

CSE 140: Components and Design Techniques for  
Digital Systems

Midterm 1

Sat 4/23 : 10AM

Lecture 6: Universal Gates Mon 4/25 : 10AM

Th 4/27 : Review

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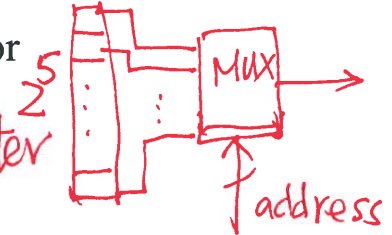
## Universal Gates

- Universal Set of Gates
  - Motivation
  - Definition
  - Examples
- Various Types of Gates
  - XOR, XNOR
  - NAND, NOR
  - Block Diagram Transfers

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# Universal Set of Gates: Motivation

- AND, OR, NOT: Logic gates related to reasoning from Aristotle (384-322BCE).
- NAND, NOR: Inverted AND, Inverted OR gates. For VLSI technologies, all gate outputs are inverted functions (AND, OR operations with a bubble at output).
- XOR: Exclusive OR gates can be used for testing, e.g. parity check.
- Multiplexer + input table: Table based logic for programmability. FPGA technology.
- Neuron and Synapse: Neural network. *Memristor*
- Quantum computing: Reversible gates.



In the future, we may have new sets of gates due to new technologies. Given a set of gates, can the gates in the set cover all possible switching functions?

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## Universal Set

Universal set is a powerful concept to identify the coverage of a set of gates afforded by a given technology.

Criterion: If the set of gates can implement AND, OR, and NOT gates, the set is universal.

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# Universal Set Definition

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

yes {AND, OR, NOT} Universal Set of Gates

yes {AND, NOT}  $a \Rightarrow b \Rightarrow a + b \Leftrightarrow a \Rightarrow b \Rightarrow (a' \cdot b')'$

yes {OR, NOT}  $a + b = (a' b')'$

$a \Rightarrow b \Rightarrow ab \Leftrightarrow a \Rightarrow b \Rightarrow (a' + b')'$

$ab = (a' + b')'$

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## Universal Set: Examples

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

y {AND, OR, NOT}

y {AND, NOT} OR can be implemented with AND & NOT gates:  $a + b = (a' b')'$

y {OR, NOT} AND can be implemented with OR & NOT gates:  $ab = (a' + b')'$

N {XOR} is not universal

y {XOR, AND} is universal

Let  $b=1$

$a \oplus b = ab' + a'b \mid b=1$

$= a \cdot 0 + a' \cdot 1 = a'$

$a \Rightarrow b \Rightarrow a'b + ab'$

$a \Rightarrow b \Rightarrow a'$

$a \Rightarrow b \Rightarrow a'$

| a | b | ab | a'b | a'b' | ab' |
|---|---|----|-----|------|-----|
| 0 | 0 | 0  | 0   | 1    | 0   |
| 0 | 1 | 0  | 1   | 0    | 1   |
| 1 | 0 | 0  | 0   | 0    | 1   |
| 1 | 1 | 1  | 0   | 0    | 0   |

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# Universal Set: Example

Is the set {AND, OR} (but no NOT gate) universal?

A. Yes

B. No

Note that the set was used in a once popular design style as **domino logic** for high performance computing.

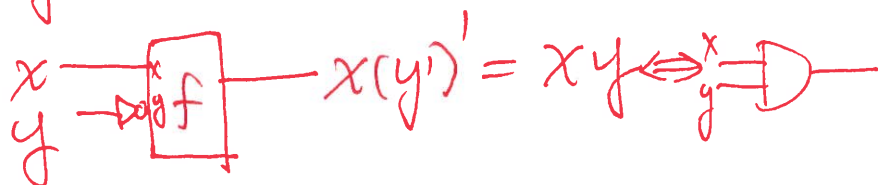
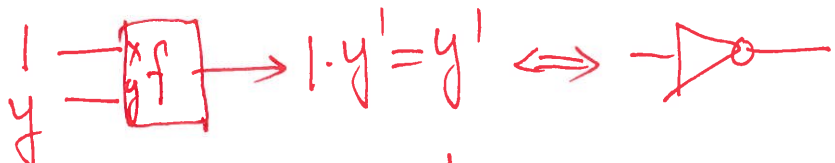
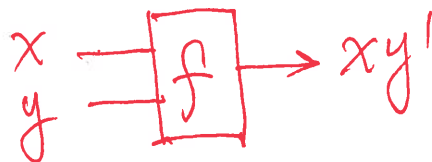
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# Universal Set: Example

Is the set  $\{f(x, y) = xy'\}$  universal?

