

Lecture 4: Four Input K-Maps

CSE 140: Components and Design Techniques for Digital Systems

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Outlines

- Switching Algebra vs. Karnaugh Maps

Algebra	Variables	Product terms	Minterms	Consensus theorem
K Map	Two-Half Planes	Rectangles	Cells	Adjacency

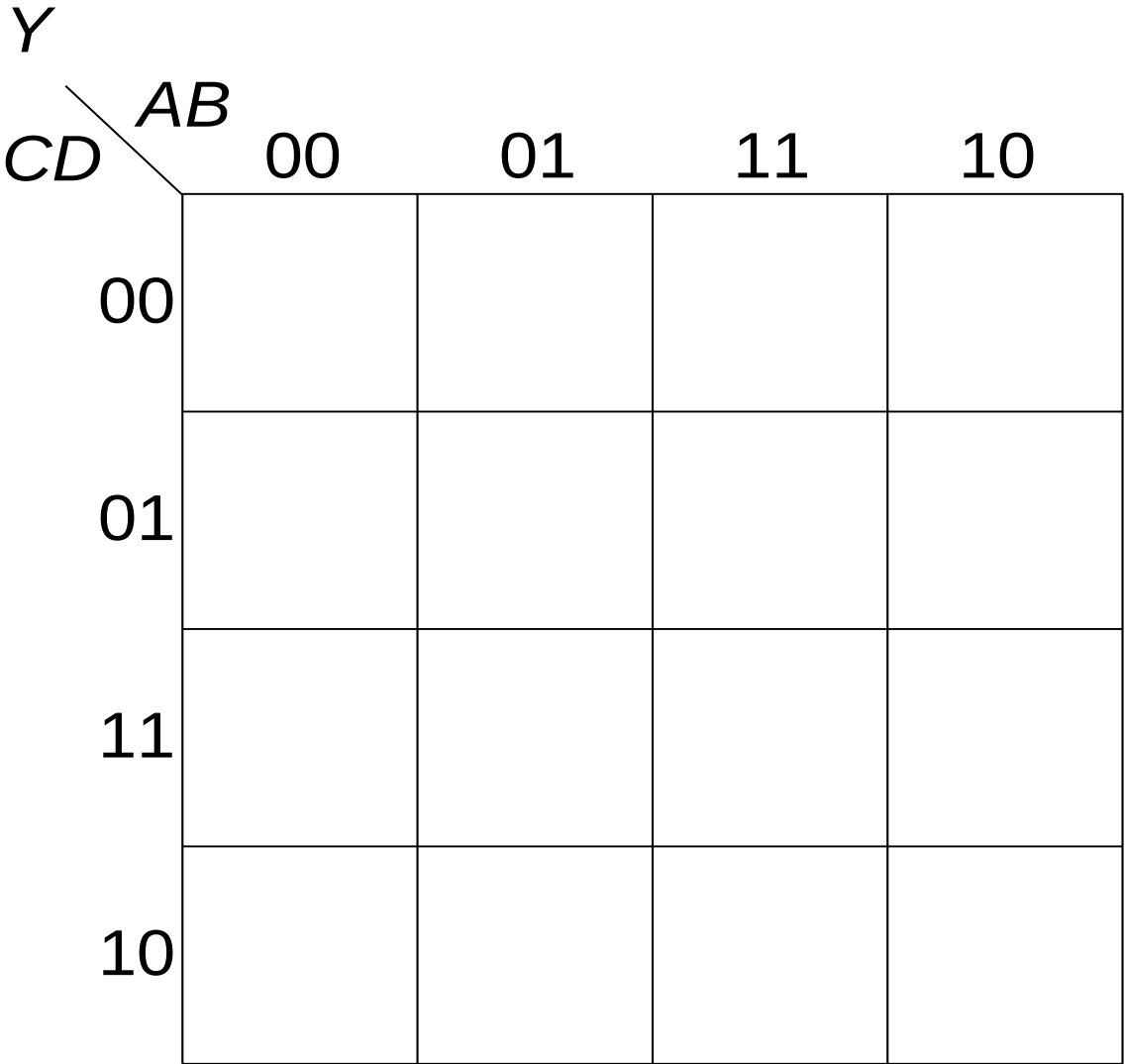
Definitions: implicants, prime implicants, essential prime implicants

