

# Image Acquisition, Geometric Transformations, and Image Interpolation

Image Processing

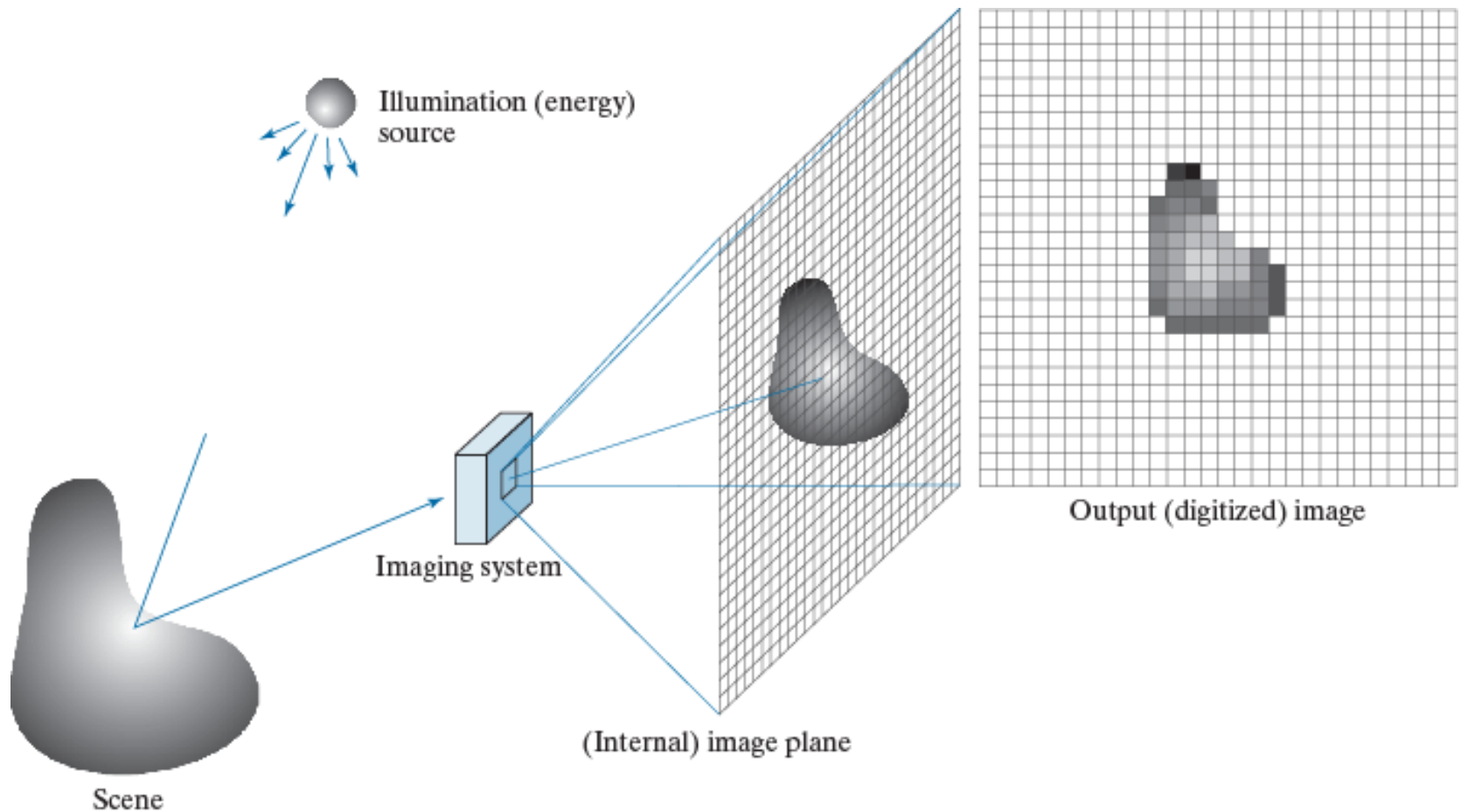
CSE 166

Lecture 2

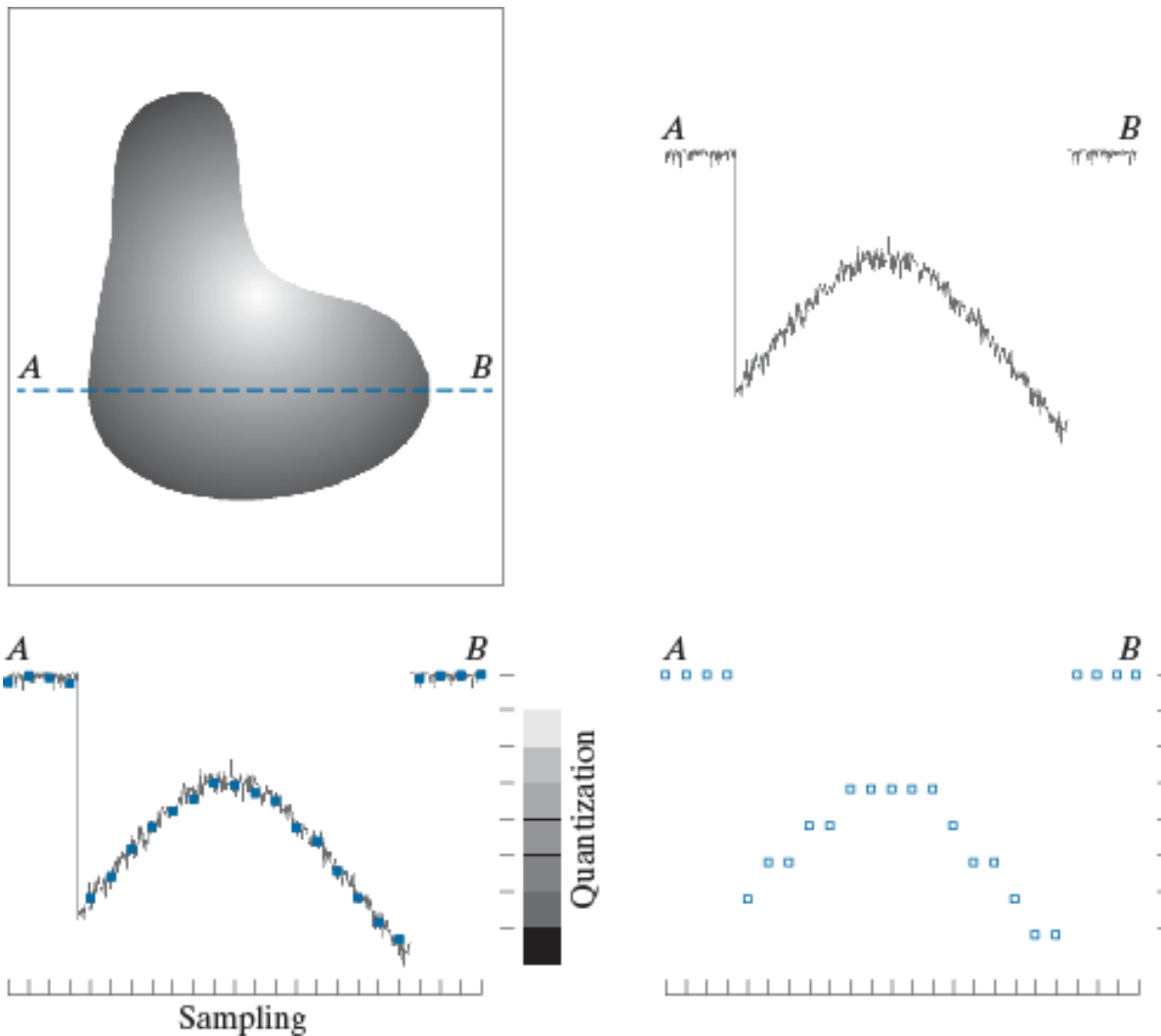
# Announcements

- Assignment 1 will be released today
  - Due Apr 6, 11:59 PM
- Reading
  - Chapter 2: Digital Image Fundamentals
    - Sections 2.3, 2.4, and 2.5

