CSE 140: Components and Design Techniques for Digital Systems

Lecture 10:
Sequential Networks: Timing and Retiming

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Timing

• Motivation

• Basic Timing Characteristic
  – Gate Delay, Interconnect Delay
  – Flip-Flop Timing Window

• Two Timing Constraints: shortest and longest timing paths
  – Examples

• Clock Skews and Retiming
  – Examples
Timing: Motivation

- Clock specifies a precise time for the next state
  - In general, we allocate one clock period for signal propagation between registers. **Goldilocks** timing.
- Too late: Fail to reach for the **setup** of the next state.
- Too early: Race to disturb the **holding** of the next state.
- Analysis: Verify the timing of the system.
- Goal: A robust design.
The Story of Goldilocks and the Three Bears

Once upon a time, there was a little girl named **Goldilocks**. She went for a walk in the forest. Pretty soon, she came upon a house. She knocked and, when no one answered, she walked right in. At the table in the kitchen, there were three bowls of porridge. Goldilocks was hungry. She tasted the porridge from the first bowl. "This porridge is **too hot**!" she exclaimed. So, she tasted the porridge from the second bowl. "This porridge is **too cold**," she said. So, she tasted the last bowl of porridge. "Ahhh, this porridge is **just right**," she said happily and she ate it all up. **DLTK's Crafts for Kids**
Motivation: So far ....

Combinational

Logic-level analysis
Motivation: This lecture …

• When does our (seemingly logically correct) design go wrong?
• How can we design a circuit that works under real constraints?
• Popular interview questions.
A typical sequential network has combinational circuit between registers (R1 to R2).
The registers are synchronized by clocks (CLK1 and CLK2).
Timing is set between clocks (CLK1 and CLK2).
The beauty of the synchronized design is that we need only to take care of the timing of the regions separated by the registers.
Timing of the System

For a synchronized digital Moore machine, we take care of the timing of the following three groups of paths between:

- Every pair of registers,
- Each input and register, and
- Each register and output.
Basic Timing Characteristics: Gate Delay

I. Min delay of a gate, also called **Contamination delay**: $t_{cd}$
   Minimum time from when an input changes until the output *starts* to change

II. Max delay of a gate, also called **Propagation delay**: $t_{pd}$
   Maximum time from when an input changes until the output *is* guaranteed to reach its final value (i.e., stop changing)
Gate Delay: Combinational Logic Timing

Different situation causes different delay at output, e.g. input patterns, process variations, noises, temperature, aging

<table>
<thead>
<tr>
<th>$A_tB_t$</th>
<th>$A_{t+1}B_{t+1}$</th>
<th>$Y = (AB)'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>11</td>
<td>1⇒0</td>
</tr>
<tr>
<td>01</td>
<td>11</td>
<td>1⇒0</td>
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<td>10</td>
<td>11</td>
<td>1⇒0</td>
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<tr>
<td>11</td>
<td>11</td>
<td>0⇒0</td>
</tr>
</tbody>
</table>
Different path causes different output transition delay.
Interconnect Delay

Speed of light: $C/\sqrt{\varepsilon} \approx 1.5 \times 10^{10} \text{ cm/s}$

For 1cm, it takes $0.7 \times 10^{-10} \text{ s} = 70\text{ps}$ (picosecond) for the light to reach from one end to the other end.

Chain of buffers: 5-40 times of speed of light, i.e. 350-2,800ps. For 5GHz, the clock period is $200\text{ps/cycle}$. 
Combinational Logic: Output timing constraints

I. Contamination delay (shortest): $t_{cd}$
Minimum time from when an input changes until any output starts to change

II. Propagation delay (longest): $t_{pd}$
Maximum time from when an input changes until the output or outputs of a combinational circuit are guaranteed to reach their final value (i.e., stop changing)
Given a flip flop, its timing behavior is simulated/tested and specified with timing characteristics:

- $t_{\text{setup}}$, $t_{\text{hold}}$: timing relation between D and CLK
- $t_{\text{ccq}}$, $t_{\text{pcq}}$: timing relation between CLK and Q

No direct timing relation between input D and output Q
FF Input Constraints: Set up and hold time

Setup time $t_{\text{setup}}$ Time *before* the clock edge that data must be stable (i.e. no change)

Setup time violation This occurs if the input signal D does not settle (set up) to the stable value at least $t_{\text{setup}}$ before the clock edge.