- Implementation Procedures

An example

id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

4-input K-map



4-input K-map

id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

Y $CD \backslash AB$					
		00	01	11	10
00	1	0	0	1	
01	0	1	0	1	
11	1	1	0	0	
10	1	1	0	1	

4-input K-map

- Arrangement of variables
- Adjacency and partition

Y CD \ AB					
		00	01	11	10
00	0 1	4 0	12 0	8 1	
01	1 0	5 1	13 0	9 1	
11	3 1	7 1	15 0	11 0	
10	2 1	6 1	14 0	10 1	

Reading the reduced K-map

Y		AB						
CD		00	01	11	10			
		0	1	2	3			
00	0	1	4	0	12	0	8	1
01	1	0	5	1	13	0	9	1
11	3	1	7	1	15	0	11	0
10	2	1	6	1	14	0	10	1

$$\sum m(2,3,6,7)$$

$$\sum m(5,7)$$

$$\sum m(8,9)$$

$$\sum m(0,2,8,10)$$

$$Y = \bar{A}\bar{C} + \bar{A}BD + A\bar{B}\bar{C} + \bar{B}\bar{D}$$

K-Map vs. Switching Algebra Expression

Algebra Expression	K-Map
Variable x_i and complement x_i'	Half planes R_i and R_i'
Product term $P = \Pi_i x_i^*$	Rectangle $R_p = \cap_{i \in P} R_i^*$
Each minterm	One element cell
Two adjacent minterms	Two neighboring cells
Each minterm has n adjacent minterms	Each cell has n neighbors
Boolean algebra	Two dimensional tool
?	<i>D</i> Don't Care set handling

Procedure for finding the minimal function via K-maps (layman's terms)

1. Truth table $\Rightarrow$ K-map: F, R, D sets
2. Find product terms p : p covers at least one $m_i \in F$ but no intersection with R . (**Implicants**)
3. Expand implicant p to its limit (**Prime Implicants**)
4. Select prime implicant p that covers an $m_i \in F$ but m_i is not contained in any other prime implicants. (**Essential Primes**)
5. Use the essential primes and a minimal set of other primes to cover F . (Challenging task)

		AB			
		00	01	11	10
CD	00				
	01				
	11				
	10				

Definitions: implicant, prime implicant, essential prime implicant

- **Implicant**: A product term that has non-empty intersection with **on-set F** and does not intersect with off-set R .
- **Prime** Implicant: An implicant that is **not covered** by any other implicant.
- **Essential** Prime Implicant: A prime implicant that has **an element in on-set F** but this element is not covered by any other prime implicants.

Examples of Primes and Essential Primes

Y		AB			
CD		00	01	11	10
	00				
	01				
	11				
	10				

Examples of Primes and Essential Primes

Y		AB			
CD		00	01	11	10
00					
01					
11					
10					

Definition: Prime Implicant

1. Implicant: A product term that has non-empty intersection with on-set F and does not intersect with off-set R .
2. Prime Implicant: An implicant that is not covered by any other implicant.

Y CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

Q: How about this one? Is it a prime implicant?

- A. Yes
B. No

Definition: Essential Prime

- Essential Prime Implicant: A prime implicant that has an element in on-set F but this element is not covered by any other prime implicants.

Y $CD \backslash AB$					
		00	01	11	10
00		1	0	0	1
01		0	1	0	1
11		1	1	0	0
10		1	1	0	1

Q: Is the blue group an essential prime?

- A. Yes
B. No

Definition: Essential Prime

Q: Which of the following product term(s) is (are) not an essential prime for the given K-map ?

		ab			
		00	01	11	10
cd	00	0 1	4	12 1	8 1
	01	1	5 1	13 1	9
	11	3	7	15 1	11 1
	10	2 1	6	14 1	10 1

- A. $bc'd$ $\sum m(5,13)$
- B. $d'b'$ $\sum m(0,2,8,10)$
- C. ac $\sum m(10,11,14,15)$
- D. ab $\sum m(12,13,14,15)$
- E. ad' $\sum m(8,10,12,14)$

Procedure for finding the minimal function via K-maps (formal terms)

1. Convert truth table to K-map
2. Include all essential primes
3. Include nonessential primes as needed to completely cover the onset (all cells of value one)

