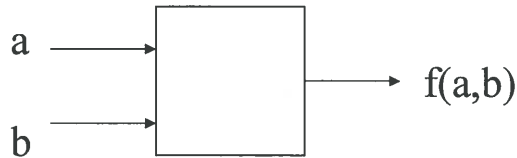


Two Variable K-maps

id	a	b	f(a, b)
0	0	0	f(0, 0)
1	0	1	f(0, 1)
2	1	0	f(1, 0)
3	1	1	f(1, 1)

possible 2-variable functions:

For 2 variables as inputs, we have $4=2^2$ entries. Each entry can be 0 or 1. Thus we have $16=2^4$ possible functions.

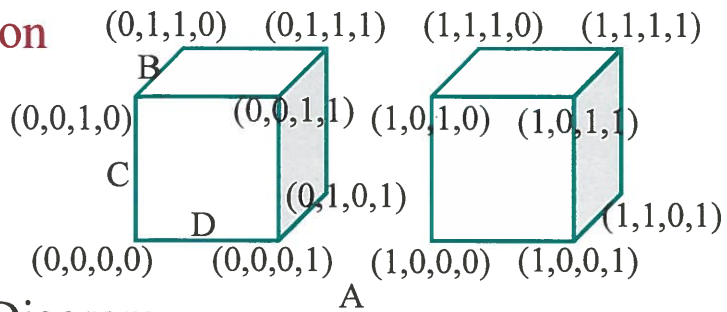


19

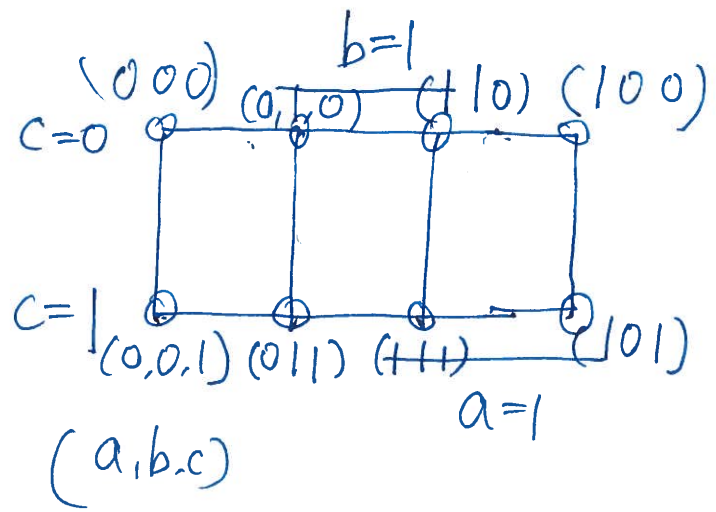
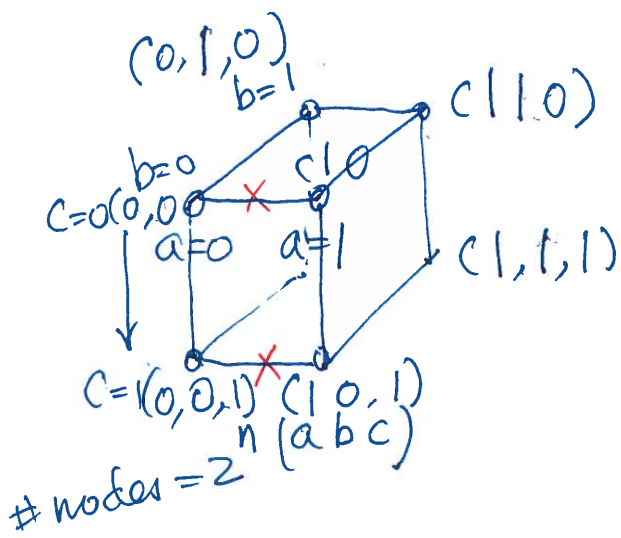
Representation of k-Variable Func.

A cube of 4 variables: (A,B,C,D)

- **Boolean Expression**
- **Truth Table**
- **Cube**
- **K Map**
- **Binary Decision Diagram**
- **And-Inverter Graph**
 - Ref: R. Brayton and A. Mishchenko, “ABC: an academic industrial-strength verification tool,” ACM Int. Conf. on Computer Aided Verification, pp. 24-40, 2010.