A. Yes

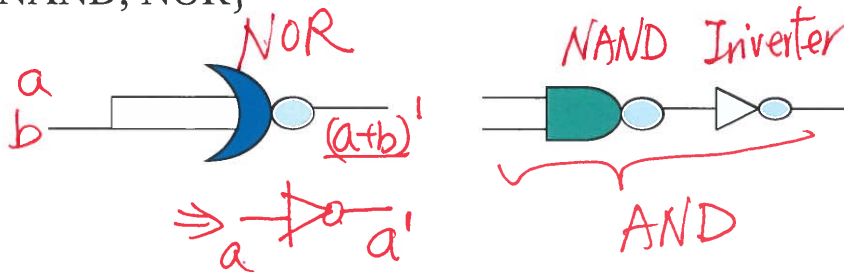
B. No



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# Universal Set: Examples

Yes {NAND, NOR}



No {XOR}

Yes {XOR, AND}  $1 \Rightarrow$

$x \oplus 1 = x1' + x'1 = x'$  if constant "1" is available.

Yes {NAND}  $a, b \Rightarrow (ab)'$   $a \rightarrow a' \Leftrightarrow$

Yes {NOR}

$a, b \Rightarrow ab \Leftrightarrow$

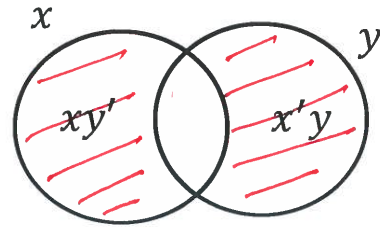
## Various Types of Gates: Properties and Usage

1. XOR (XNOR)  $x \oplus y = xy' + x'y$
2. NAND, NOR
3. Block Diagram Transfers

## Other Types of Gates: XOR

$$x \oplus y = xy' + x'y$$

It is a parity function (examples)  
useful for testing because the  
flipping of a single input changes the  
output.



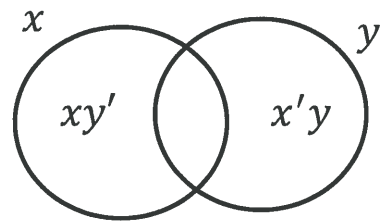
| id | x | y | $x \oplus y$ |
|----|---|---|--------------|
| 0  | 0 | 0 | 0            |
| 1  | 0 | 1 | 1            |
| 2  | 1 | 0 | 1            |
| 3  | 1 | 1 | 0            |

|     | x=0 | x=1 |
|-----|-----|-----|
| y=0 | 0   | 1   |
| y=1 | 1   | 0   |

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## Other Types of Gates: XOR

1) XOR  $x \oplus y = xy' + x'y$



(a) Commutative  $x \oplus y = y \oplus x$

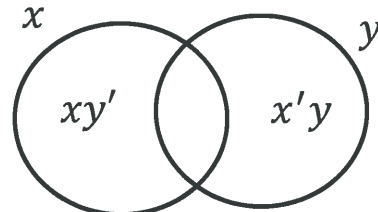
(b) Associative  $(x \oplus y) \oplus z = x \oplus (y \oplus z)?$

$$\begin{aligned}
 & (xy' + x'y)' \\
 &= (x' + y)(x + y') \\
 &= \cancel{x'x} + x'y' + xy + \cancel{yy'} \\
 &= xy + x'y' \\
 & \text{LHS} = (xy' + x'y) \oplus z \\
 &= (xy' + x'y)z' + (xy' + x'y)'z \\
 &= xy'z' + x'yz' + (xy + x'y')z \\
 &= xy'z' + x'yz' + xyz + x'y'z
 \end{aligned}$$