Y $CD \backslash AB$					
		00	01	11	10
CD	00	1	0	0	1
	01	0	1	0	1
	11	1	1	0	0
	10	1	1	0	1

K-maps with Don't Cares

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Y</i>
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

<i>Y</i> \ <i>CD</i> \ <i>AB</i>					
		<i>AB</i>			
		00	01	11	10
<i>CD</i>	00				
	01				
	11				
	10				

K-maps with Don't Cares

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Y</i>
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

<i>Y</i>		<i>AB</i>			
<i>CD</i>		00	01	11	10
00		1	0	X	1
01		0	X	X	1
11		1	1	X	X
10		1	1	X	X

K-maps with Don't Cares

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Y</i>
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

<i>Y</i>		<i>AB</i>			
<i>CD</i>		00	01	11	10
		00	01	11	10
00		1	0	X	1
01		0	X	X	1
11		1	1	X	X
10		1	1	X	X

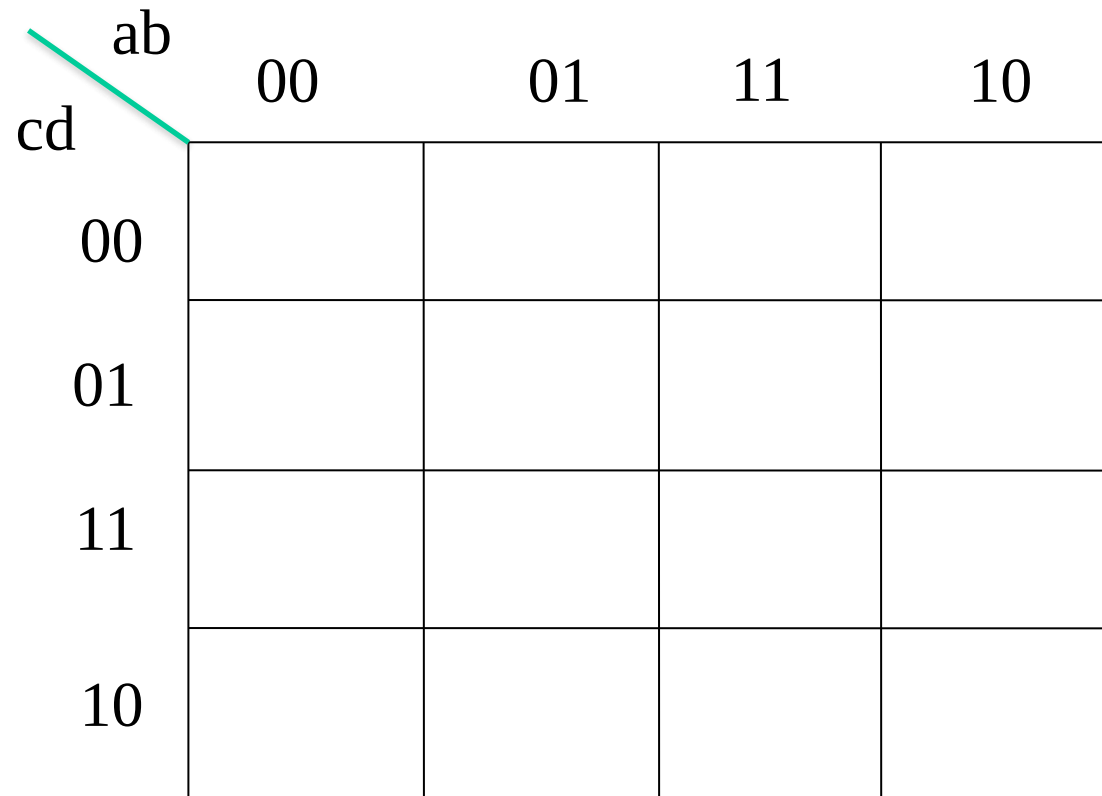
$$Y = A + \bar{B}\bar{D} + C$$

Reducing Canonical expressions

Given $F(a,b,c,d) = \sum m(0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \sum m(9, 10)$

1. Draw K-map



ab	00	01	11	10
cd				
00				
01				
11				
10				

Reducing Canonical Expressions

Given $F(a,b,c,d) = \sum m(0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \sum m(9, 10)$

