Introduction

The purpose of this assignment is to check Moore’s law, practice CMOS logic design style and Boolean algebra. For CMOS logic design style, we implement a Boolean function and show the design with a schematic diagram. For Boolean algebra, first, the exercise is to help you practice the application of Boolean algebra theorems to transform and reduce Boolean expressions. The second goal is to help you learn how to go from the world of Boolean expressions to the world of digital circuits. The final goal is to help you translate a problem described in Boolean algebraic expression to digital logic. We hope you can think of why each of these exercises is useful when designing digital circuits.

1 Moore’s Law

List three largest VLSI products (chips) in the last three years. Check if the transistor counts of these three products fit the curve in Lecture 1, Slide #19 according to Moore’s law.

2 Implementation Of Switching Function Using CMOS Logic

Given a circuit with three binary inputs \((a, b, c, d)\) and a binary output function \(y(a, b, c, d) = ((a + b)c + d)’\), implement the function using CMOS logic technology. Draw the design with a schematic diagram.

3 Application Of Boolean Algebra And Shannon Expansion

3.1 Prove the following using Boolean algebra.

A. \(a + 1 = 1\).

B. Following the proof of A, show that using the duality of Boolean algebra, you can derive \(a \cdot 0 = 0\).

C. \((be + d)(a + c + d')(a + be + c) = (be + d)(a + c + d')\)
3.2 Prove the above three equations using Shannon’s expansion.

4 Prove Or Disprove The Following Statements In Boolean Expressions

A. If $a = bc$ and $a = b + c$, then $b = c$.
B. If $a = bc$ and $a' = b' + c'$, then $b = c$.
C. $(a + bc + d)(b' + c' + ef + g)(ef + g) = (a + d)(ef + g)$

5 Translate From A Digital Problem To Boolean Expressions

A node has four connecting pipes. The four pipes are controlled by four switches $(a_3, a_2, a_1, a_0)$. If $a_i = 1$, pipe $i$ is open, otherwise $a_i = 0$ and pipe $i$ is closed.

A. For all the time, two and exactly two pipes are open, and the other two are closed. Write a Boolean expression to formulate the constraint.
B. For all the time, the number of pipes that are open cannot be an odd number. Write a Boolean expression to formulate the constraint.

6 Boolean Algebra And Implementation

A. Simplify each of the following two Boolean equations (using Boolean algebra, in particular consensus theorem). Write the results in Boolean expressions.
B. Convert the simplified expressions with logic using $AND$, $OR$ gates with bubbles for (True, False) inversion. Draw the logic diagram.
C. List the numbers of literals and operators versus the numbers of gates, nets, and pins in the schematic diagrams
   i. $y(a, b, c, d, e) = a(a' + b)(a' + b' + c)(a' + b' + c' + d)(a' + b' + c' + d' + e)$
   ii. $y(a, b, c, d, e, f, g, h) = a'b + ab'(c'd + cd')(ef' + ef(gh' + gh'))$

Assignment Turn-in

Once you’re finished with the assignment, turn it in as a pdf on Gradescope.