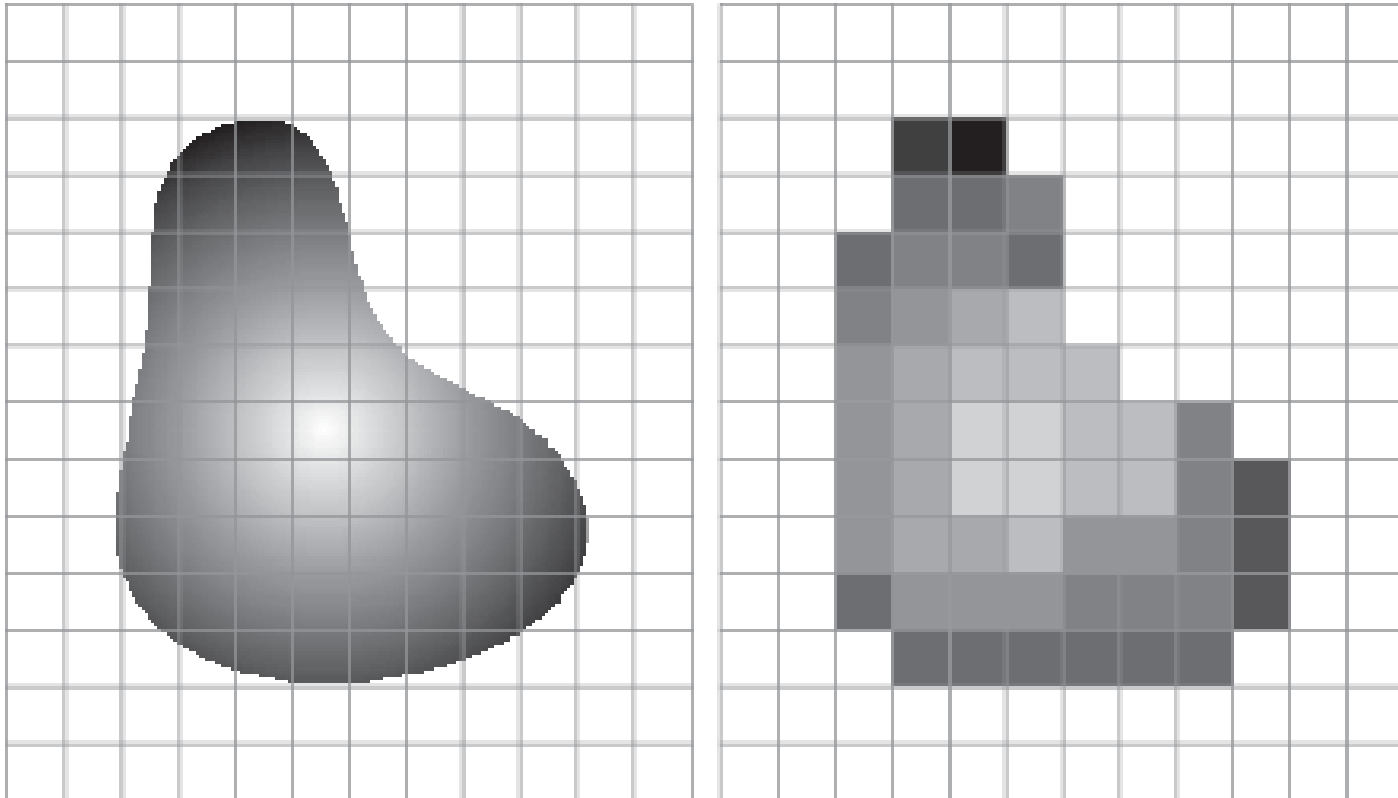
# Image acquisition



# Digitization, one row of image



# Digitization, whole image



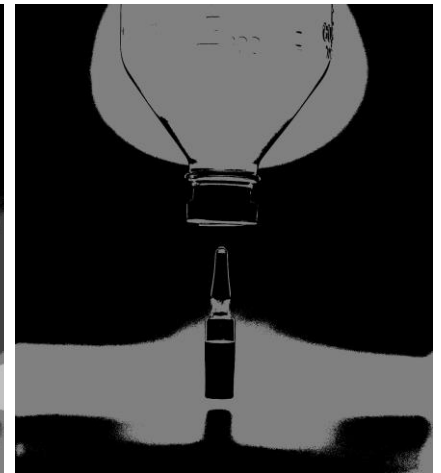
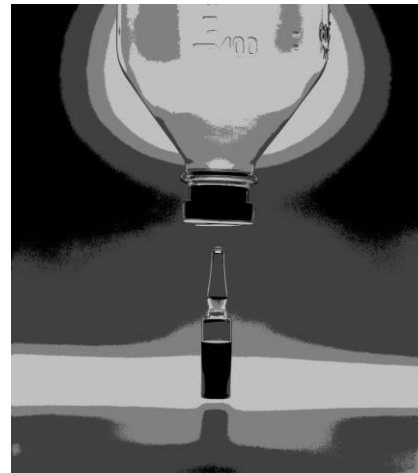
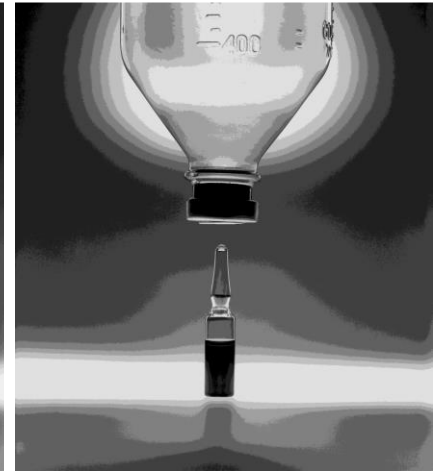
# Number of quantization levels

