

# CSE 140 Discussion 2

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# Announcements

- HW #1 due next Wednesday 4/13 at 11:59 PM
- HW #2 out, due Wednesday 4/20 at 11:59 PM
- Midterm 1 coming up soon

# Today's Agenda

- Consensus Theorem
- Shannon's Expansion
- Full Adder and Subtractor
- Karnaugh Maps

# Consensus Theorem

$$AB + A'C + BC = AB + A'C$$

$$\text{Duality: } (A + B)(A' + C)(B + C) = (A + B)(A' + C)$$

$$\text{Proof: } AB + A'C + BC = AB + A'C + (1)BC$$

$$= AB + A'C + (A + A')BC$$

$$= AB + A'C + ABC + A'BC$$

$$= AB(1 + C) + A'C(1 + B)$$

$$= AB + A'C$$

# Consensus Examples

1.  $abe + a'bcd + b'd$ 
  - Consensus of  $a$  and  $a'$  ( $abe$ ,  $a'bcd$ ) is  $bcde$
  - Consensus of  $b$  and  $b'$  ( $abe$ ,  $b'd$ ) is  $ade$
  - Consensus of  $b$  and  $b'$  ( $a'bcd$ ,  $b'd$ ) is  $a'cd$
2.  $(a + c'd')(a' + bc'd')(bc'd' + c'd') = (a + c'd')(a' + bc'd')$ 
  - Recall  $(A + B)(A' + C)(B + C) = (A + B)(A' + C)$
  - $A = a$ ,  $B = c'd'$ ,  $A' = a'$ ,  $C = bc'd'$
  - Consensus of  $a$  and  $a'$  is  $(bc'd' + c'd')$

# Shannon's Expansion

- Choose a variable to decompose the equation into simpler terms

Sum of Products Form:  $f(x, y, z) = x f(1, y, z) + x' f(0, y, z)$

Product of Sums Form:  $f(x, y, z) = (x + f(0, y, z))(x' + f(1, y, z))$

- If we were to choose  $y$ , we would have:
  - $f(x, y, z) = y f(x, 1, z) + y' f(x, 0, z)$
  - $f(x, y, z) = (y + f(x, 0, z))(y' + f(x, 1, z))$

# Shannon's Expansion Example #1

$$f(x, y, z) = xyz + x'y + z'y$$

$$f(x, y, z) = x f(1, y, z) + x' f(0, y, z)$$

$$= x(yz + 0 + z'y) + x'(0 + y + z'y)$$

$$= x(y(z+z')) + x'(y(1+z'))$$

$$= xy + x'y$$

$$= y$$

# Shannon's Expansion Example #2

$$f(x, y, z) = (x + y + z') (x' + y' + z) (z + y)$$

$$f(x, y, z) = (x + f(0, y, z))(x' + f(1, y, z))$$

$$= (x + (y + z')(1)(z + y))(x' + (1)(y' + z)(z + y))$$

$$= (x + (y + z')(y + z))(x' + (y' + z)(y + z))$$

$$= (x + y)(x' + z)$$

# Full Adder

- Takes 1-bit A, B,  $C_{in}$
- Outputs  $C_{out}$ , S
- $C_{in}$  = Carry In
- $C_{out}$  = Carry Out
- $S = (A+B+C_{in})\%2$ 
  - If S exceeds 1,  
then  $C_{out} = 1$

| id | A | B | $C_{in}$ | $C_{out}$ | S |
|----|---|---|----------|-----------|---|
| 0  | 0 | 0 | 0        | 0         | 0 |
| 1  | 0 | 0 | 1        | 0         | 1 |
| 2  | 0 | 1 | 0        | 0         | 1 |
| 3  | 0 | 1 | 1        | 1         | 0 |
| 4  | 1 | 0 | 0        | 0         | 1 |
| 5  | 1 | 0 | 1        | 1         | 0 |
| 6  | 1 | 1 | 0        | 1         | 0 |
| 7  | 1 | 1 | 1        | 1         | 1 |