Hold time $t_{\text{hold}}$ Time *after* the clock edge that data must be stable

Hold time violation This occurs if the input signal D does not remain unchanged (hold) for at least $t_{\text{hold}}$ after the clock edge.
FF Output Timing Constraints

- Propagation delay: \( t_{pcq} = \) time after clock edge that the output \( Q \) is guaranteed to be stable (i.e., to stop changing)
- Contamination delay: \( t_{ccq} = \) time after clock edge that \( Q \) might be unstable (i.e., start changing)
Two Timing Constraints

$\begin{align*}
t_{cq} + t_{comb} + t_{setup} & \leq T \\
t_{hold} & < t_{cq} + t_{comb}
\end{align*}$

$t_{cq}$: time from rising edge of clock to Q update ($CLK1 \Rightarrow B$)
$t_{comb}$: time of combinational logic delay ($B \Rightarrow C$)
$t_{setup}$: setup time before rising edge of clock ($C \Rightarrow CLK2$)
$t_{hold}$: hold time after the rising edge of clock
$T$: clock period ($CLK1 \Rightarrow CLK2$)
Two Timing Constraints

Setup time constraint
\[ t_{cq} + t_{comb} + t_{setup} \leq T \]

Longest delay from CLK1 to CLK2
\[ \max(t_{cq} + t_{comb} + t_{setup}) \leq T \]

Hold time constraint
\[ t_{hold} < t_{cq} + t_{comb} \]

Shortest delay from CLK1 to CLK2
\[ t_{hold} < \min(t_{cq} + t_{comb}) \]
Two Timing Constraints

\[ t_{cq} + t_{comb} + t_{setup} \leq T \]

\[ t_{hold} < t_{cq} + t_{comb} \]
Two Timing Constraints

\[ t_{cq} + t_{comb} + t_{setup} \leq T \]

\[ t_{hold} < t_{cq} + t_{comb} \]

Too long

Just right

Too short

CLK1

CLK2
Example: The timing of which of the following signals can cause a setup-time violation?

A. Signal D arrives too early
B. Signal D arrives too late
C. Clock CLK arrives too late
D. Output Q(t) responds too early
E. None of the above
Example: A hold time violation is likely to occur when

A. Signal D changes too early
B. Signal D changes too late
C. Clock CLK arrives too early
D. Clock CLK arrives too late
An alternate view of the sequential circuit

Combinational

R1
CLK

D1
R1
CLK

Q1

Combinational

D2

R2
CLK
What should happen within a clock cycle for correct functionality?
The delay between registers has a **minimum** and **maximum** delay, dependent on the delays of the circuit elements.

![Diagram of circuit elements](Image)
The delay between registers has a **minimum** and **maximum** delay, dependent on the delays of the circuit elements.

![Diagram showing the delay between registers with minimum and maximum delays.](image)
PI Q: Suppose CLK rises at \( t_1 \), what is the maximum delay (from \( t_1 \)) after which D2 reaches a stable value?

A. Setup time of R1 +
   Propagation delay of CL +
   Propagation delay of R2
B. Hold time of R1 + Propagation delay of CL + setup time of R1
C. Propagation delay of R1 +
   Propagation delay of CL +
   Propagation delay of R2
D. Propagation delay of R1 +
   Propagation delay of CL
E. Propagation delay of CL +
   Propagation delay of R2
Setup Time Constraint

- The setup time constraint depends on the maximum delay from register R1 through the combinational logic.
- The input to register R2 must be stable at least $t_{\text{setup}}$ before the clock edge.

Maximum delay, $t_{\text{max}}$

\[ t_{\text{max}} = \]

Setup Time Constraint:
Setup Time Constraint

\[ T \geq t_{pcq} + t_{pd} + t_{\text{setup}} \]

PI Q: As a designer, which of the following parameters would you modify to meet the set up time constraint?

A. The clock period, \( T \)
B. The prop. delay of R1, \( t_{pcq} \)
C. The prop. delay of CL, \( t_{pd} \)
D. The setup time of R2, \( t_{\text{setup}} \)
E. All of the above
Setup Time Constraint

\[ T \geq t_{pcq} + t_{pd} + t_{\text{setup}} \]
\[ t_{pd} \leq T - (t_{pcq} + t_{\text{setup}}) \]

PI Q: As a designer, which of the following parameters would you modify to meet the setup time constraint?

