Feature Extraction
(Part 1)

Image Processing
CSE 166
Lecture 17
Announcements

• Assignment 7 is due Jun 1, 11:59 PM

• Reading
  – Chapter 12: Feature extraction
    • Sections 12.1, 12.4, and 12.6
Feature extraction

• Feature extraction is comprised of
  – Feature detection
  – Feature description

  • A feature descriptor is
    – Invariant with respect to a set of transformations if its value remains unchanged after the application of any transformation from the family
    – Covariant with respect to a set of transformations if applying any transformation from the set produces the same result in the descriptor
Features

- Features
  - Local (a member of a set)
  - Global (the entire set)

- Categories
  - Boundaries
  - Regions
  - Whole images
Region feature descriptors

- Basic descriptors
  - Compactness
  - Circularity
  - Effective diameter
    - Diameter of circle with same area
  - Eccentricity (of an approximating ellipse)

FIGURE 12.21
(a) An ellipse in standard form.
(b) An ellipse approximating a region in arbitrary orientation.

\[ c^2 = a^2 - b^2 \]
Feature space

- \( n \)-dimensional feature vector
  - “Packaged” form of features
  - Example
    - Compactness, circularity, and eccentricity form a 3-vector
Texture as a region descriptor

- Statistical texture measures
  - Statistical moments
    - The $n$-th moment of $z$ about the mean is
      \[
      \mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)
      \]
      where the mean
      \[
      m = \sum_{i=0}^{L-1} z_i p(z_i)
      \]
    - $p(z_i)$ is the normalized histogram and $L$ is the number of gray levels
    - The second, third, and fourth moments are the variance, skewness, and kurtosis, respectively
Texture as a region descriptor

• Statistical texture measures
  – Relative intensity smoothness
    \[ R(z) = 1 - \frac{1}{1 + \sigma^2(z)} \]
  – Uniformity
    \[ U(z) = \sum_{k=0}^{L-1} p^2(z_i) \]
  – Average entropy
    \[ e(z) = -\sum_{k=0}^{L-1} p(z_i) \log_2(p(z_i)) \]
Statistical texture measures

**Figure 12.29**
The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

**Table 12.2**
Statistical texture measures for the subimages in Fig. 12.29.

<table>
<thead>
<tr>
<th>Texture</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>$R$ (normalized)</th>
<th>3rd moment</th>
<th>Uniformity</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>82.64</td>
<td>11.79</td>
<td>0.002</td>
<td>−0.105</td>
<td>0.026</td>
<td>5.434</td>
</tr>
<tr>
<td>Coarse</td>
<td>143.56</td>
<td>74.63</td>
<td>0.079</td>
<td>−0.151</td>
<td>0.005</td>
<td>7.783</td>
</tr>
<tr>
<td>Regular</td>
<td>99.72</td>
<td>33.73</td>
<td>0.017</td>
<td>0.750</td>
<td>0.013</td>
<td>6.674</td>
</tr>
</tbody>
</table>
Texture as a region descriptor

• Graylevel co-occurrence matrix
  – Let $Q$ be an operator that defines the position of two pixels relative to each other in image $f$ with number of gray levels $L$
  – Let $G$ be a matrix whose element $g_{ij}$ is the number of times that pixel pairs with intensities $z_i$ and $z_j$ occur in image $f$ in the position specified by $Q$
  • Probability a pair of points satisfy $Q$ is

$$p_{ij} = \frac{g_{ij}}{n}$$

where $n$ is the number of pixels pairs that satisfy $Q$ (i.e., $n$ is the sum of the elements in $Q$)
Graylevel co-occurrence matrix

- Number of gray levels $L = 8$
- Position operator $Q$ is “one pixel immediately to the right”
Graylevel co-occurrence matrix

Images

Graylevel co-occurrence matrices
Texture as a region descriptor

- Graylevel co-occurrence matrix-based feature descriptors

**TABLE 12.3**
Descriptors used for characterizing co-occurrence matrices of size \( K \times K \). The term \( p_{ij} \) is the \( ij \)-th term of \( G \) divided by the sum of the elements of \( G \).

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>Explanation</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum probability</td>
<td>Measures the strongest response of ( G ). The range of values is ([0, 1]).</td>
<td>( \max_{i,j}(p_{ij}) )</td>
</tr>
<tr>
<td>Correlation</td>
<td>A measure of how correlated a pixel is to its neighbor over the entire image. The range of values is ([-1, 1]) corresponding to perfect positive and perfect negative correlations. This measure is not defined if either standard deviation is zero.</td>
<td>( \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{(i - m_r)(j - m_c)p_{ij}}{\sigma_r \sigma_c} ) where ( \sigma_r \neq 0 ) and ( \sigma_c \neq 0 )</td>
</tr>
<tr>
<td>Contrast</td>
<td>A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is ( [0, 1] ) when ( G ) is constant.</td>
<td>( \sum_{i=1}^{K} \sum_{j=1}^{K} (i - j)^2 p_{ij} )</td>
</tr>
<tr>
<td>Uniformity (also called Energy)</td>
<td>A measure of uniformity in the range ([0, 1]). Uniformity is 1 for a constant image.</td>
<td>( \sum_{i=1}^{K} \sum_{j=1}^{K} p_{ij}^2 )</td>
</tr>
<tr>
<td>Homogeneity</td>
<td>Measures the spatial closeness to the diagonal of the distribution of elements in ( G ). The range of values is ([0, 1]), with the maximum being achieved when ( G ) is a diagonal matrix.</td>
<td>( \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{p_{ij}}{1 +</td>
</tr>
<tr>
<td>Entropy</td>
<td>Measures the randomness of the elements of ( G ). The entropy is 0 when all ( p_{ij} )'s are 0, and is maximum when the ( p_{ij} )'s are uniformly distributed. The maximum value is thus ( 2 \log_2 K ).</td>
<td>( -\sum_{i=1}^{K} \sum_{j=1}^{K} p_{ij} \log_2 p_{ij} )</td>
</tr>
</tbody>
</table>