1. Draw K-map

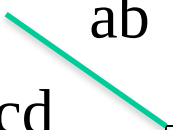
ab \ cd	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Reducing Canonical Expressions

Given $F(a,b,c,d) = \sum m(0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \sum m(9, 10)$

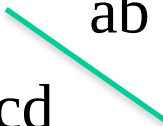
1. Draw K-map



ab		00	01	11	10
cd	00	⁰ 1	⁴ 0	¹² 0	⁸ 1
	01	¹ 1	⁵ 0	¹³ 0	⁹ X
	11	³ 0	⁷ 0	¹⁵ 0	¹¹ 0
	10	² 1	⁶ 0	¹⁴ 1	¹⁰ X

Reducing Canonical Expressions

1. Draw K-map
2. Identify Prime implicants
3. Identify Essential Primes



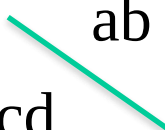
ab		00	01	11	10
cd	00	⁰ 1	⁴ 0	¹² 0	⁸ 1
	01	¹ 1	⁵ 0	¹³ 0	⁹ X
	11	³ 0	⁷ 0	¹⁵ 0	¹¹ 0
	10	² 1	⁶ 0	¹⁴ 1	¹⁰ X

Q: How many primes (P) and essential primes (EP) are there?

- A. Four (P) and three (EP)
- B. Three (P) and two (EP)
- C. Three (P) and three (EP)
- D. Four (P) and Four (EP)

Reducing Canonical Expressions

1. Prime implicants: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
2. Essential Primes: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$



		ab			
		00	01	11	10
cd	00	0 1	4 0	12 0	8 1
	01	1 1	5 0	13 0	9 X
	11	3 0	7 0	15 0	11 0
	10	2 1	6 0	14 1	10 X

Q: Do the E-primes cover the entire on set?

A. Yes

B. No

Reducing Canonical Expressions

1. Prime implicants: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
2. Essential Primes: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
3. Min exp: $\Sigma (\text{Essential Primes}) = \Sigma m(0, 1, 8, 9) + \Sigma m(0, 2, 8, 10) + \Sigma m(10, 14)$

$$f(a,b,c,d) = ?$$

ab					
		00	01	11	10
cd	00	⁰ 1	⁴ 0	¹² 0	⁸ 1
	01	¹ 1	⁵ 0	¹³ 0	⁹ X
	11	³ 0	⁷ 0	¹⁵ 0	¹¹ 0
	10	² 1	⁶ 0	¹⁴ 1	¹⁰ X

Q: Do the E-primes cover the entire on set?

A. Yes

B. No

Reducing Canonical Expressions

1. Prime implicants: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
2. Essential Primes: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
3. Min exp: $\Sigma(\text{Essential Primes}) = \Sigma m(0, 1, 8, 9) + \Sigma m(0, 2, 8, 10) + \Sigma m(10, 14)$

$$f(a,b,c,d) = b'c' + b'd' + acd'$$

		ab			
		00	01	11	10
cd	00	0 1	4 0	12 0	8 1
	01	1 1	5 0	13 0	9 X
	11	3 0	7 0	15 0	11 0
	10	2 1	6 0	14 1	10 X

Q: Do the E-primes cover the entire on set?

