associative, correlation is not

TABLE 3.5
Some fundamental properties of convolution and correlation. A dash means that the property does not hold.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \,☆\, (g + h) = (f \,☆\, g) + (f \,☆\, h)$

Smoothing kernels

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Average
(box kernel)

$$\frac{1}{4.8976} \times \begin{bmatrix} 0.3679 & 0.6065 & 0.3679 \\ 0.6065 & 1.0000 & 0.6065 \\ 0.3679 & 0.6065 & 0.3679 \end{bmatrix}$$

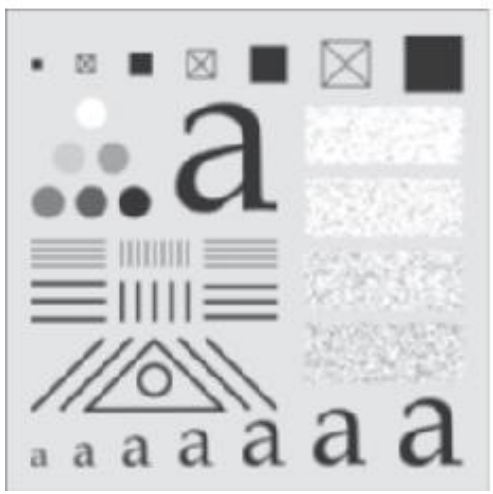
Weighted average
(Gaussian kernel)

Smoothing with box kernel

Input image

3x3

11x11

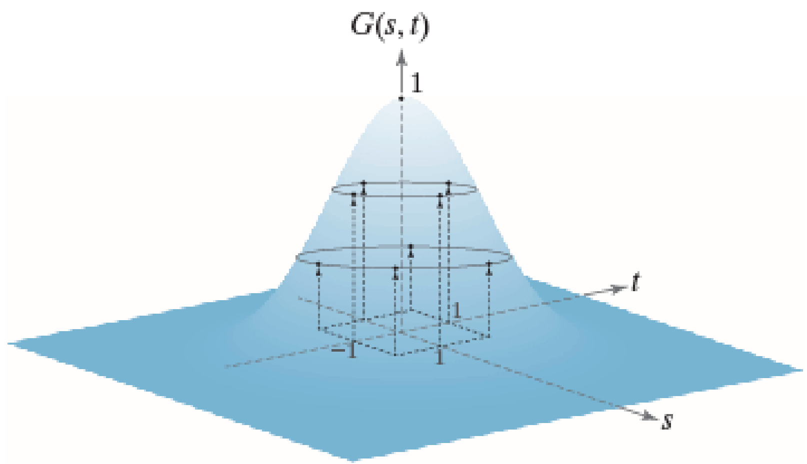


21x21

Smoothing with Gaussian kernel

a b

FIGURE 3.41 (a) Sampling a Gaussian function to obtain a discrete Gaussian kernel. The values shown are for $K = 1$ and $\sigma = 1$. (b) Resulting 3×3 kernel [this is the same as Fig. 3.37(b)].



$\frac{1}{4.8976} \times$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

Standard deviation σ	Percent of total volume under surface
1	39.35
2	86.47
3	98.89

Volume under surface greater than 3σ is negligible

Smoothing with Gaussian kernel

$$\text{minimum size is} \begin{cases} \lceil 6\sigma \rceil & \text{if } \lceil 6\sigma \rceil \text{ is odd} \\ \lceil 6\sigma \rceil + 1 & \text{otherwise} \end{cases}$$

$\sigma = 7$
43x43

$\sigma = 7$
85x85

Difference

Smoothing with Gaussian kernel

Input image

$\sigma = 3.5$

21x21

$\sigma = 7$

43x43

Border padding

v	v	v	v	v	v	v	v	v
v	v	v	v	v	v	v	v	v
v	v	1	2	3	4	5	v	v
v	v	6	7	8	9	10	v	v
v	v	11	12	13	14	15	v	v
v	v	16	17	18	19	20	v	v
v	v	v	v	v	v	v	v	v

Zero padding when v = 0

Constant padding

1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
6	6	6	7	8	9	10	10	10
11	11	11	12	13	14	15	15	15
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20

Replicate padding

13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8
3	2	1	2	3	4	5	4	3
8	7	6	7	8	9	10	9	8
13	12	11	12	13	14	15	14	13
18	17	16	17	18	19	20	19	18
13	12	11	12	13	14	15	14	13