20



23

K-map

- Circle 1's in adjacent squares
- Find rectangles which correspond to product terms in Boolean expression

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Y C	AB			
	00	01	11	10
0	1	0	0	0
1	1	0	0	0

$$y(A,B,C) = A' B' C' + A' B' C = A' B' (C' + C) = A' B'$$

24

Truth table

Id	a	b	c	f(a,b,c)
0	0	0	0	1 $a'b'c'$
1	0	0	1	0
2	0	1	0	1 $a'bc'$
3	0	1	1	0
4	1	0	0	1 $ab'c'$
5	1	0	1	0
6	1	1	0	1 abc'
7	1	1	1	0

$$\Sigma m(0,2) = a'c'$$

$$\Sigma m(4,6) = ac'$$

$$\Sigma m(0,2,4,6) : c'$$

Corresponding three variable K-map

Gray code (a,b)

Gray code

	(0,0)	(0,1) b	(1,1)	(1,0)
c = 0	0 1	2 1	6 1	4 1
c = 1	1 0	3 0	7 0	5 0

a

$$f(a,b,c) = c'$$

21

Karnaugh Maps (K-Maps)

- K-maps minimize equations graphically
- Note that the label decides the order in the map

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

	AB	00	01	11	10
C	0	1	0	0	0
	1	1	0	0	0

	AB	00	01	11	10
C	0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
	1	$\bar{A}BC$	$\bar{A}BC$	ABC	$A\bar{B}C$

$$a'b'c' + a'b'c = a'b'$$

22

Another 3-Input truth table

Id	a	b	c	f(a,b,c)
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	X
7	1	1	1	1

Ex:

	c			
x:	0	1	3	2
	0	0	0	1
a	4	5	7	6
	1	1	1	X

$$f(a,b,c) = a + bc'$$

Another 3-Input truth table

Id	a	b	c	f(a,b,c)
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	X
7	1	1	1	1

Corresponding K-map

$m_2: a'bc'$
 $m_6: abc'$
 $\Sigma m(2,6): bc'$

	(0,0)	(0,1)	(1,1)	(1,0)
c = 0	0	1	X	1
c = 1	0	0	1	1

a

Ex

~~Ex~~

$$f(a,b,c) = a + bc'$$

	b			
	0	1	0	1
c	0	0	1	1

25

Corresponding K-map

b = 1

	(0,0)	(0,1)	(1,1)	(1,0)
c = 0	0	1	X	1
c = 1	0	0	1	1

a = 1

$$f(a,b,c) = a + bc'$$

3-input truth table

Id	a	b	c	f(a,b,c,d)
0	0	0	0	1
1	0	0	1	1
2	0	1	0	X
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Corresponding K-map

$$f(a,b,c) = b'$$

	(0,0)	(0,1)	(1,1)	(1,0)
c = 0	0 1	2 X	6 0	4 1
c = 1	1 1	3 0	7 0	5 1

a

Q: In the above case the minimal Boolean expression is obtained when the don't care term is:

- A. Included i.e. Circled in with one or more minterms that evaluate to 1
- B. Excluded when grouping the minterms that evaluate to one
- C. Either approach gives the minimal expression

27

3-Input Truth table

Id	a	b	c	f(a,b,c,d)
0	0	0	0	1
1	0	0	1	1
2	0	1	0	X
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Corresponding K-map

	(0,0)	(0,1)	(1,1)	(1,0)
c = 0	0 1	2 X	6 0	4 1
c = 1	1 1	3 0	7 0	5 1

Q: In the above case the minimal Boolean expression is obtained when the don't care term is:

- A. Included i.e. Circled in with one or more minterms that evaluate to 1
- B. Excluded when grouping the minterms that evaluate to one
- C. Either approach gives the minimal expression

28

3-input truth table with Don't cares

Id	a	b	c	f(a,b,c,d)
0	0	0	0	1
1	0	0	1	1
2	0	1	0	X
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Corresponding K-map

	$b = 1$			
	(0,0)	(0,1)	(1,1)	(1,0)
$c = 0$	⁰ 1	² X	⁶ 0	⁴ 1
$c = 1$	¹ 1	³ 0	⁷ 0	⁵ 1
	$a = 1$			

$$f(a,b,c) = b'$$

29

Consensus Theorem in View of K Maps

Consensus Theorem: $A'B + AC + BC = A'B + AC$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

	AB	00	01	11	10
C	0	0	1	0	0
1	0	0	1	1	1

	AB	00	01	11	10
C	0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	0	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

$$\left. \begin{array}{l} \Sigma m(2,3) = a'b \\ \Sigma m(3,7) = \underline{bc} \\ \Sigma m(5,7) = ac \end{array} \right\} \Rightarrow f(a,b,c) = a'b + ac$$