# Full Subtractor

- Takes 1-bit A, B,  $B_{in}$
- Outputs  $B_{out}$ , D
- $B_{in}$  = Borrow In
- $B_{out}$  = Borrow Out
- $D = A - B - B_{in}$ 
  - If we ever have to perform  $0 - 1$ , then  $B_{out} = 1$

| id | A | B | $B_{in}$ | $B_{out}$ | D |
|----|---|---|----------|-----------|---|
| 0  | 0 | 0 | 0        | 0         | 0 |
| 1  | 0 | 0 | 1        | 1         | 1 |
| 2  | 0 | 1 | 0        | 1         | 1 |
| 3  | 0 | 1 | 1        | 1         | 0 |
| 4  | 1 | 0 | 0        | 0         | 1 |
| 5  | 1 | 0 | 1        | 0         | 0 |
| 6  | 1 | 1 | 0        | 0         | 0 |
| 7  | 1 | 1 | 1        | 1         | 1 |

# Karnaugh Maps

- Reduce boolean expressions by writing the entries of the truth table and reducing the expression to either SOP or POS terms
- Row and column entries ordered in **gray code** (i.e. 00, 01, 11, 10)
- **Prime Implicant**: Obtained by combining all possible maximum number of squares/cells
- **Essential Prime Implicant**: Prime Implicant is essential if it is the only prime implicant that only covers the minterm
  - **Must be included in every simplified boolean expression**

Practice Resource: <http://www.32x8.com/index.html>

# 3 way K-map

$$f(a,b,c) = \sum_m(0, 2, 4, 5) = ab' + a'c' + b'c'$$

| C/AB | 00 | 01 | 11 | 10 |
|------|----|----|----|----|
| 0    | 1  | 1  | 0  | 1  |
| 1    | 0  | 0  | 0  | 1  |

Prime  
Implicants:

$AB'$ ,  $A'C'$ ,  $B'C'$

Essential Prime  
Implicants:

$AB'$ ,  $A'C'$

# 3 way K-map with Don't Cares

$$f(a,b,c) = \sum_m(0, 2, 4, 5) + \sum_d(1, 6) = b' + c'$$

| C/AB | 00 | 01 | 11 | 10 |
|------|----|----|----|----|
| 0    | 1  | 1  | X  | 1  |
| 1    | X  | 0  | 0  | 1  |

Prime  
Implicants:  
 $b', c'$

Essential Prime  
Implicants:  
 $b', c'$

# 3 way K-map for Full Adders

K-map for S:

| $C_{in}/AB$ | 00 | 01 | 11 | 10 |
|-------------|----|----|----|----|
| 0           | 0  | 1  | 0  | 1  |
| 1           | 1  | 0  | 1  | 0  |

$$S = A'B'C_{in} + A'BC_{in}' + ABC_{in} + AB'C_{in}'$$

K-map for  $C_{out}$ :

| $C_{in}/AB$ | 00 | 01 | 11 | 10 |
|-------------|----|----|----|----|
| 0           | 0  | 0  | 1  | 0  |
| 1           | 0  | 1  | 1  | 1  |