256

128

16

8



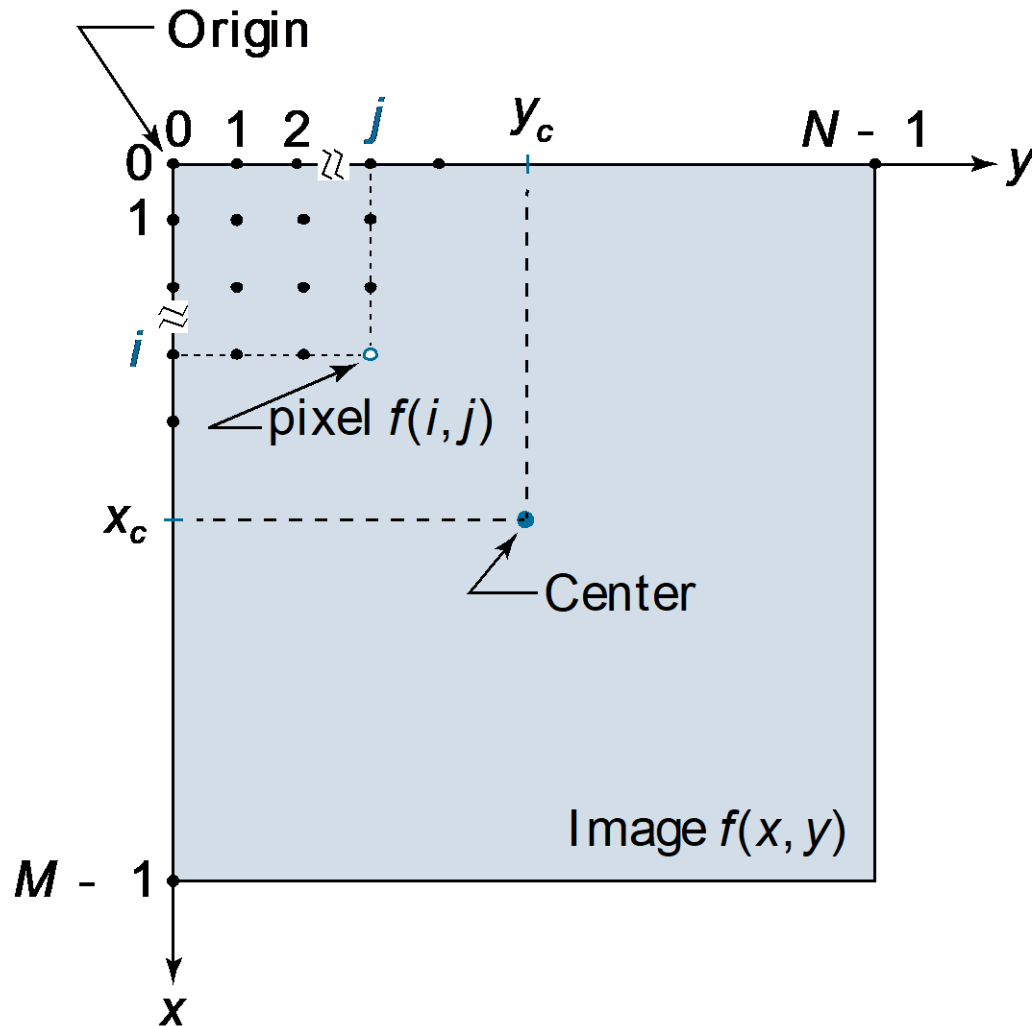
64

32

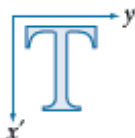
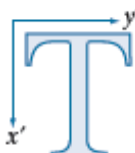
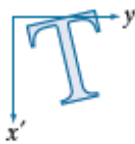
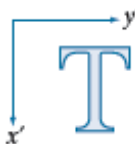
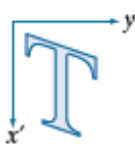
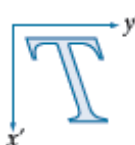
4

2

# Image coordinates



# Geometric transformations

Transformation Name	Affine Matrix, A	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x' &= x \\ y' &= y \end{aligned}$	
Scaling/Reflection (For reflection, set one scaling factor to -1 and the other to 0)	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x' &= c_x x \\ y' &= c_y y \end{aligned}$	
Rotation (about the origin)	$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x' &= x \cos \theta - y \sin \theta \\ y' &= x \sin \theta + y \cos \theta \end{aligned}$	
Translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}$	
Shear (vertical)	$\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x' &= x + s_v y \\ y' &= y \end{aligned}$	
Shear (horizontal)	$\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x' &= x \\ y' &= s_h x + y \end{aligned}$	



# Geometric transformations

Euclidean  
transformation

$$\begin{aligned}x' &= x \cos \theta - y \sin \theta + t_x \\y' &= x \sin \theta + y \cos \theta + t_y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & t_x \\ \sin \theta & \cos \theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Similarity  
transformation

$$\begin{aligned}x' &= sx \cos \theta - sy \sin \theta + t_x \\y' &= sx \sin \theta + sy \cos \theta + t_y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s \cos \theta & -s \sin \theta & t_x \\ s \sin \theta & s \cos \theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine  
transformation

$$\begin{aligned}x' &= a_{11}x + a_{12}y + a_{13} \\y' &= a_{21}x + a_{22}y + a_{23}\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Composition and inversion of transformations

First transformation

$$H_1 : \mathbf{x} \mapsto \mathbf{x}'$$
$$\mathbf{x}' = H_1 \mathbf{x}$$

Second transformation

$$H_2 : \mathbf{x}' \mapsto \mathbf{x}''$$
$$\mathbf{x}'' = H_2 \mathbf{x}'$$

Third transformation

$$H_3 : \mathbf{x}'' \mapsto \mathbf{x}'''$$
$$\mathbf{x}''' = H_3 \mathbf{x}''$$