30

Lecture 4: Four Input K-Maps

CSE 140: Components and Design Techniques for Digital Systems

CK Cheng

Dept. of Computer Science and Engineering
University of California, San Diego

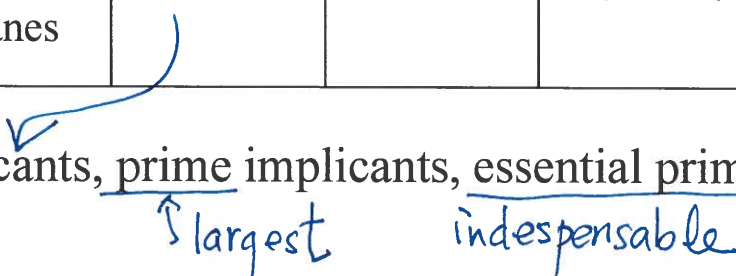
1

Outlines

- Switching Algebra vs. Karnaugh Maps

Algebra	Variables	Product terms	Minterms	Consensus theorem
K Map	Two-Half Planes	Rectangles	Cells	Adjacency

Definitions: implicants, prime implicants, essential prime implicants



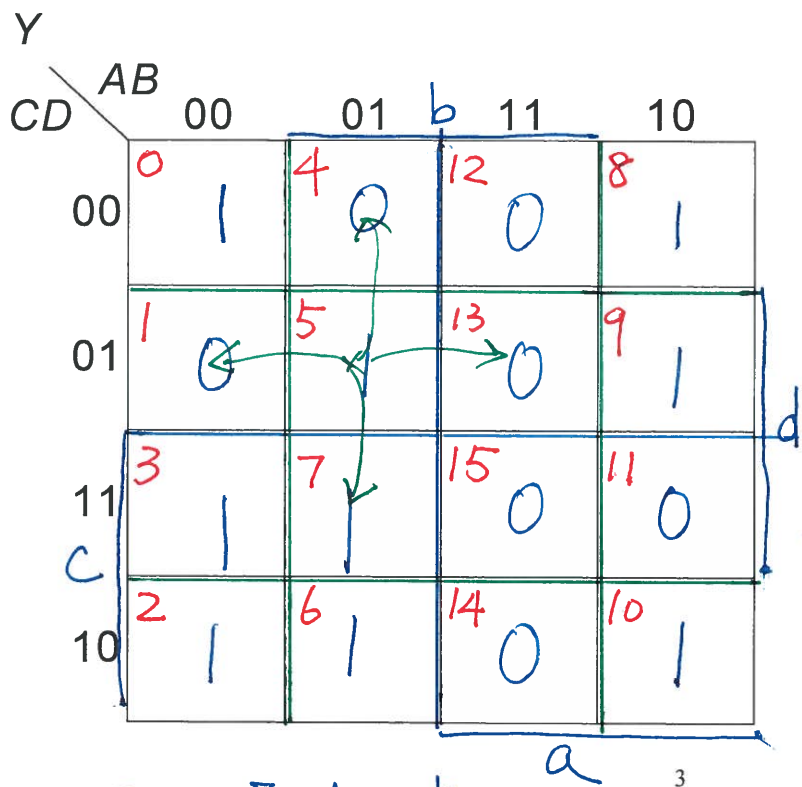
- Implementation Procedures

2

An example

id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

4-input K-map

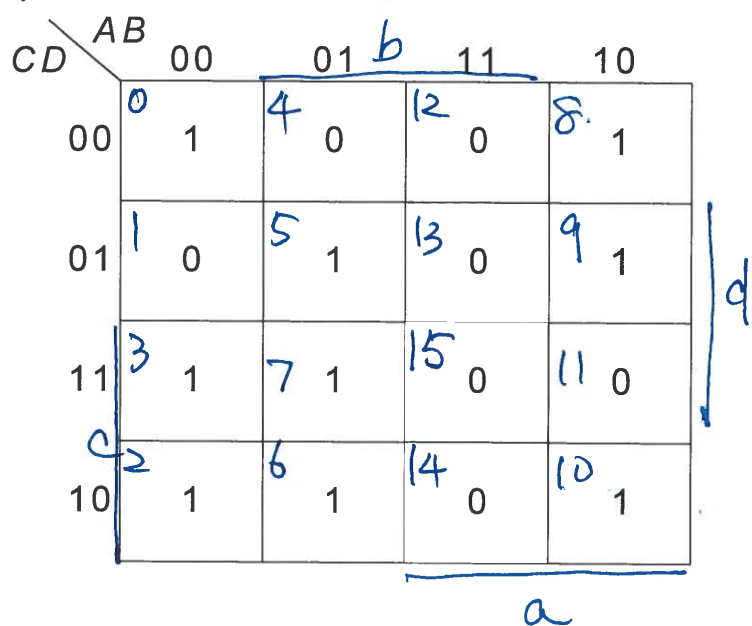


Prime Implicants

$$E: \sum m(0, 2, 8, 10), \sum m(2, 3, 6, 7)$$

4-input K-map

$$Y: \sum m(2, 5, 7), \sum m(9, 8)$$



id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

4-input K-map

- Arrangement of variables
- Adjacency and partition

Y CD \ AB		00		01		11		10	
00	0	1	4	0	12	0	8	1	
01	1	0	5	1	13	0	9	1	
11	3	1	7	1	15	0	11	0	
10	2	1	6	1	14	0	10	1	

5

Reading the reduced K-map

Y CD \ AB		00	01	11	10			
		00	01	11	10			
00	0	1	4	0	12	0	8	1
01	1	0	5	1	13	0	9	1
11	3	1	7	1	15	0	11	0
10	2	1	6	1	14	0	10	1

$$\begin{aligned}
 \Sigma m(2,3,6,7) & \text{ } a'c \\
 \Sigma m(5,7) & \text{ } a'bd \\
 \Sigma m(8,9) & \text{ } ab'c' \\
 \Sigma m(0,2,8,10) & \text{ } b'd'
 \end{aligned}$$

$$Y = \bar{A}\bar{C} + \bar{A}BD + A\bar{B}\bar{C} + \bar{B}\bar{D}$$

6

K-Map vs. Switching Algebra Expression

Algebra Expression	K-Map
Variable x_i and complement x_i'	Half planes R_i and R_i'
Product term $P = \prod_i x_i^*$	Rectangle $R_p = \cap_{i \in P} R_i^*$
Each minterm	One element cell
Two adjacent minterms	Two neighboring cells
Each minterm has n adjacent minterms	Each cell has n neighbors
Boolean algebra	Two dimensional tool
?	D Don't Care set handling

7

Procedure for finding the minimal function via K-maps (layman's terms)

1. Truth table $\Rightarrow$ K-map: F, R, D sets
