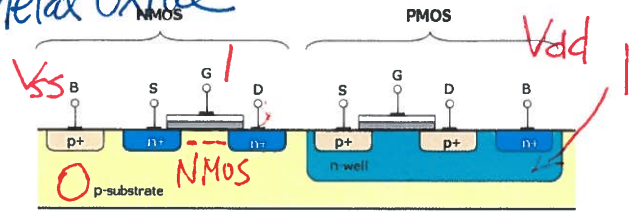
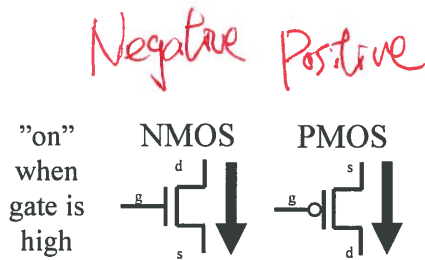


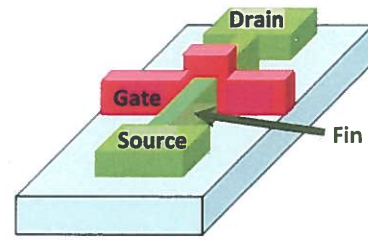
Complementary Basic CMOS Semiconductor



https://en.wikipedia.org/wiki/CMOS#/media/File:Cmos_impurity_profile.PNG

- Complementary Metal Oxide Semiconductor
- Invented in the 1960's, but took over in the 80's
- "on" means low resistance, "off" means high resistance
- Logic "1" and Logic "0" values are arbitrary
e.g. logic "1" == 1.0 V, logic "0" == 0.0 V

Planar technology

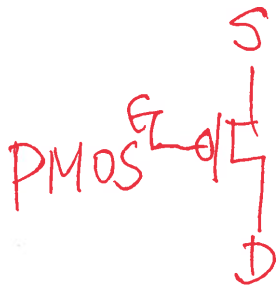


FINFET technology

By Irene Ringworm. CC BY-SA 3.0.
<https://commons.wikimedia.org/w/index.php?curid=3833512>

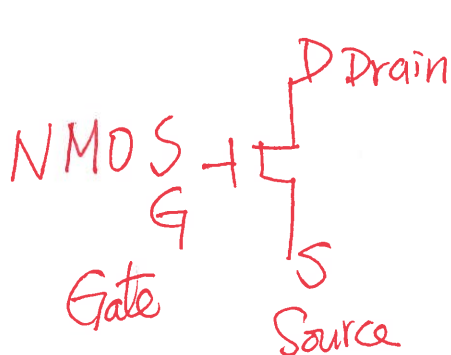
35

Transistors as Switches



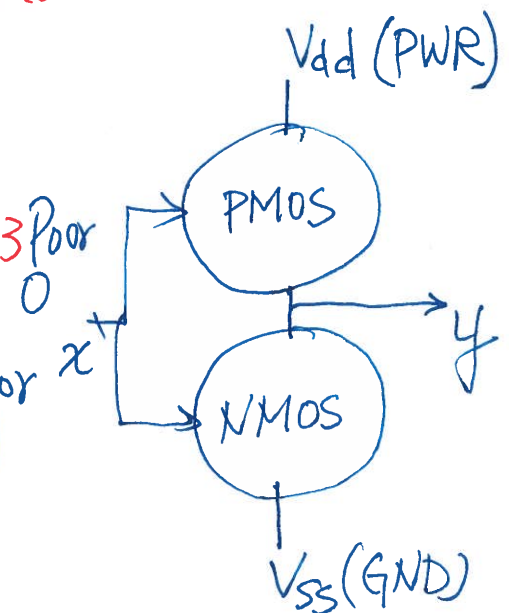
$G=1$ S & D are disconnected.

$G=0$ { If S=1, D=1
If S=0, D=0.3 Poor



$G=1$ { If S=1, D=0.7 Poor
If S=0, D=0

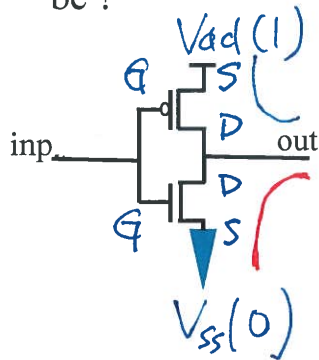
$G=0$ { S & D are disconnected.



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The most basic CMOS gate - inverter

- When "inp" is "1", then the nmos is on and the pmos is off – output will be ?
- When "inp" is "0", then the nmos is off and the pmos is on – output will be ?