\( m_r = \sum_{i=1}^{K} \sum_{j=1}^{K} p_{ij} \)

\( m_c = \sum_{j=1}^{K} \sum_{i=1}^{K} p_{ij} \)

\( \sigma_r^2 = \sum_{i=1}^{K} (i - m_r)^2 \sum_{j=1}^{K} p_{ij} \)

\( \sigma_c^2 = \sum_{j=1}^{K} (i - m_c)^2 \sum_{i=1}^{K} p_{ij} \)
Graylevel co-occurrence matrix-based feature descriptors

Table 12.4
Descriptors evaluated using the co-occurrence matrices displayed as images in Fig. 12.32.

<table>
<thead>
<tr>
<th>Normalized Co-occurrence Matrix</th>
<th>Maximum Probability</th>
<th>Correlation</th>
<th>Contrast</th>
<th>Uniformity</th>
<th>Homogeneity</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1/n_1$</td>
<td>0.00006</td>
<td>-0.0005</td>
<td>10838</td>
<td>0.00002</td>
<td>0.0366</td>
<td>15.75</td>
</tr>
<tr>
<td>$G_2/n_2$</td>
<td>0.01500</td>
<td>0.9650</td>
<td>00570</td>
<td>0.01230</td>
<td>0.0824</td>
<td>06.43</td>
</tr>
<tr>
<td>$G_3/n_3$</td>
<td>0.06860</td>
<td>0.8798</td>
<td>01356</td>
<td>0.00480</td>
<td>0.2048</td>
<td>13.58</td>
</tr>
</tbody>
</table>
Whole-image features

• Feature descriptors applicable to entire images that are members of a large family of images

• Detection
  – Corner-like features
  – Regions
    • Could be described using the descriptors covered earlier
Detection of corner-like features

• Corner
  – A rapid change of direction in a curve
  – A highly effective feature
    • Distinctive
    • Reasonably invariant to viewpoint
Detection of corner-like features

• Examine a small window over an image
Detection of corner-like features

• For each window location, compute the spatial gradient matrix

\[
M = \begin{bmatrix}
\sum_s \sum_t f_x(s, t)^2 & \sum_s \sum_t f_x(s, t)f_y(s, t) \\
\sum_s \sum_t f_x(s, t)f_y(s, t) & \sum_s \sum_t f_y(s, t)^2
\end{bmatrix}
\]

where \( f_x \) is the gradient in the \( x \)-direction and \( f_y \) is the gradient in the \( y \)-direction

• Then, compute eigenvalues of spatial gradient matrix
Eigenvalues of spatial gradient matrix

**Figure 12.46** (a)—(c) Noisy images and image patches (small squares) encompassing image regions similar in content to those in Fig. 12.45. (d)—(f) Plots of value pairs \((f_x, f_y)\) showing the characteristics of the eigenvalues of \(M\) that are useful for detecting the presence of a corner in an image patch.
Detection of corner-like features

- Harris-Stephens corner detector
  - Run a small window over an image and compute spatial gradient matrix $\mathbf{M}$
  - Approximate the minor eigenvalue of $\mathbf{M}$ to compute measurement image $R$
    - Include sensitivity factor $k$
      \[
      R(x, y) = \det(\mathbf{M}) - k \text{Tr}^2(\mathbf{M})
      \]
    - Threshold measurement image $R$ using global threshold $T$
      - Corner at coordinates corresponding to $R > T$
Harris-Stephens corner detector

$\text{Image + noise}$

$k = 0.04, T = 0.01$

$k = 0.1, T = 0.01$

$k = 0.1, T = 0.1$

$k = 0.04, T = 0.1$

$k = 0.04, T = 0.3$
Harris-Stephens corner detector

Image + more noise

\( k = 0.249 \)
\( T = 0.01 \)

\( k = 0.04 \)
\( T = 0.15 \)
Harris-Stephens corner detector

Image

$\begin{array}{ll}
\text{a} & k = 0.04, \ T = 0.01 \\
\text{b} & k = 0.249, \ T = 0.01 \\
\text{c} & k = 0.17, \ T = 0.05 \\
\text{d} & k = 0.04, \ T = 0.05 \\
\end{array}$
Harris-Stephens corner detector

![Image](image.png)

Image

Image rotated 5 degrees

$\begin{align*}
  k &= 0.04 \\
  T &= 0.07
\end{align*}$

Partial rotation invariance
Detection of corner-like features

• Shi-Tomasi corner detector
  – Run a small window over an image and compute spatial gradient matrix $\mathbf{M}$
  – Compute the minor eigenvalue of $\mathbf{M}$ to compute measurement image $R$
  – Apply nonmaximal suppression to the measurement image $R$
    • Prevents corners from being too close to each other
  – Threshold resulting image $R$ using global threshold $T$
    • Corner at coordinates corresponding to $R > T$
Next Lecture

- Feature extraction
- Reading
  - Chapter 12: Feature extraction
    - Section 12.7