$$C_{out} = BC_{in} + AB + AC_{in}$$

# Simplify Summation Term

Recall

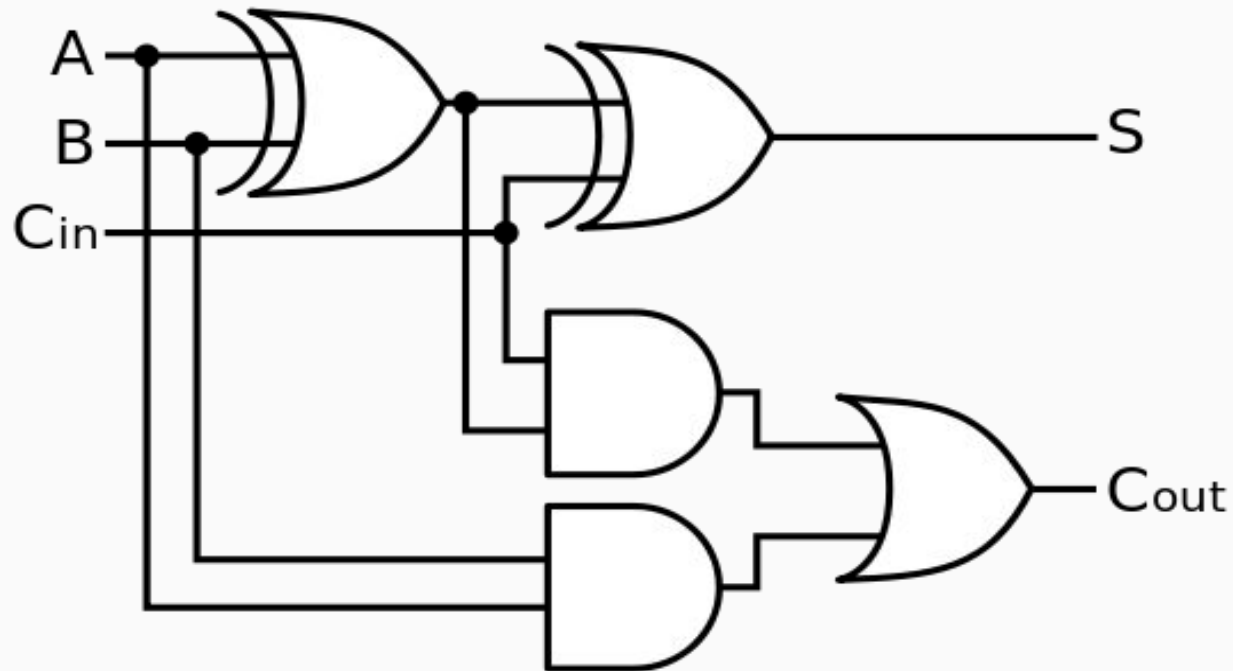
$$AB' + A'B = A \text{ XOR } B,$$

$$AB + A'B' = A \text{ XNOR } B = (A \text{ XOR } B)'$$

Then:

$$\begin{aligned} S &= A'B'C_{in} + A'BC_{in}' + ABC_{in} + AB'C_{in}' \\ &= B(AC + A'C_{in}') + B'(A'C_{in} + AC_{in}') \\ &= B(A \text{ XOR } C_{in})' + B'(A \text{ XOR } C_{in}) \\ &= B \text{ XOR } (A \text{ XOR } C_{in}) \\ &= B \text{ XOR } A \text{ XOR } C_{in} \end{aligned}$$

# Full Adder Diagram



# 3 way K-map for Full Subtractors

K-map for D:

| $B_{in}/AB$ | 00 | 01 | 11 | 10 |
|-------------|----|----|----|----|
| 0           | 0  | 1  | 0  | 1  |
| 1           | 1  | 0  | 1  | 0  |

$$D = A'B'B_{in} + A'BB_{in}' + ABB_{in} + AB'B_{in}'$$

K-map for  $B_{out}$ :

| $B_{in}/AB$ | 00 | 01 | 11 | 10 |
|-------------|----|----|----|----|
| 0           | 0  | 1  | 0  | 0  |
| 1           | 1  | 1  | 1  | 0  |