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## Other Types of Gates: XOR

1) XOR  $x \oplus y = xy' + x'y$



a) Commutative  $x \oplus y = y \oplus x$

b) Associative  $(x \oplus y) \oplus z = x \oplus (y \oplus z)$

c)  $1 \oplus x = x'$ ,  $0 \oplus x = x$

d)  $x \oplus x = 0$ ,  $x \oplus x' = 1$

$$f(x, y) = x'y + xy'$$

$$f(x, 1) = x' \cdot 1 + x \cdot 1' = x'$$

$$f(x, 0) = x' \cdot 0 + x \cdot 0' = x$$

$$f(x, x) = x'x + xx' = 0 + 0 = 0$$

$$f(x, x') = x'x' + xx'' = x' + x = 1$$

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## Other Types of Gates: Properties and Usage

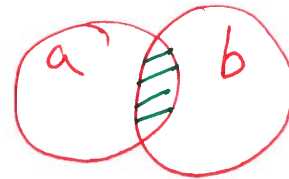
e) If  $ab = 0$ , then  $a + b = a \oplus b$

Proof: If  $ab = 0$ , then

$$a = a(b + b') = \cancel{ab} + ab' = ab'$$

$$b = b(a + a') = \cancel{ab} + a'b = a'b$$

Therefore,  $a + b = ab' + a'b = a \oplus b$



Note that in full adder, we have

$$s = a \oplus b \oplus c_{in} = (a \oplus b) \oplus c_{in}$$

$$c_{out} = ab + bc + ac = \underline{ab} + c(a + b) = ab + c(a \oplus b)$$

From property e), we can write

$$c_{out} = ab + c(a \oplus b)$$

## Other Types of Gates: XOR

f)  $f(x, y) = x \oplus xy' \oplus x'y \oplus (x + y) \oplus x = ?$

$$1 \oplus x = 1'x + 1x' = x'$$

(Priority of operations: AND,  $\oplus$ , OR)

Hint: We apply Shannon's Expansion.

$$\begin{aligned} \textcircled{1} x=1, f(x, y) \Big|_{x=1} &= 1 \oplus 1y' \oplus 1'y \oplus (1+y) \oplus 1 \\ &= 1 \oplus y' \oplus 0 \oplus 1 \oplus 1 \\ &= 1 \oplus y' = 1 \cdot (y')' + 1' \cdot y' = y + 0 = y \end{aligned}$$

$$\begin{aligned} \textcircled{2} x=0, f(x, y) \Big|_{x=0} &= 0 \oplus 0y' \oplus 0'y \oplus (0+y) \oplus 0 \\ &= 0 \oplus 0 \oplus y \oplus y \oplus 0 \stackrel{15}{=} 0 \end{aligned}$$

$$f(x, y) = x f(1, y) + x' f(0, y) = x \cdot y + x' \cdot 0 = xy$$

## Various Types of Gates: XOR

### Shannon's Expansion (for switching functions)

**Formula:**  $f(x, y) = x f(1, y) + x' f(0, y)$

Proof: We prove Shannon's expansion by enumeration:

If  $x = 1$ , we have  $f(x, y) = f(1, y)$ .

For Shannon's expansion, we have

$$x f(1, y) + x' f(0, y) = 1 f(1, y) + 1' f(0, y) = f(1, y).$$

If  $x = 0$ , we have  $f(x, y) = f(0, y)$ :