Mirror padding

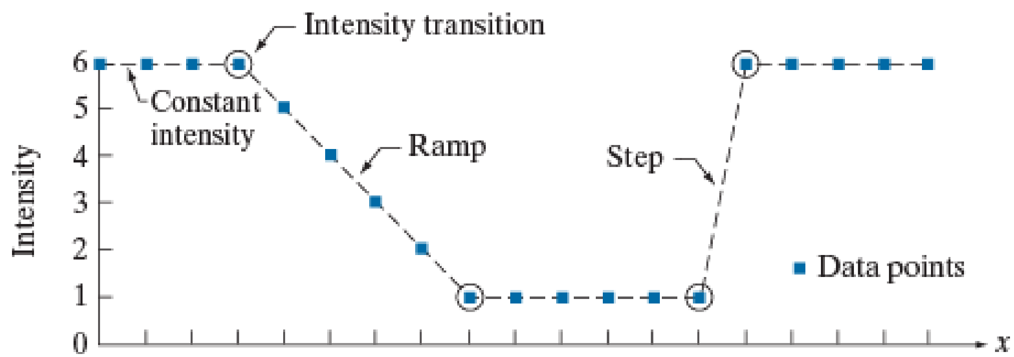
Border padding

Zero padding

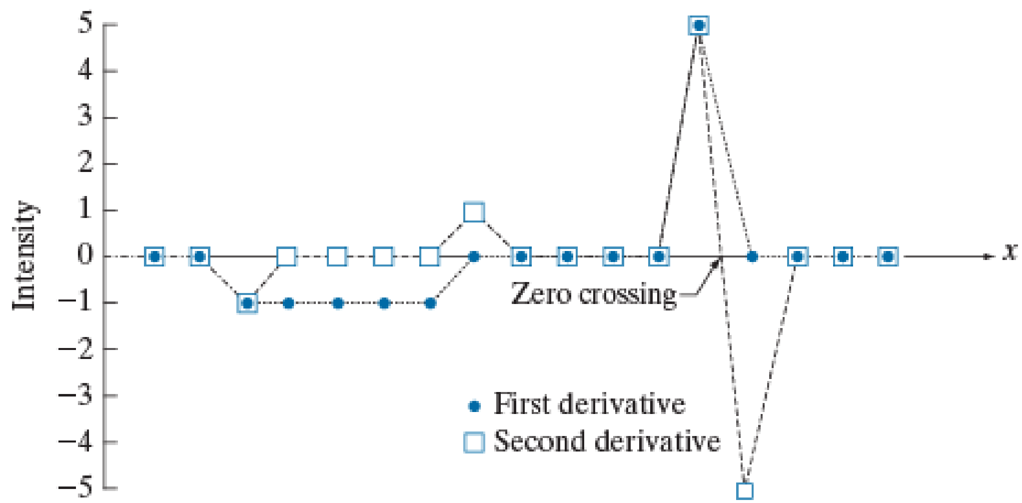
Mirror padding

Replicate padding

Derivatives



Values of scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



a
b
c

FIGURE 3.50
(a) A section of a horizontal scan line from an image, showing ramp and step edges, as well as constant segments.
(b) Values of the scan line and its derivatives.
(c) Plot of the derivatives, showing a zero crossing. In (a) and (c) points were joined by dashed lines as a visual aid.