Composition of transformations

$$\mathbf{x}''' = H_3 \mathbf{x}''$$

$$\mathbf{x}''' = H_3 H_2 \mathbf{x}'$$

$$\mathbf{x}''' = H_3 H_2 H_1 \mathbf{x}$$

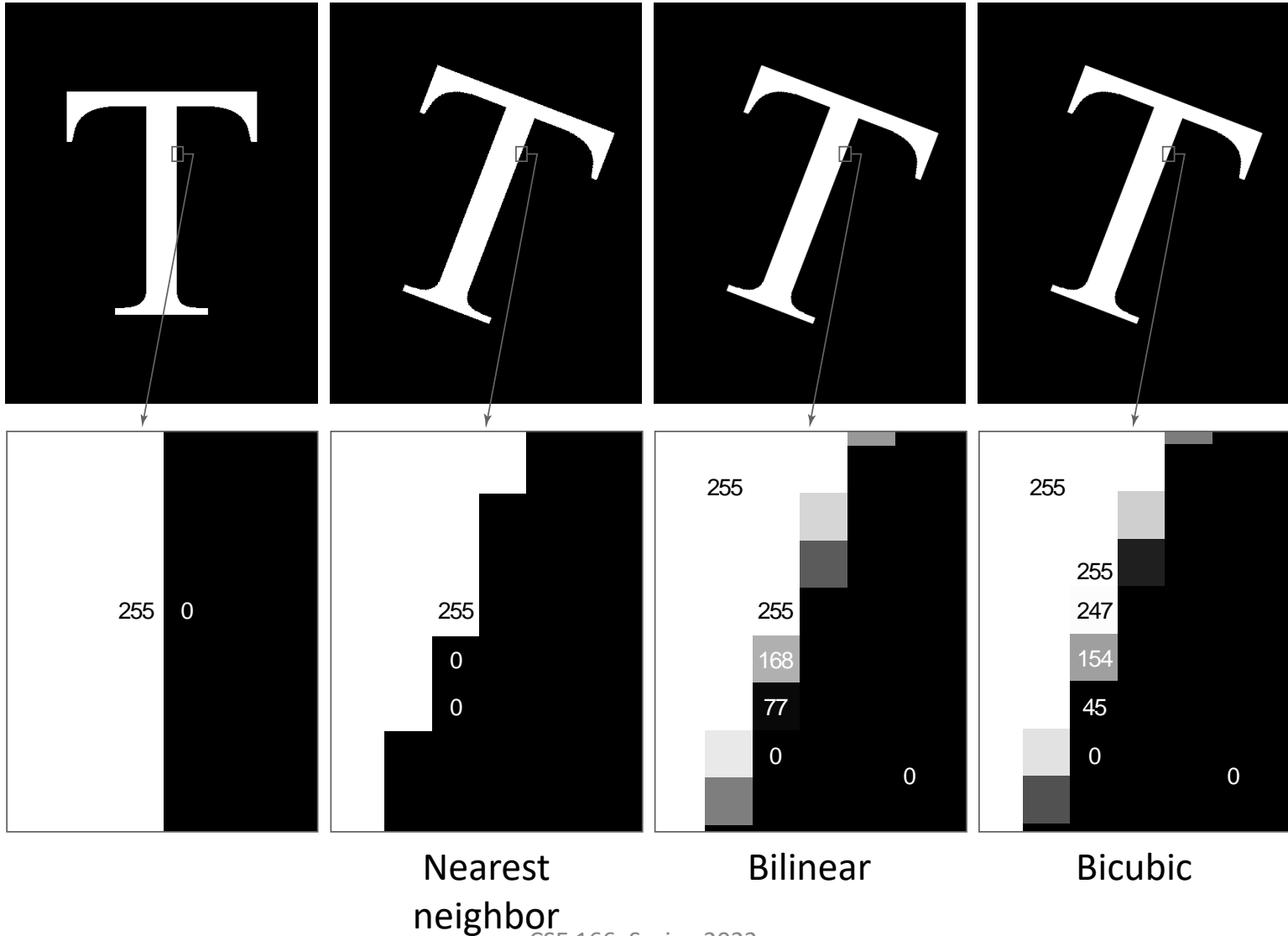
$$\mathbf{x}''' = H \mathbf{x} \text{ where } H = H_3 H_2 H_1$$