For Shannon's expansion, we have

$$x f(1, y) + x' f(0, y) = 0 f(x, y) + 0' f(0, y) = f(0, y).$$

## Other types of gates: XOR

Simplify the function (Priority of operations: AND,  $\oplus$ , OR)

$$f(x, y) = x \oplus xy' \oplus x'y \oplus (x + y) \oplus x$$

$$\text{Case } x = 1: f(1, y) = 1 \oplus y' \oplus 0 \oplus 1 \oplus 1 = y$$

$$\text{Case } x = 0: f(0, y) = 0 \oplus 0 \oplus y \oplus y \oplus 0 = 0$$

Thus, using Shannon's expansion, we have

$$f(x, y) = xf(1, y) + x'f(0, y) = xy + x'0 = xy$$

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## XOR gates: Examples

Is the equation

$$a+(b\oplus c) = (a+b)\oplus(a+c) \text{ true?}$$

A. Yes

B. No

$$(a, b, c) = (1, 0, 0)$$

$$\text{LHS} = 1 + (0 \oplus 0) = 1 + 0 = 1.$$

$$\text{RHS} = (1 + 0) \oplus (1 + 0) = 1 \oplus 1 = 0.$$

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# XOR gates: Examples

XNOR: Exclusive NOR gate

$$f(x, y) = (x \oplus y)'$$

$$= (xy' + x'y)'$$

$$= xy + x'y'$$

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## Other Types of Gates: NAND, NOR

2) NAND, NOR gates

NAND (NOR) gates are not associative

$$\text{Let } a \mid b = (ab)'$$

$$(a \mid b) \mid c \neq a \mid (b \mid c)$$

$$\text{LHS } [(ab)'c]' = ab + c'$$

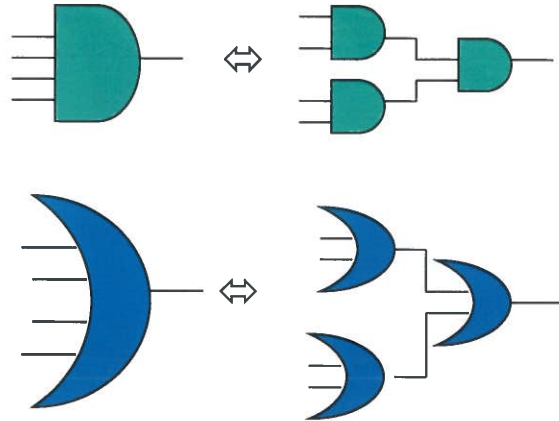
$$\text{RHS } [a(bc)']' = a' + bc.$$

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## Other Types of Gates: Block Diagram Transform

### 3) Block Diagram Transformation

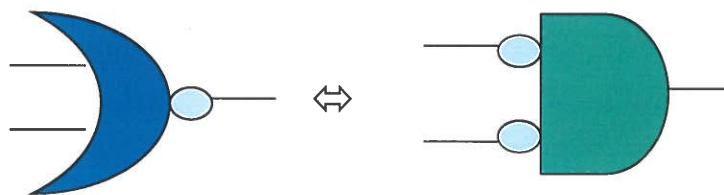
a) Reduce # of inputs.



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## Other Types of Gates: Block Diagram Transform