Sharpening filters

a b
c d

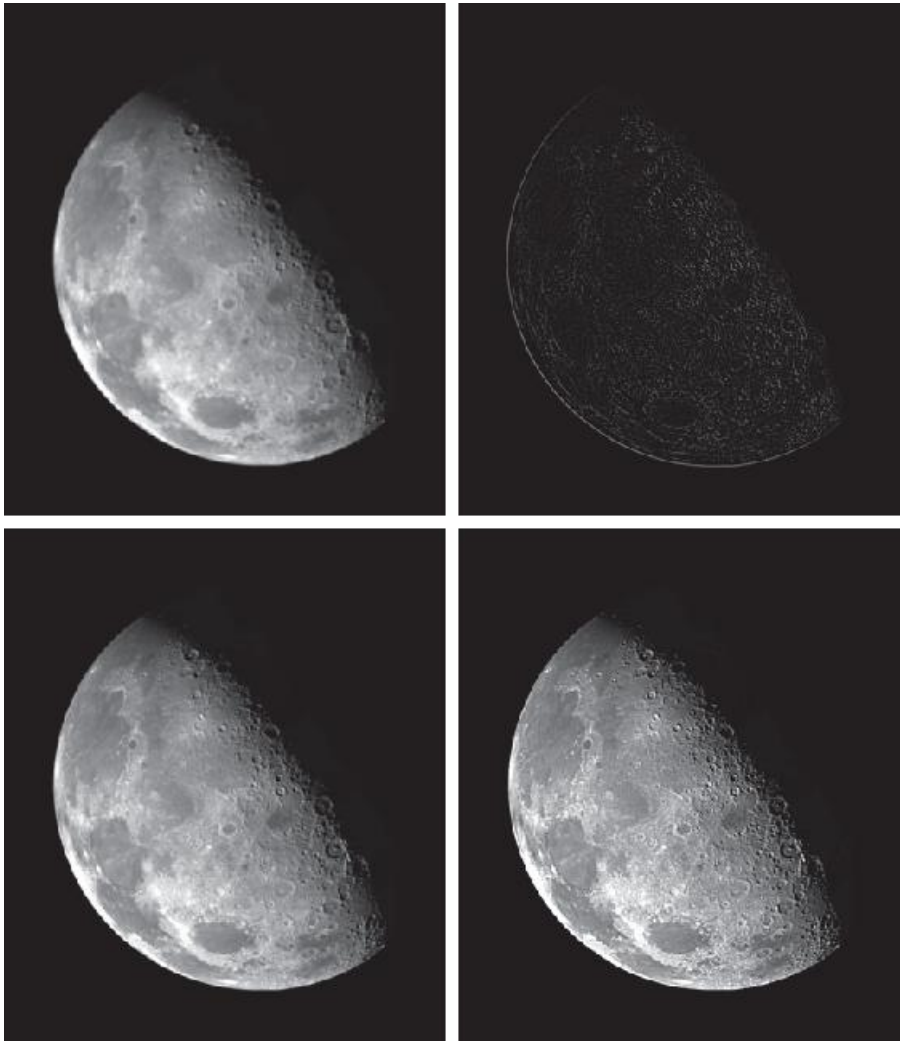


FIGURE 3.52
(a) Blurred image of the North Pole of the moon.
(b) Laplacian image obtained using the kernel in Fig. 3.51(a).
(c) Image sharpened using Eq. (3-63) with $c = -1$.
(d) Image sharpened using the same procedure, but with the kernel in Fig. 3.51(b).
(Original image courtesy of NASA.)

FIGURE 3.53
The Laplacian image from Fig. 3.52(b), scaled to the full [0, 255] range of intensity values. Black pixels correspond to the most negative value in the unscaled Laplacian image, grays are intermediate values, and white pixels corresponds to the highest positive value.

Gradient (first derivatives)

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

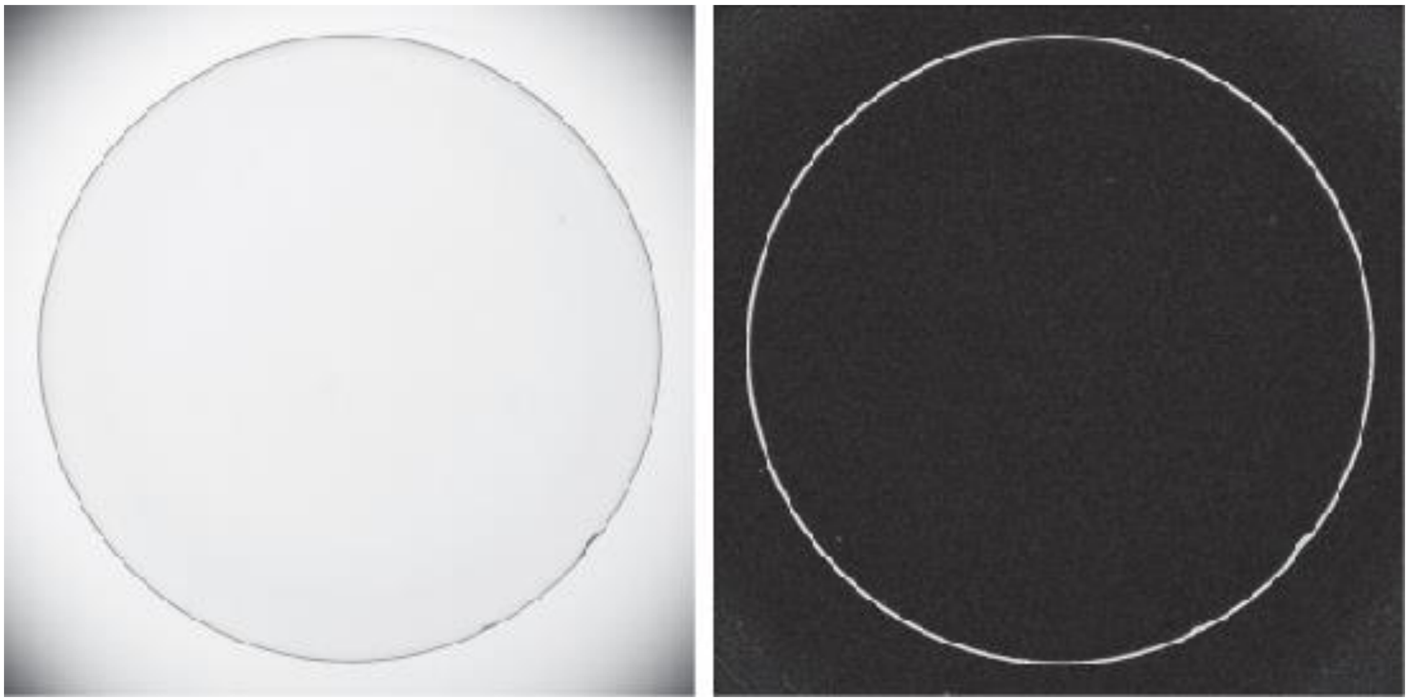
−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

a
b c
d e

FIGURE 3.56
(a) A 3×3 region of an image, where the zs are intensity values. (b)–(c) Roberts cross-gradient operators. (d)–(e) Sobel operators. All the kernel coefficients sum to zero, as expected of a derivative operator.