Schematic



Truth Table

in	out
0	1
1	0

Equation

$$\begin{cases} \text{out} = \text{in}' \\ \text{out} = \overline{\text{in}} \end{cases}$$

$$\begin{aligned} \text{Inp} = 1 &\Rightarrow \text{out} = 0 \\ \text{Inp} = 0 &\Rightarrow \text{out} = 1. \end{aligned}$$

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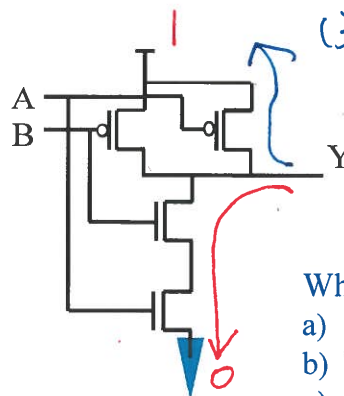
Basic Gates – (N)AND gate

AND $Y = A \& B$ $Y = AB$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

NAND $Y = (A \& B)'$ $Y = (AB)'$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



(1). NMOS trans in series
(2) PMOS trans in parallel
(3). y connects to either $V_{dd}(1)$ or $V_{ss}(0)$ but not both.

What kind of gate is this?

- AND
- NAND
- Inverter
- None of the above

If $AB = 1$, $y = 0$

If $A = 0$ or $B = 0$, $y = 1$.

$$A' = 1 \text{ or } B' = 1 \quad (A' + B' = 1) \\ AB = 0$$

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Basic Gates – (N)OR gate

OR $Y = A + B$ $Y = A | B$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



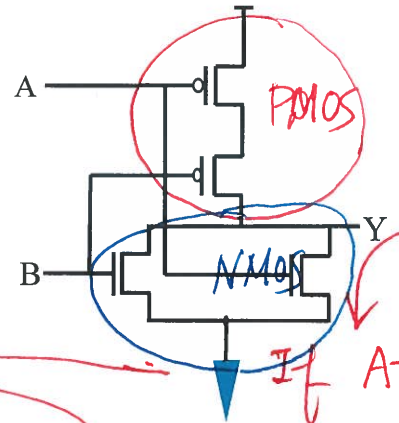
NOR $Y = (A + B)'$ $Y = (A | B)'$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0



Bubble means Invert

NOR is universal gate



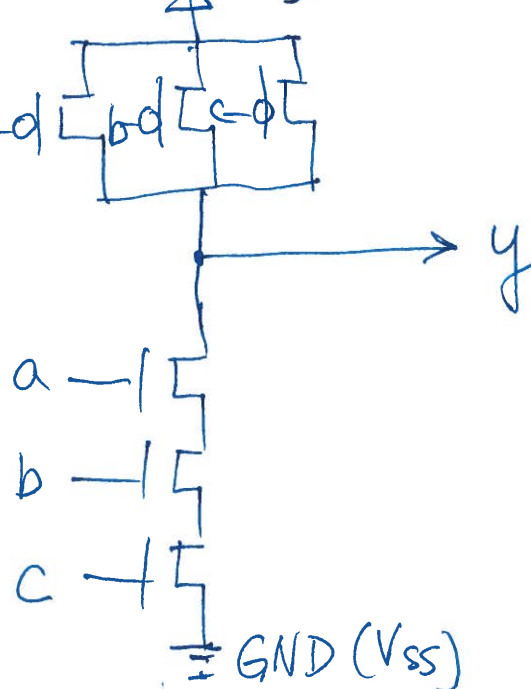
If $A+B=1$, $y=0$.
 If $A=0$ & $B=0$, $y=1$
 $(A'=1 \text{ \& } B'=1)$
 $(A'B'=1)$
 $(A+B=0)$

Transistors as Switches: CMOS

a	b	c	y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

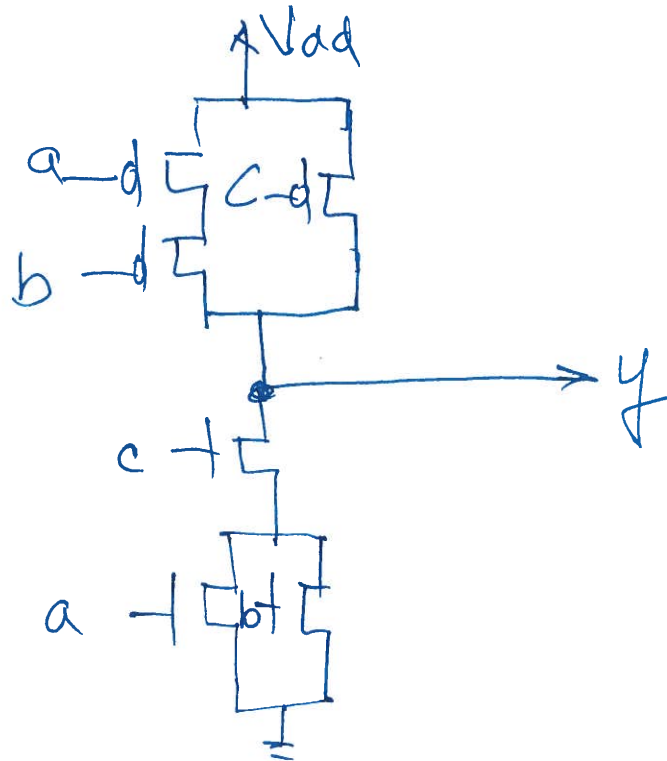
NAND(a,b,c)
 $= (abc)'$

PWR (V_{dd})



