

Combinational Logic.

✓ 1. CMOS Logic.

2. Boolean Algebra ✓ Consensus Theorem

3. Switching Functions Shannon's Expansion

Truth Table

✓ K-Map

Universal Gates

✓ Various types of Gates

Adder / Subtractor

Bit Counter

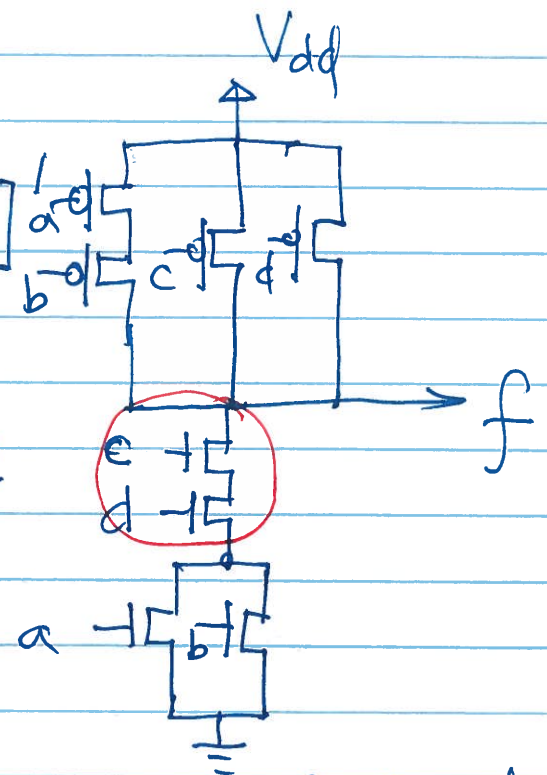
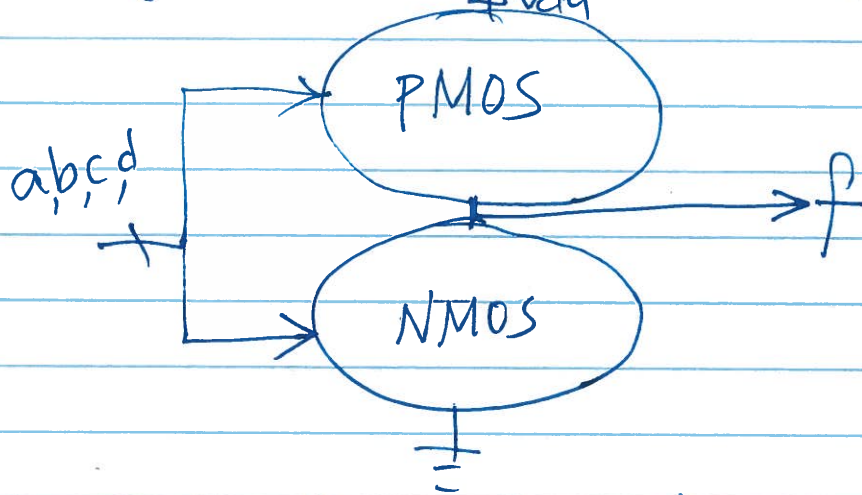
Majority Functions

NAND/NOR

XOR/XNOR

CMOS Logic

① $f(a,b,c,d) = [(a+b)cd]'$



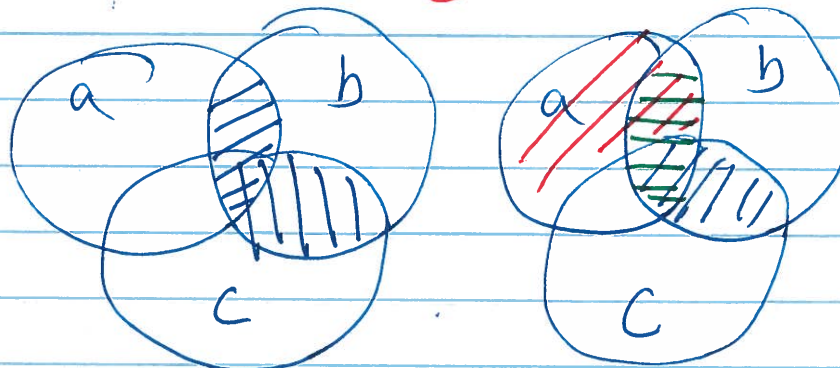
$$f=0 \text{ if } cd=1 \text{ and } a \text{ or } b=1 \Rightarrow (a+b)cd=1$$

$$f=1 \text{ if } a' \cdot b' = 1 \text{ or } c'=1 \text{ or } d'=1$$

$$d' \cdot b' + c' + d' = [(a+b)cd]'$$

Consensus Theorem

$$f(a,b,c) = ab + \underline{bc} + \underline{ac'}$$



$$(a'+b)c + (d+b)c'$$

consensus is $(a'+b)(d+b)$

Ex: $a + \underline{a'b} + \underline{b} = a + b$, Ex: $(a+b+c')(a+d+c) = (a+b+c')(a+d+c)(\underline{a+b+d})$