Magnitude of gradient vector

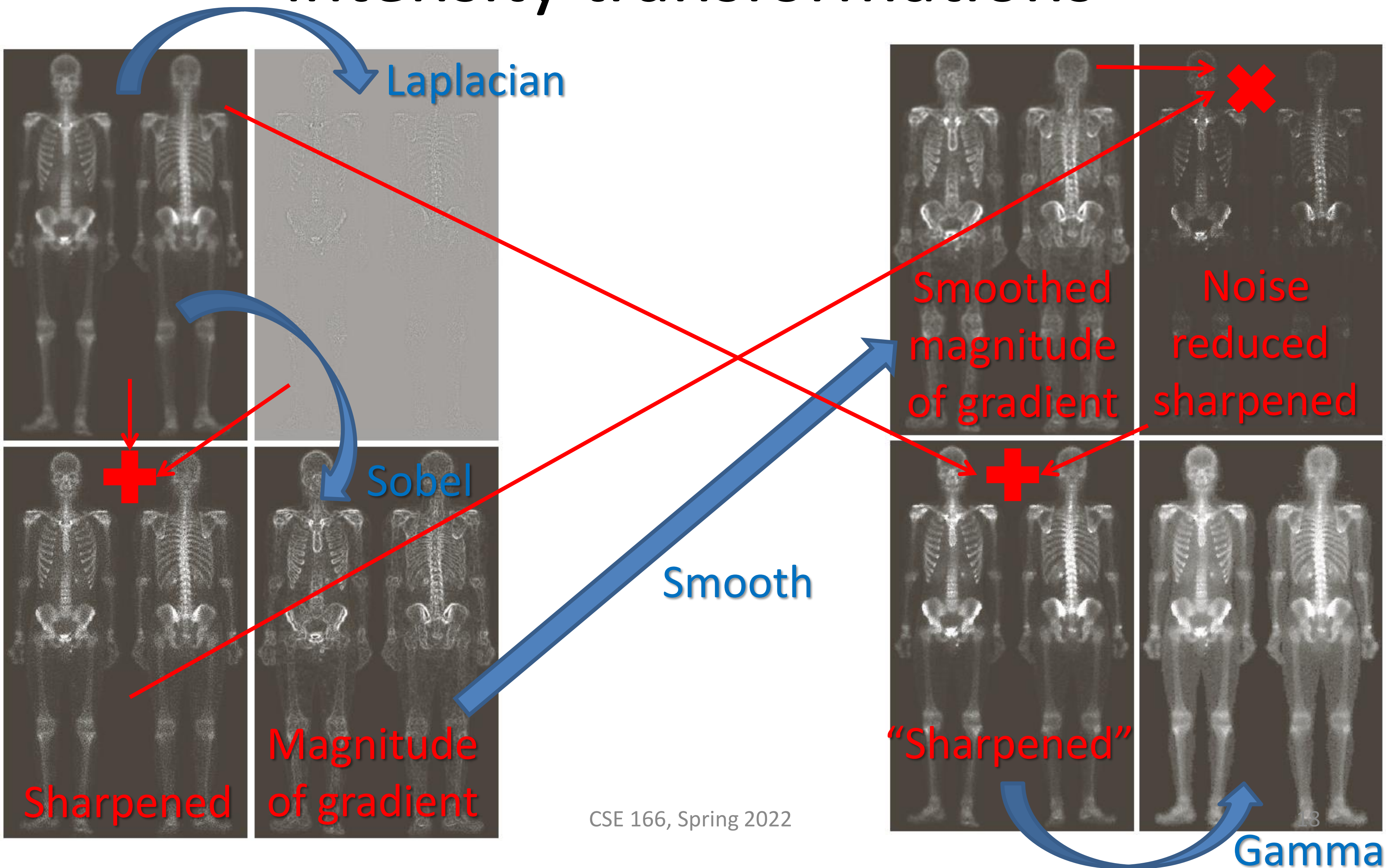


a b

FIGURE 3.57
(a) Image of a contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Perceptics Corporation.)

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Combining spatial filtering and intensity transformations



Next Lecture

- Continuous Fourier transform
- Reading
  - Chapter 4: Filtering in the Frequency Domain
    - Sections 4.1 and 4.2