CSE140 Discussion 1

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Agenda

- CMOS and Logic Gates
- Boolean Algebra
- Practice Examples
CMOS

- Acts like a switch in electrical circuit
## Boolean Algebra

<table>
<thead>
<tr>
<th></th>
<th>Expression</th>
<th>Identity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Associative Law</strong></td>
<td>((a+b)+c = a+(b+c))</td>
<td>((ab)c = a(bc))</td>
</tr>
<tr>
<td><strong>Commutative Law</strong></td>
<td>(a+b = b+a)</td>
<td>(ab = ba)</td>
</tr>
<tr>
<td><strong>Distributive Law</strong></td>
<td>((a+b)(a+c) = a+bc)</td>
<td>((ab)+(ac) = a(b+c))</td>
</tr>
<tr>
<td><strong>Identity Law</strong></td>
<td>(a+0 = a)</td>
<td>(a\cdot1 = a)</td>
</tr>
<tr>
<td><strong>Complement Law</strong></td>
<td>(a+a' = 1)</td>
<td>(a\cdot a' = 0)</td>
</tr>
</tbody>
</table>

**Duality (swap +/- s and 1/0s)**
Example 1

Draw both a logic gate diagram and CMOS circuit for \( A + BC \)
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Example 2

How many transistors are in this circuit? How many transistors are in NAND, AND, NOR, OR, and NOT gates?
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Total: 12T
Example 2

Can we improve the transistor count?
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Can we improve the transistor count?

Can try to reduce further, but try to keep pMOS/VDD on top and nMOS/Vss on bottom per convention.
Example 3

Still using the boolean expression from earlier: $A + BC$

- What are the literals?
- Is this an implicant (product of literals)? Implicate (sum of literals)?
- Could it be considered a minterm or maxterm of $f(A,B,C)$?
Example 3

Still using the boolean expression from earlier: $A + (BC)$

- What are the literals? 
  
  $A, B, C$

- Is this an implicant (product of literals)? Implicate (sum of literals)?

- Could it be considered a minterm or maxterm of $f(A, B, C)$?
Example 3

Still using the boolean expression from earlier: $A + BC$

- What are the literals?
  - $A, B, C$

- Is this an implicant (product of literals)? Implicate (sum of literals)?
  - Neither, as this expression contains both + and ∙.

- Could it be considered a minterm or maxterm of $f(A, B, C)$?
Example 3

Still using the boolean expression from earlier: A+BC

- What are the literals?
  - A, B, C

- Is this an implicant (product of literals)? Implicate (sum of literals)?
  - Neither, as this expression contains both + and \cdot

- Could it be considered a minterm or maxterm of f(A,B,C)?
  - No, minterms or maxterms are implicants or implicates containing all literals
Example 4

Show that $YQX + X'QYZ' + XZQY' = (Y(X+Z') + XZ)Q$
Example 4

Show that $YQX+X'QYZ'+XZQY'=(Y(X+Z')+XZ)Q$

Right hand side (RHS)

$=QY(X+Z')+QXZ$ (Distributivity)

$=QXY+QYZ'+QXZ$ (Distributivity)
Example 4

Show that $YQX+X'QYZ'+XZQY'=\overline{(Y(X+Z')+XZ)Q}$

Right hand side (RHS)

$=QY(X+Z')+QXZ$ (Distributivity)

$=QXY+QYZ'+QXZ$ (Distributivity)

$=QXY(1)+Q(1)YZ'+QX(1)Z$ (Identity)
Example 4

Show that \( YQX + X'QYZ' + XZQY' = (Y(\overline{X} + Z') + XZ)Q \)
Right hand side (RHS)
=\( QY(\overline{X} + Z') + QXZ \) (Distributivity)
=\( QXY + QYZ' + QXZ \) (Distributivity)
=\( QXY(1) + Q(1)YZ' + QX(1)Z \) (Identity)
=\( QXY(\overline{Z} + Z') + Q(X + X')YZ' + QX(Y + Y')Z \) (Complements)
Example 4

Show that $YQX + X'QYZ' + XZQY' = (Y(X+Z') + XZ)Q$

Right hand side (RHS)

$= QY(X+Z') + QXZ$ (Distributivity)

$= QXY + QYZ' + QXZ$ (Distributivity)

$= QXY(1) + Q(1)YZ' + QX(1)Z$ (Identity)

$= QXY(Z+Z') + Q(X+X')YZ' + QX(Y+Y')Z$ (Complements)

$= QXYZ + QXYZ' + QXYZ' + QX'YZ' + QXYZ + QXY'Z$ (Distributivity)
Example 4

Show that \( YQX + X'QYZ' + XZQY' = (Y(X+Z')+XZ)Q \)

Right hand side (RHS)
\[ = QY(X+Z')+QXZ \text{ (Distributivity)} \]
\[ = QXY+QYZ'+QXZ \text{ (Distributivity)} \]
\[ = QXY(1)+Q(1)YZ'+QX(1)Z \text{ (Identity)} \]
\[ = QXY(Z+Z')+Q(X+X')YZ'+QX(Y+Y')Z \text{ (Complements)} \]
\[ = QXYZ+QXYZ'+QXYZ'+QX'YZ'+QX'Y'Z \text{ (Distributivity)} \]
\[ = QXYZ+QXYZ'+QX'YZ'+QXY'Z \text{ (Idempotency)} \]

LHS = \( YQX + X'QYZ' + XZQY' \)
\[ QXY + QX'YZ' + QXY'Z \]
Example 4

Show that \( YQX + X'QYZ' + XZQY' = (Y(X+Z') + XZ)Q \)

Right hand side (RHS)

\[
= QY(X+Z')+QXZ \quad \text{(Distributivity)} \\
= QXY + QYZ'+QXZ \quad \text{(Distributivity)} \\
= QXY(1)+Q(1)YZ'+QX(1)Z \quad \text{(Identity)} \\
= QXY(Z+Z')+Q(X+X')YZ'+QX(Y+Y')Z \quad \text{(Complements)} \\
= QXYZ+QXYZ'+QX'YZ'+QXYZ+QXY'Z \quad \text{(Distributivity)} \\
= QXYZ+QXYZ'+QX'YZ'+QXY'Z \quad \text{(Idempotency)} \\
= QXY+QX'YZ'+QXY'Z \quad \text{(Combining)} \\
= \text{LHS}
\]
Questions