b. DeMorgan's Law



$$(a+b)' = a' b'$$

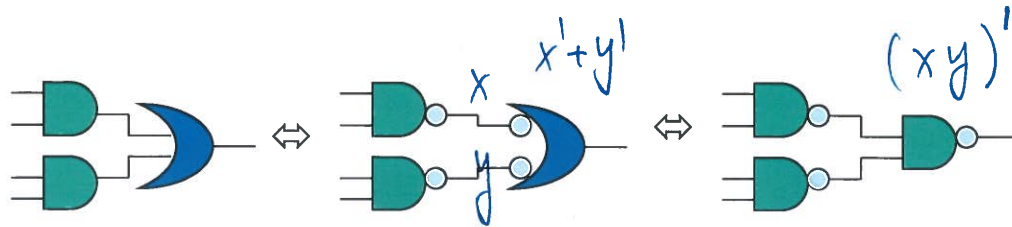


$$(ab)' = a' + b'$$

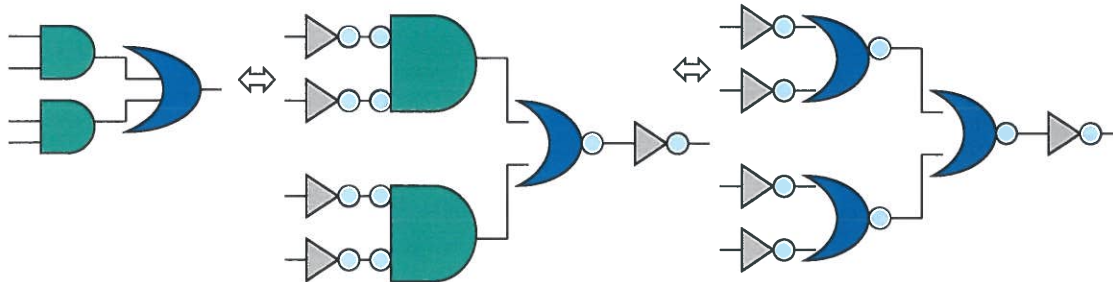
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## Other Types of Gates: Block Diagram Transform

### c. Sum of Products (Using only NAND gates)



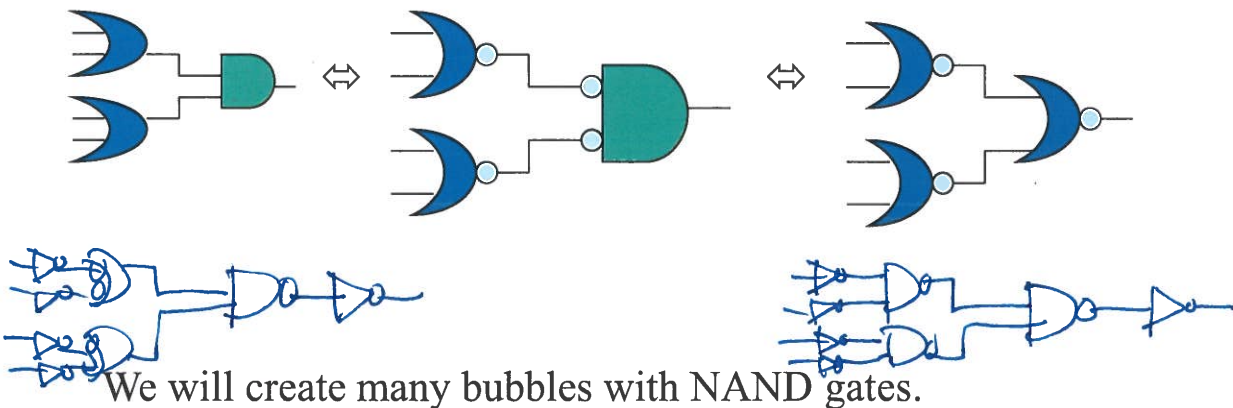
Sum of Products (We create many bubbles with NOR gates)



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## Other Types of Gates: Block Diagram Transform

### d. Product of Sums (NOR gates only)



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## Other Types of Gates: Block Diagram Transform

### NAND, NOR gates

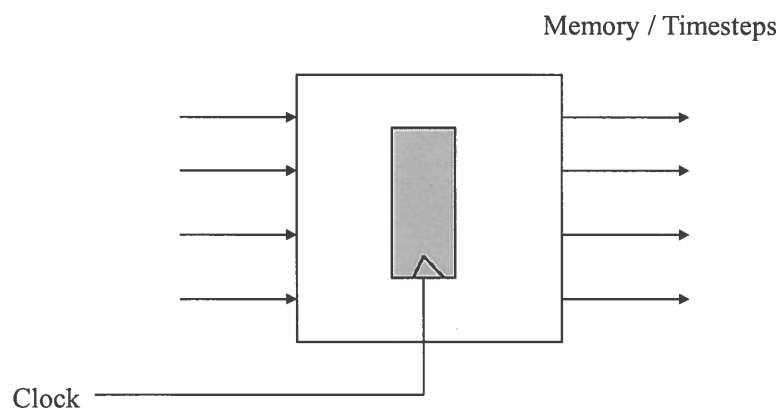
Remark:

Two level NAND gates: Sum of Products

Two level NOR gates: Product of Sums

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## Part II. Sequential Networks



Flip flops  
Specification  
Implementation

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