A. Yes

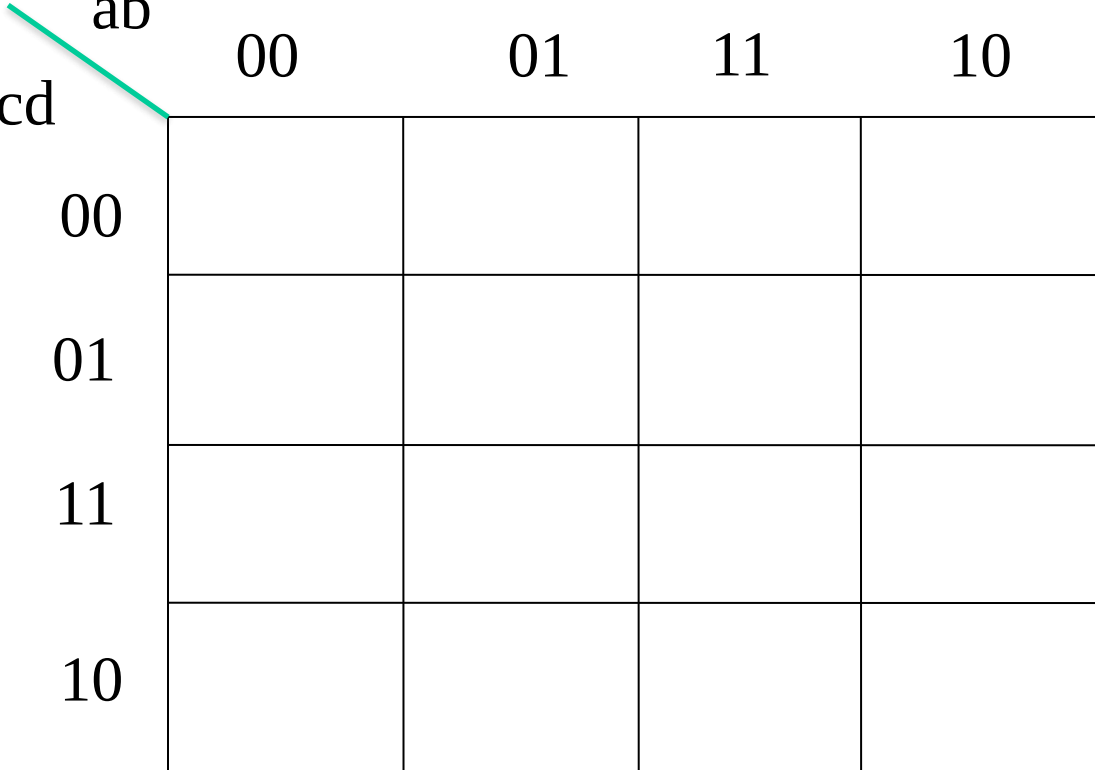
B. No

Another example

Given $F(a,b,c,d) = \sum m(0, 3, 4, 14, 15)$

$D(a,b,c,d) = \sum m(1, 11, 13)$

1. Draw the K-Map

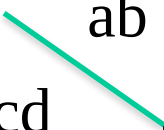


ab		00	01	11	10
cd	00				
	01				
	11				
	10				

Another example

Given $F(a,b,c,d) = \sum m(0, 3, 4, 14, 15)$

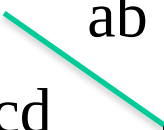
$D(a,b,c,d) = \sum m(1, 11, 13)$



		ab			
		00	01	11	10
cd	00	0 1	4 1	12 0	8 0
	01	1 X	5 0	13 X	9 0
	11	3 1	7 0	15 1	11 X
	10	2 0	6 0	14 1	10 0

Reducing Canonical Expressions

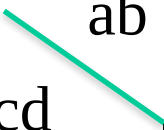
1. Prime implicants: $\Sigma m(0, 4), \Sigma m(0, 1), \Sigma m(1, 3), \Sigma m(3, 11), \Sigma m(14, 15), \Sigma m(11, 15), \Sigma m(13, 15)$
2. Essential Primes: $\Sigma m(0, 4), \Sigma m(14, 15)$



		ab			
		00	01	11	10
cd	00	0 1	4 1	12 0	8 0
	01	1 X	5 0	13 X	9 0
	11	3 1	7 0	15 1	11 X
	10	2 0	6 0	14 1	10 0

Reducing Canonical Expressions

1. Prime implicants: $\Sigma m(0, 4), \Sigma m(0, 1), \Sigma m(1, 3), \Sigma m(3, 11), \Sigma m(14, 15), \Sigma m(11, 15), \Sigma m(13, 15)$
2. Essential Primes: $\Sigma m(0, 4), \Sigma m(14, 15)$
3. Min exp: $\Sigma m(0, 4), \Sigma m(14, 15), (\Sigma m(3, 11) \text{ or } \Sigma m(1, 3))$
4. $f(a,b,c,d) = a'c'd' + abc + b'cd \text{ (or } a'b'd)$



		ab			
		00	01	11	10
cd	00	0 1	4 1	12 0	8 0
	01	1 X	5 0	13 X	9 0
	11	3 1	7 0	15 1	11 X
	10	2 0	6 0	14 1	10 0 ₂₉