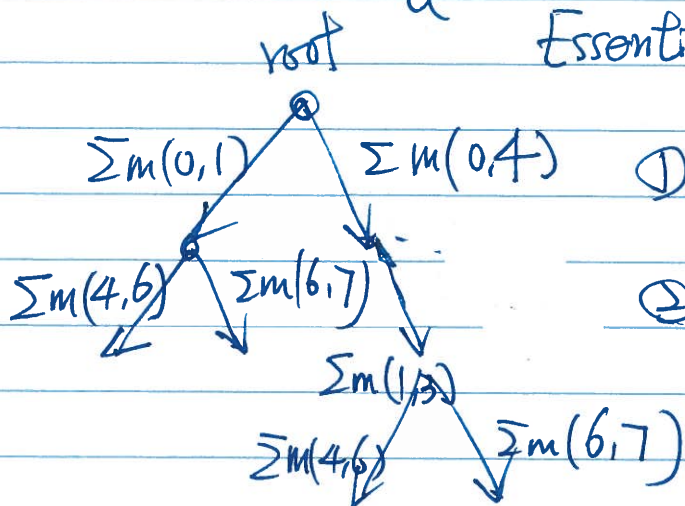
K-Map: $f(a,b,c) = \sum m(0,1,6) + \sum d(3,4,7)$

SOP

	<u>b</u>				
	0	1	2	3	4
c	1	1	3	7	5
	<u>a</u>				

Primes: $\sum m(0,1), \sum m(0,4)$
 $\sum m(4,6), \sum m(6,7)$
 $\sum m(1,3)$

Essential Primes: None.



① $\sum m(0,1), \sum m(4,6)$
 $f_1(a,b,c) = a'b' + ac'$

② $\sum m(0,1), \sum m(6,7)$
 $f_2(a,b,c) = a'b' + ab$

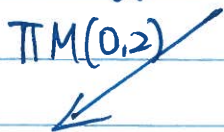
K-Map: $f(a,b,c) = \sum m(1,3,4) + \sum d(2,5)$

POS

	<u>b</u>				
	0	1	2	3	4
c	1	1	3	7	5
	<u>a</u>				

Primes: $\pi M(0,2), \pi M(2,6)$
 $\pi M(6,7), \pi M(5,7)$

Essential Prime: $\pi M(0,2)$



$\pi M(0,2), \pi M(6,7)$
 $(a+c)(a' + b')$

$\oplus$ gates

$$f(x, y) = (xy) \oplus (x' + y') \oplus y \oplus x \oplus (xy) \oplus (x'y')$$

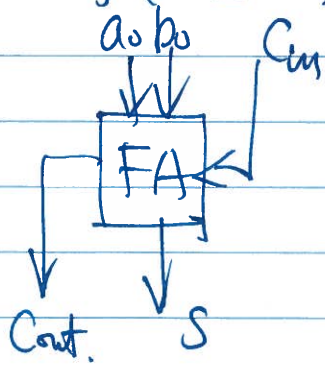
$$f(1, y) = (1 \cdot y) \oplus (1' + y') \oplus y \oplus 1 \oplus (1 \cdot y) \oplus (1'y')$$
$$= y \oplus y' \oplus \cancel{y} \oplus 1 \oplus \cancel{y} \oplus \cancel{0}$$