$$B_{out} = A'B_{in} + A'B + BB_{in}$$

# Simplify Difference Term

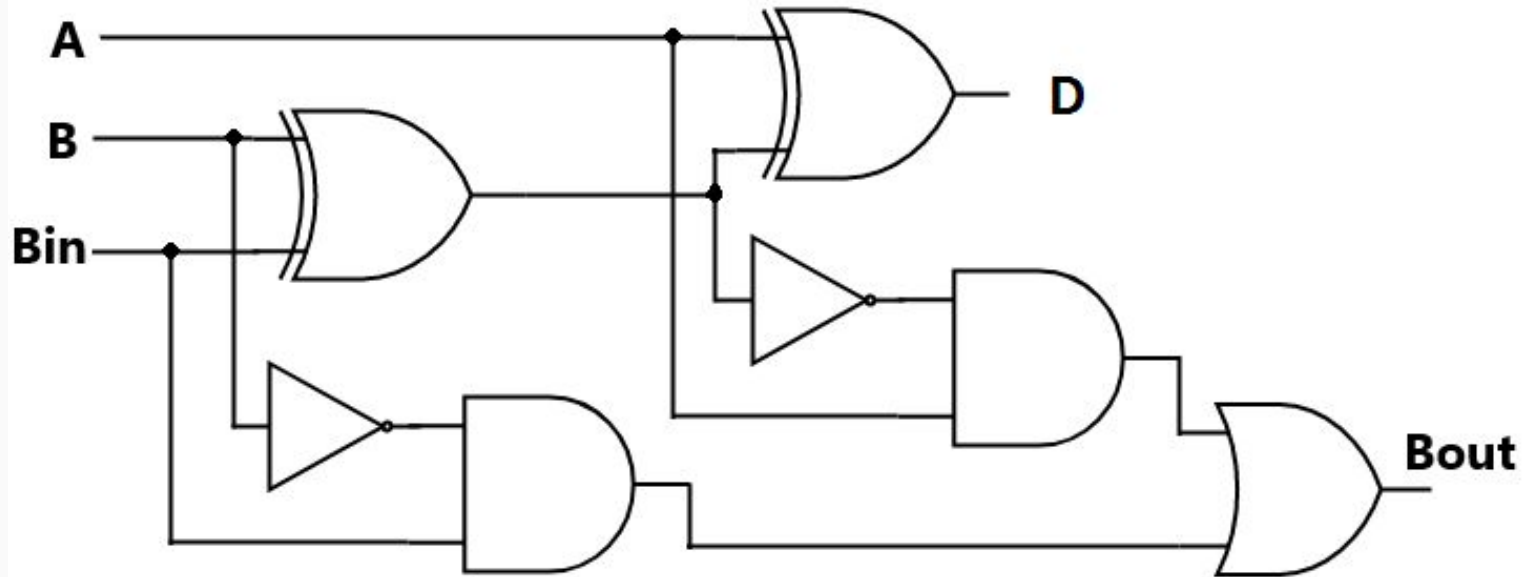
Recall

$$AB' + A'B = A \text{ XOR } B,$$

$$AB + A'B' = A \text{ XNOR } B = (A \text{ XOR } B)'$$

$$\begin{aligned} D &= A'B'B_{in} + A'BB_{in}' + ABB_{in} + AB'B_{in}' \\ &= A(BB_{in} + B'B_{in}') + A'(B'B_{in} + BB_{in}') \\ &= A(B \text{ XOR } B_{in})' + A'(B \text{ XOR } B_{in}) \\ &= A \text{ XOR } B \text{ XOR } B_{in} \end{aligned}$$

# Full Subtractor Diagram



# 4 way K-map

$$f(a,b,c, d) = \sum_m(0, 1, 8, 9, 10) + \sum_d(2, 4, 5, 11, 13)$$

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | X  | 0  | 1  |
| 01      | 1  | X  | X  | 1  |
| 11      | 0  | 0  | 0  | X  |
| 10      | X  | 0  | 0  | 1  |

Prime

Implicants:

$a'c'$ ,  $b'd'$ ,  $b'c'$ ,  $c'd$ ,  
 $ab'$

Essential Prime

Implicants:

None

# 4 way K-map

$$f(a,b,c, d) = \sum_m(0, 1, 8, 9, 10) + \sum_d(2, 4, 5, 11, 13)$$

| <b>cd \ ab</b> | 00 | 01 | 11 | 10 |
|----------------|----|----|----|----|
| 00             | 1  | X  | 0  | 1  |
| 01             | 1  | X  | X  | 1  |
| 11             | 0  | 0  | 0  | X  |
| 10             | X  | 0  | 0  | 1  |

$$f = b'd' + b'c'$$

$$f = b'c' + ab'$$

$$f = c'd + b'd'$$

$$f = ab' + b'c'$$