Transistors as Switches

$$y = [(a+b)c]'$$



If $(a+b)c=1, y=0$

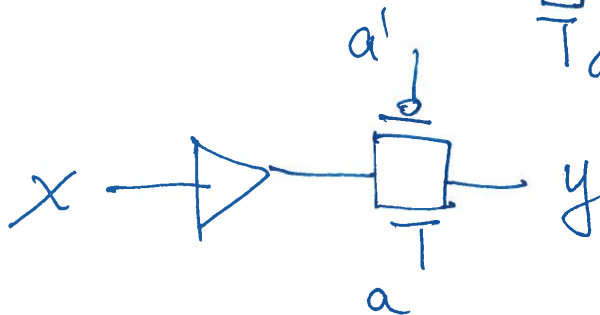
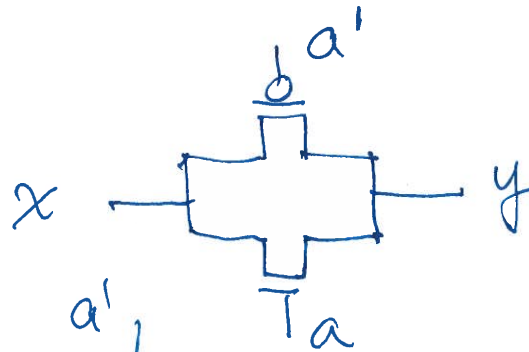
If $(a+b)c=0, y=1$

$(a=0 \& b=0) \text{ or } c=0 \Rightarrow y=1$

$(a'=1 \& b'=1) \text{ or } c'=1 \Rightarrow (a'b') + c' = 1 \Rightarrow (a+b)c = 0$

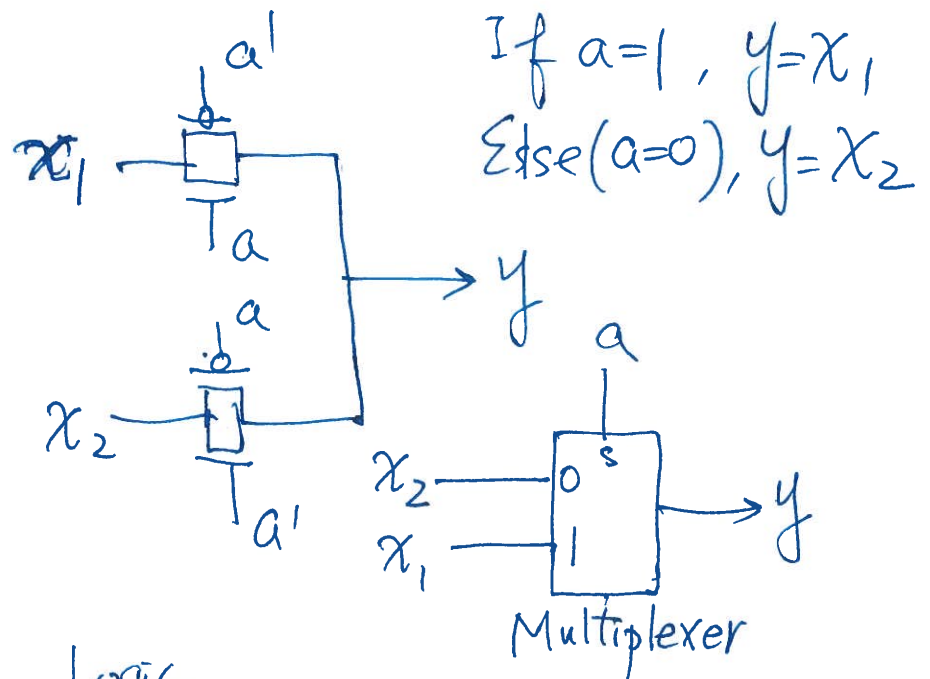
Transistors as Switches: Transmission Gate

If $a=1, y=x$



tristate buffer
y=0
floating

Transistors as Switches: Transmission Gate



Logic

0 false empty + OR union
 1 true universe • AND intersect

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Boolean Algebra

A branch of algebra in which the values of the variables belong to a set B (e.g. $\{0, 1\}$), has two operations $\{+, \cdot\}$ that satisfy the following four sets of laws.