$$= 1 \oplus 1 = 0$$

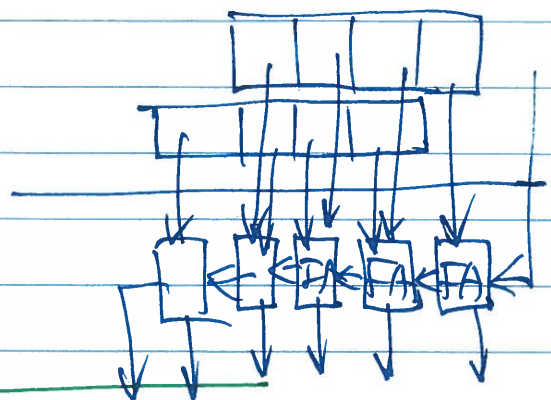
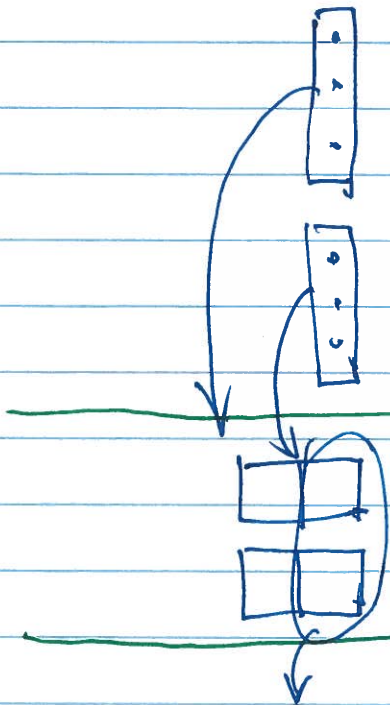
$$f(0, y) = (0 \cdot y) \oplus (0' + y') \oplus y \oplus 0 \oplus (0 \cdot y) \oplus (0'y')$$
$$= \cancel{0} \oplus 1 \oplus y \oplus \cancel{0} \oplus \cancel{0} \oplus y'$$
$$= 1 \oplus 1 = 0$$

$$f(x, y) = x f(1, y) + x' f(0, y) = 0.$$

Bit Counter $f(a_2, a_1, a_0) = (y_1, y_0)$



$$\begin{array}{r}
 10101 \\
 \times 11001 \\
 \hline
 10101 \\
 000000 \\
 000000 \\
 101011 \\
 101011 \\
 + 101011 \\
 \hline
 \hline
 \end{array}$$



Majority Function

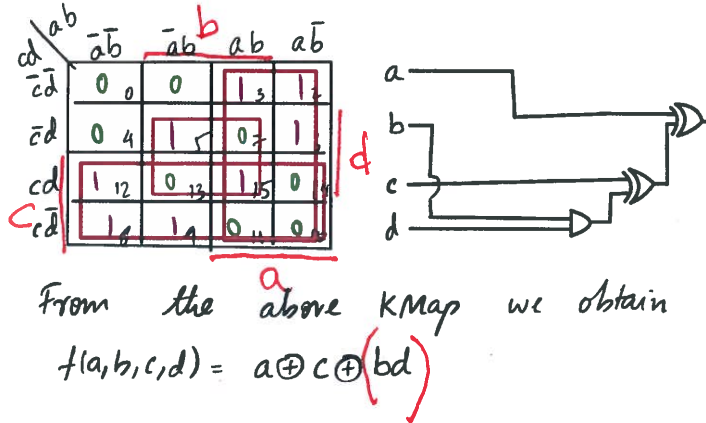
$f(a_1, a_2, \dots, a_n) = 1$ if majority of a_i are true.

$= 0$ otherwise.

Universal Set $\{ \text{Majority function, Inverter} \}$

B3

B.3. Karnaugh Map: Given a four input combination function $f(a, b, c, d)$, we have On-Set $F = \sum m(2, 3, 5, 6, 8, 9, 12, 15)$ and an empty Don't-Care set. Implement the function with {AND, XOR, NOT} gates only (no other gates). Assuming that we can have multiple (more than 2) input gates and NOT gates are free. Design with a minimal networks in terms of the total number of AND, XOR gates. Draw a logic diagram to illustrate the design. (5pts)



B4

3.4.1. Given a three-input, three-output quantum gate $f(a, b, c) = (a, b, (ab) \oplus c)$, can $\{f(a, b, c)\}$ be a universal set? State your choice of Yes or No. Explain your choice.

Yes

- AND: $f(a, b, 0) = (a, b, ab)$
- NOT: $f(1, 1, c) = (1, 1, 1 \oplus c) = (1, 1, c')$
- OR: De Morgan's Laws.

3.4.2. Given the output of the quantum gate in B.5.1, can we recover the input? State your choice of Yes or No. If Yes, provide your formula to recover the input. If No, explain your choice.

Yes. $f(a, b, c) = (x, y, z) \implies g(x, y, z) = (x, y, (xy) \oplus z)$.

Note here, $x=a$, $y=b$, $z=ab \text{ xor } c$, thus $xy \text{ xor } z = c$.