A. The clock period, \( T \)
B. The prop. delay of R1, \( t_{pcq} \)
C. The prop. delay of CL, \( t_{pd} \)
D. The setup time of R2, \( t_{\text{setup}} \)
E. All of the above
PI Q: Suppose CLK rises at $t_1$, what is the minimum delay (from $t_1$) after which D2 starts to change?

A. Setup time of R1 + propagation delay of CL + propagation of R2
B. Hold time of R1 + propagation time of CL + setup time of R1
C. Hold time of R1 + Contamination delay of CL + Propagation time of R2
D. Contamination delay of R1 + Contamination delay of CL
E. Contamination delay of CL + Contamination delay of R2
Hold Time Constraint

- The hold time constraint depends on the minimum delay from register R1 through the combinational logic.
- The input to register R2 must be stable for at least $t_{\text{hold}}$ after the clock edge.

**Minimum delay, $t_{\text{min}}$**

**Hold Time Constraint:**
Hold Time Constraint

\[ t_{\text{hold}} < t_{\text{ccq}} + t_{\text{cd}} \]

\[ t_{\text{cd}} > t_{\text{hold}} - t_{\text{ccq}} \]
Timing Analysis: Example

Timing Characteristics

FFs  \( t_{ccq} = 30 \text{ ps} \)
\( t_{pcq} = 50 \text{ ps} \)
\( t_{setup} = 60 \text{ ps} \)
\( t_{hold} = 70 \text{ ps} \)

Gates \( t_{pd}(g) = 35 \text{ ps} \)
\( t_{cd}(g) = 25 \text{ ps} \)

\( t_{pd}(\text{com}) = \)
\( t_{cd}(\text{com}) = \)

Setup time constraint: \( T_c \geq \)
\( f_c = 1/T_c = \)

Hold time constraint: \( t_{ccq} + t_{cd}(\text{com}) > t_{hold} ? \)
Timing Analysis: Example

FFs
- $t_{ccq} = 30 \text{ ps}$
- $t_{pcq} = 50 \text{ ps}$
- $t_{setup} = 60 \text{ ps}$
- $t_{hold} = 70 \text{ ps}$

Gates
- $t_{pd}(g) = 35 \text{ ps}$
- $t_{cd}(g) = 25 \text{ ps}$

Setup time constraint:

$$T \geq t_{pcq} + t_{pd}(\text{com}) + t_{setup}$$

$$= 50 + 105 + 60 = 215 \text{ ps}$$

$$f_c = 1/T_c = 4.65 \text{ GHz}$$

Hold time constraint:

$$t_{ccq} + t_{cd}(\text{com}) > t_{hold}?$$

$$(30 + 25) \text{ ps} > 70 \text{ ps} ?$$

No!
Example: Fix Hold Time Violation

Add buffers to the short paths:

Timing Characteristics

**FFs**
- \( t_{ccq} = 30 \text{ ps} \)
- \( t_{pcq} = 50 \text{ ps} \)
- \( t_{setup} = 60 \text{ ps} \)
- \( t_{hold} = 70 \text{ ps} \)

**Gates**
- \( t_{pd}(g) = 35 \text{ ps} \)
- \( t_{cd}(g) = 25 \text{ ps} \)

Setup time constraint:
- \( T \geq \)

Hold time constraint:
- \( t_{ccq} + t_{cd}(com) > t_{hold} \) ?
Example: Fix Hold Time Violation

Add buffers to the short paths:

\[ t_{pd}(\text{com}) = 3 \times 35 = 105 \text{ ps} \]
\[ t_{cd}(\text{com}) = 2 \times 25 = 50 \text{ ps} \]

Setup time constraint:
\[ T \geq 50 + 105 + 60 = 215 \text{ ps} \]
\[ f_c = 1/T_c = 4.65 \text{ GHz} \]

Gates
\[ t_{pd}(g) = 35 \text{ ps} \]
\[ t_{cd}(g) = 25 \text{ ps} \]

FFs
\[ t_{ccq} = 30 \text{ ps} \]
\[ t_{pcq} = 50 \text{ ps} \]
\[ t_{\text{setup}} = 60 \text{ ps} \]
\[ t_{\text{hold}} = 70 \text{ ps} \]

Hold time constraint:
\[ t_{ccq} + t_{cd}(\text{com}) > t_{\text{hold}} \]
\[ (30 + 50) \text{ ps} > 70 \text{ ps} \]
Yes!
Clock Skew

The clock doesn’t arrive at all registers at the same time. The difference between two clock edges is skew.