2. Find product terms p : p covers at least one $m_i \in F$ but no intersection with R . (Implicants) $\exists m_i \in F$ s.t. $m_i \in p$. $p \cap R = \emptyset$.
3. Expand implicant p to its limit (Prime Implicants)
4. Select prime implicant p that covers an $m_i \in F$ but m_i is not contained in any other prime implicants. $\exists m_i \in F$, s.t. $m_i \in p$ but $m_i \notin p_j$. (Essential Primes) $p_j \neq p$.
5. Use the essential primes and a minimal set of other primes to cover F . (Challenging task)

		AB			
		00	01	11	10
Y CD	00				
	01				
	11				
	10				

Definitions: implicant, prime implicant, essential prime implicant

- **Implicant:** A product term that has non-empty intersection with **on-set F** and does not intersect with off-set R .
- **Prime Implicant:** An implicant that is **not covered** by any other implicant.
- **Essential Prime Implicant:** A prime implicant that has **an element in on-set F** but this element is not covered by any other prime implicants.

9

Examples of Primes and Essential Primes

$$f(a,b,c,d) = \sum m(0,4), \sum m(10,11)$$

$$(1) \sum m(4,5) \sum m(13,15)$$

$$(2) \sum m(5,13) \sum m(13,15)$$

$$(3) \sum m(5,13) \sum m(11,15)$$

$$\begin{aligned} &\sum m(\textcircled{0},4) \\ &\sum m(4,5) \\ &\sum m(5,13) \\ &\sum m(13,15) \\ &\sum m(11,15) \\ &\sum m(\textcircled{10},11) \end{aligned}$$

Y CD \ AB		(31)Σm(5,13)Σm						
		00	01	11	10			
00	~~0~~	~~1~~	~~4~~	12	8			
01	1	0	5	1	13	9		
11	3	0	7	0	15	1	11	
10	2	0	6	0	14	0	10	~~7~~

Examples of Primes and Essential Primes

Y CD \ AB				
	00	01	11	10
00				
01				
11				
10				

11

Definition: Prime Implicant

1. Implicant: A product term that has non-empty intersection with on-set F and does not intersect with off-set R .
2. Prime Implicant: An implicant that is not covered by any other implicant.

Y CD \ AB				
	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

Q: How about this one? Is it a prime implicant?

- A. Yes
B. No

12