Inverse of transformation

$$\mathbf{x}''' = H \mathbf{x}$$

$$\mathbf{x} = H^{-1} \mathbf{x}'''$$

# Transformation with interpolation



# Transformation with interpolation

- Nearest neighbor interpolation

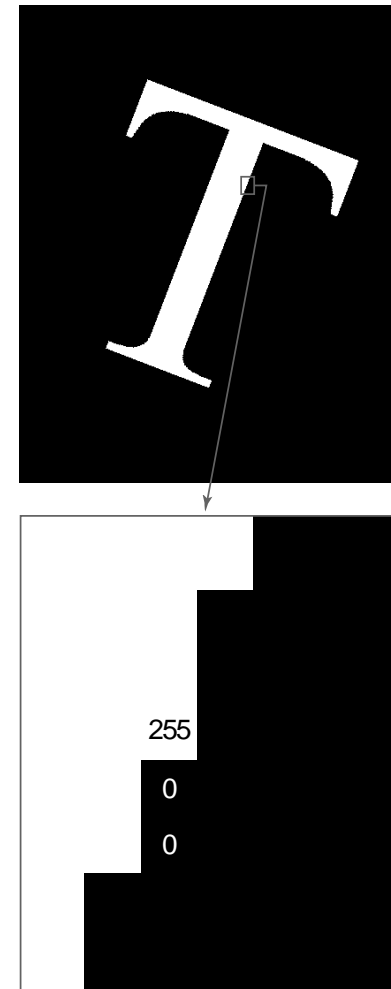
$$H : I(x, y) \mapsto J(x', y')$$

for each  $(x', y')$  in  $J$

$$\mathbf{x} = H^{-1}\mathbf{x}'$$

$$J(x', y') = I(\text{round}(x), \text{round}(y))$$

end



# Transformation with interpolation

- Linear interpolation

$$H : I(x, y) \mapsto J(x', y')$$

for each  $(x', y')$  in  $J$

$$\mathbf{x} = H^{-1}\mathbf{x}'$$

$$x_0 = \lfloor x \rfloor$$

$$x_1 = x_0 + 1$$

$$y_0 = \lfloor y \rfloor$$

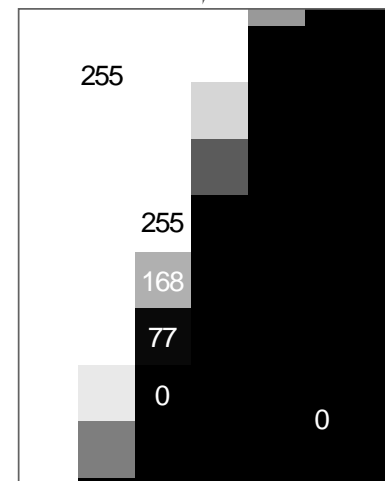
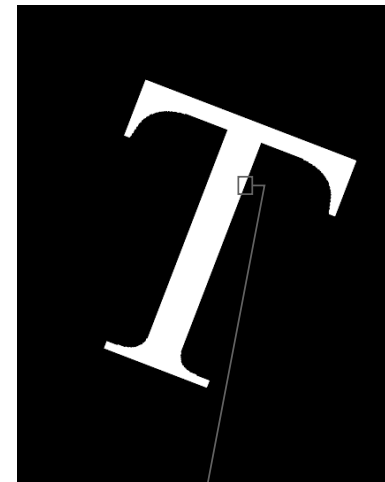
$$y_1 = y_0 + 1$$

$$a = (x_1 - x)I(x_0, y_0) + (x - x_0)I(x_1, y_0)$$

$$b = (x_1 - x)I(x_0, y_1) + (x - x_0)I(x_1, y_1)$$

$$J(x', y') = (y_1 - y)a + (y - y_0)b$$

end



# Next Lecture

- Intensity transformations
- Reading
  - Chapter 3: Intensity Transformations and Spatial Filtering
    - Sections 3.1, 3.2, and 3.3