• Associative laws: $(a+b)+c = a+(b+c)$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

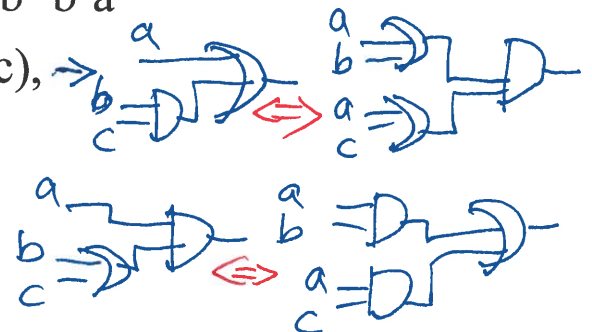
• Commutative laws: $a+b = b+a$, $a \cdot b = b \cdot a$

• Distributive laws: $a+(b \cdot c) = (a+b) \cdot (a+c)$, $a \cdot (b+c) = a \cdot b + a \cdot c$

• Identity laws: $a+0=a$, $a \cdot 1=a$

• Complement laws: $a+a'=1$, $a \cdot a'=0$

(x' : the complement element of x)



Duality

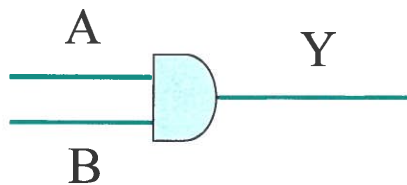
- Swap $(+, \cdot)$ and complement all 0's and 1's
- If we can prove a statement using laws of Boolean algebra true, then the duality of the statement is also true.

Laws and their duals

Associative	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(a + b) + c = a + (b + c)$
Commutative	$a \cdot b = b \cdot a$	$a + b = b + a$
Distributive*	$a \cdot (b + c) = a \cdot b + a \cdot c$	$a + (b \cdot c) = (a + b) \cdot (a + c)$
Identity	$a \cdot 1 = a$	$a + 0 = a$
Compliment	$a \cdot a' = 0$	$a + a' = 1$

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Representations of combinational circuits: The Schematic



- What is the simplest combinational circuit that you know?

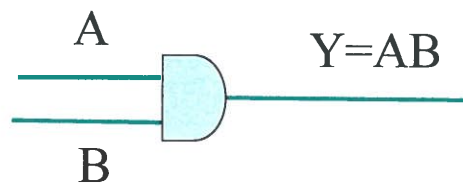
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Representations of combinational circuits

Truth Table: Enumeration of all combinations

Example: AND

id	A	B	Y
0	0	0	0
1	0	1	0
2	1	0	0
3	1	1	1



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Boolean Algebra

Similar to regular algebra but defined on sets with only three basic ‘logic’ operations:

1. Intersection: AND (2-input); Operator: $\cdot$, $\&$
2. Union: OR (2-input); Operator: $+$, $|$
3. Complement: NOT (1-input); Operator: $'$, $!$

“ $\&$, $|$, $!$ ” Symbols in BSV

Some Def's

- Complement : variable with a "BAR" over it or ' after it

A'

#variable = 4

- Literal : variable or its complement

#literals = 12

- Implicant: product of literals

ABC

$$f(a,b,c,d) = ab' + (a+b+c+d)(abcd + c'd)$$

- Implicate: sum of literals

(A+B+C)

maxterm

minterm

Minterm, maxterm (implicant or implicate that includes all the inputs)

f(A,B,C,D): ABCD, (A+B+C+D)

To determine $f=1$ we check each term
 $f=0$ it is a SAT problem
 Satisfiability
 NP Complete

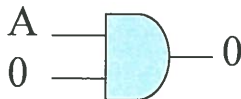
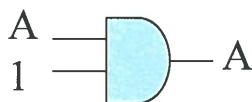
$$f(a,b,c,d) = ab' + a'b + cde$$

$$g(a,b,c,d) = (a+b')(a'+b)(c+d+e)$$

Boolean algebra and switching functions

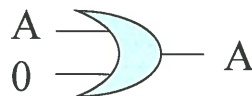
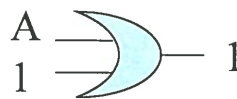
Two-input AND (·)

AND	A	B	Y
	0	0	0
	0	1	0
	1	0	0
	1	1	1



Two-input OR (+)

OR	A	B	Y
	0	0	0
	0	1	1
	1	0	1
	1	1	1



One-input NOT

(Complement, ')

NOT	A	Y
	0	1
	1	0

For an AND gate,
 0 at input blocks the other inputs
 and dominates the output
 1 at input passes signal A

For an OR gate,
 1 at input blocks the other inputs
 and dominates the output
 0 at input passes signal A