- **Skew as Noise:** Caused by process variation, voltage fluctuation, crosstalks (PVC). Examine the worst case to guarantee that the timing is right.

- **Designated Skew:** Make skew by design to improve the performance.
Time Constraint with Clock Skew (Noise)

In the worst case, the CLK2 is:
- Earlier than CLK1 for setup time
- Later than CLK1 for hold time.

\[ T_c \geq t_{pcq} + t_{pd}(\text{com}) + t_{\text{setup}} + t_{\text{skew}} \]

\[ t_{ccq} + t_{cd}(\text{com}) > t_{\text{hold}} + t_{\text{skew}} \]
Timing Analysis with Clock Skew: Example

Timing Characteristics

- \( t_{ccq} = 30 \text{ ps} \)
- \( t_{pcq} = 50 \text{ ps} \)
- \( t_{setup} = 60 \text{ ps} \)
- \( t_{hold} = 70 \text{ ps} \)
- \( t_{pd} = 35 \text{ ps} \)
- \( t_{cd} = 25 \text{ ps} \)
- \( t_{skew} = 10 \text{ ps} \)

\( t_{pd} = 3 \times 35 \text{ ps} = 105 \text{ ps} \)

\( t_{cd} = 25 \text{ ps} \)

Setup time constraint:

\( T_c \geq 225 \text{ ps} \)

\( f_c = \frac{1}{T_c} = 4.44 \text{ GHz} \)

Without skew we got \( f_c = 4.65 \text{ GHz} \)
Time Constraint with Clock Skew: Example

- In the worst case for hold time, CLK2 is later than CLK1

\[ t_{ccq} + t_{cd}(\text{com}) > t_{\text{hold}} + t_{\text{skew}} \]
Clock Skew: Example

Timing Characteristics

- $t_{ccq} = 30$ ps
- $t_{pcq} = 50$ ps
- $t_{setup} = 60$ ps
- $t_{hold} = 70$ ps
- $t_{pd}(g) = 35$ ps
- $t_{cd}(g) = 25$ ps

$t_{skew} = 50$ ps

$\begin{align*}
t_{pd}(\text{com}) &= 3 \times 35 \text{ ps} = 105 \text{ ps} \\
t_{cd}(\text{com}) &= 2 \times 25 \text{ ps} = 50 \text{ ps}
\end{align*}$

Hold time constraint:

$\begin{align*}
t_{ccq} + t_{cd}(\text{com}) &> t_{hold} + t_{skew} \\
(30 + 50) &> (70 + 50) \text{ ps}
\end{align*}$
Retiming with **Designated Skew**

\[
T \geq t_{pcq} + t_{pd}(\text{com}) + t_{\text{setup}} + t_{\text{skew}}
\]

\[
t_{ccq} + t_{cd}(\text{com}) > t_{\text{hold}} + t_{\text{skew}}
\]

**Skew as noise (worst case)**

\[
t_{CLK2} = t_{CLK1} \pm t_{\text{skew}}
\]

**Designated skew**

\[
t_{CLK2} = t_{CLK1} + t_{\text{skew}}
\]

\[
t_{ccq} + t_{cd}(\text{com}) > t_{\text{hold}} + t_{\text{skew}}
\]
Retiming: Example

\[ T_c \geq t_{pcq} + t_{pd}(\text{com}) + t_{setup} - t_{skew} \]

\[ t_{ccq} + t_{cd}(\text{com}) \geq t_{hold} + t_{skew} \]

\[ T_c \geq 50 + 105 + 60 - t_{skew} \]

\[ 30 + 50 \geq 70 + t_{skew} \]

Example: The minimum clock period \( T \) can be:

A. 195
B. 205
C. 215
D. None of the above

\( t_{ccq} = 30 \text{ ps} \)
\( t_{pcq} = 50 \text{ ps} \)
\( t_{setup} = 60 \text{ ps} \)
\( t_{hold} = 70 \text{ ps} \)
\( t_{pd}(g) = 35 \text{ ps} \)
\( t_{cd}(g) = 25 \text{ ps} \)
Timing and Retiming

• Retiming: Adjust the clock skew so that the clock period can be reduced.
• Add a few more examples on timing and retiming.
Conclusion

- Clock to Clock: Range of shortest and longest paths
- Design revision and retiming to adjust the constraints
- Research: Variation aware designs

Extra materials: