

Lecture 5: K-Map minimization in larger input dimensions and K-map minimization using max terms

CSE 140: Components and Design Techniques for Digital Systems

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Part I. Combinational Logic

1. Specification
2. Implementation

K-map:

Sum of products

Product of sums

Definitions: Sum of Products and Product of Sums Expressions

Sum of Products Expression

Implicant: A **product term** that has non-empty intersection with on-set $\mathcal{F}$ and does not intersect with off-set $\mathcal{R}$.

Prime Implicant: An **implicant** that is not a proper subset of any other **implicant**.

Essential Prime Implicant: A prime **implicant** that has an element in on-set $\mathcal{F}$ but this element is not covered by any other prime **implicants**.

Product of Sums Expression

Implicate: A **sum term** that has non-empty intersection with off-set $\mathcal{R}$ and does not intersect with on-set $\mathcal{F}$.

Prime Implicate: An **implicate** that is not a proper subset of any other implicate.

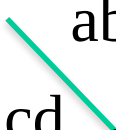
Essential Prime Implicate: A prime **implicate** that has an element in off-set $\mathcal{R}$ but this element is not covered by any other prime **implicates**.

K-Map to Minimized Product of Sums

- Sometimes it is easier to reduce the K-map by considering the off-set

$$F_1 = a'b'c'd + abc'd + a'b'cd' + abcd'$$

$$F_2 = (a + b')(a' + b)(c' + d')(c + d)$$



ab \ cd	00	01	11	10
00	0	0	0	0
01	1	0	1	0
11	0	0	0	0
10	1	0	1	0

Q: Which function is simpler for a two-level logic implementation?

A. F_1

B. F_2

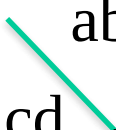
C. Two are the same

K-Map to Minimized Product of Sums

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ab \ cd	00	01	11	10
00	0	0	0	0
01	1	0	1	0
11	0	0	0	0
10	1	0	1	0

Another min product of sums example

Given $\mathcal{R}(a,b,c,d) = \Sigma m(3, 11, 12, 13, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m(4, 8, 10)$

K-map

		ab			
		00	01	11	10
cd	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Another min product of sums example

Given $\mathcal{R}(a,b,c,d) = \Sigma m(3, 11, 12, 13, 14)$
 $\mathcal{D}(a,b,c,d) = \Sigma m(4, 8, 10)$

Is $\pi M(12,14)$ a prime implicate?

A: Yes

B: No

		ab			
		00	01	11	10
cd	00	⁰ 1	⁴ X	¹² 0	⁸ X
	01	¹ 1	⁵ 1	¹³ 0	⁹ 1
	11	³ 0	⁷ 1	¹⁵ 1	¹¹ 0
	10	² 1	⁶ 1	¹⁴ 0	¹⁰ X
		a			

d

Prime Implicates:

$\Pi M(0,8)$, $\Pi M(11,15)$, $\Pi M(12,13,14,15)$, $\Pi M(6,14)$

Q: Which of the following is a not an essential prime implicate?

A. $\Pi M(0,8)$

B. $\Pi M(11,15)$

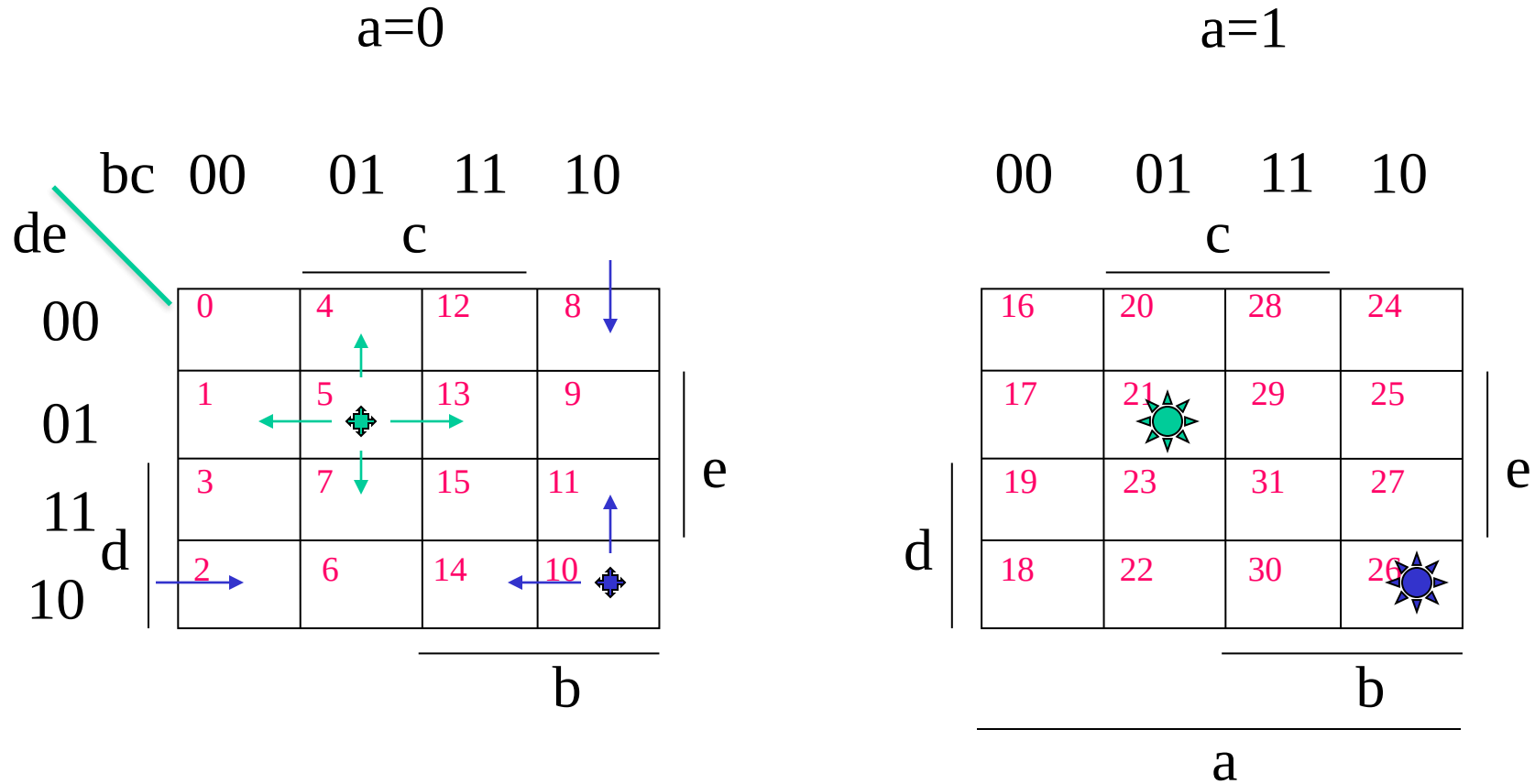
C. $\Pi M(12,13,14,15)$

D. $\Pi M(6,14)$

		ab			
cd		00	01	11	10
	00	0 ⁰ 0	4 ⁴ 1	12 ¹² 0	8 ⁸ X
	01	1 ¹ 1	5 ⁵ 1	13 ¹³ 0	9 ⁹ 1
	11	3 ³ 1	7 ⁷ 1	15 ¹⁵ 0	11 ¹¹ 0
	10	2 ² 1	6 ⁶ X	14 ¹⁴ 0	10 ¹⁰ 1
		a			

d

Five variable K-map



Neighbors of m_5 are: minterms 1, 4, 7, 13, and 21

Neighbors of m_{10} are: minterms 2, 8, 11, 14, and 26

Reading a Five variable K-map: An example

a=0

		bc			
		00	01	11	10
		c			
de	00	0 1	4	12 1	8 1
	01	1	5 1	13 1	9
	11	3	7	15 1	11 1
	10	2 1	6	14 1	10 1
		b			

e

d

a=1

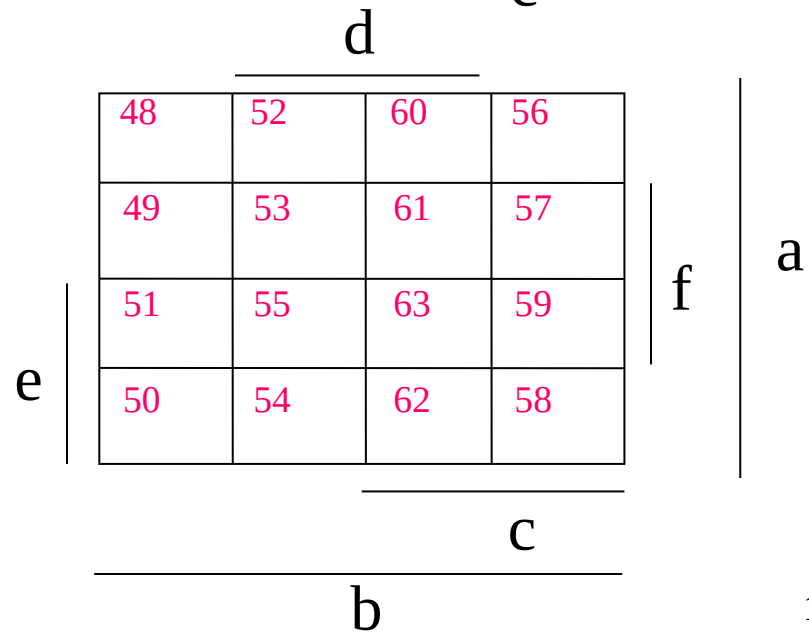
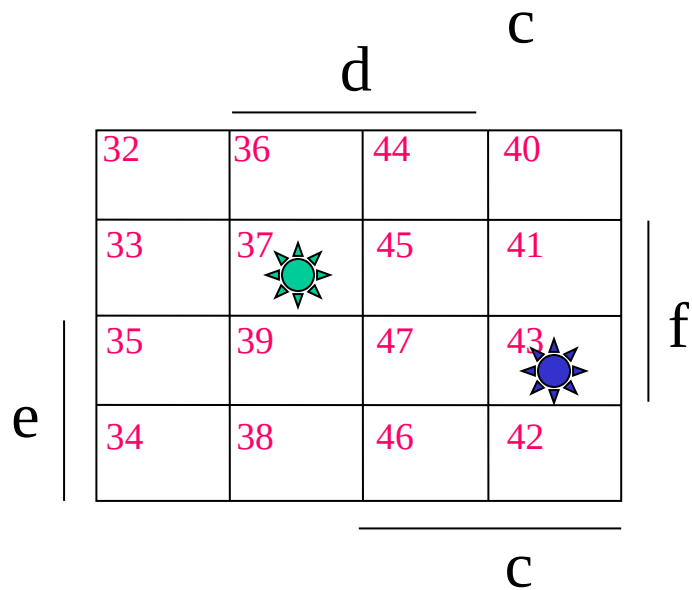
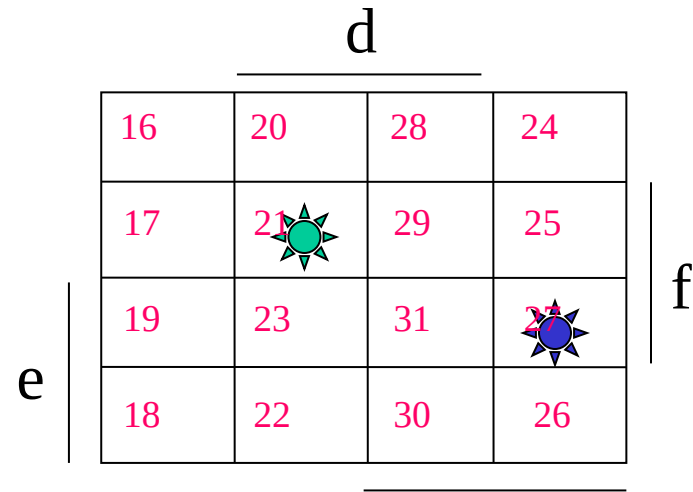
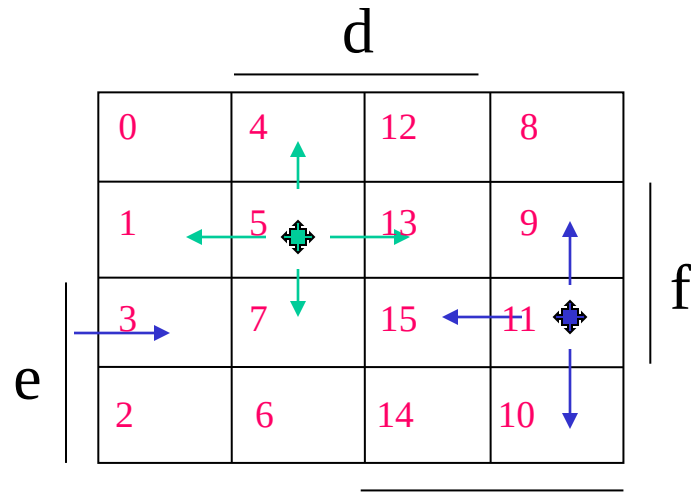
		bc			
		00	01	11	10
		c			
de	00	16 1	20	28	24 1
	01	17	21	29	25
	11	19	23 1	31 1	27 1
	10	18 1	22	30 1	26 1
		b			

e

d

a

Six variable K-map



7 Variables and Beyond

1. Procedure
 1. Specification
 2. Prime enumeration
 3. Essential prime selection
 4. Logic synthesis
2. Theory
3. Methodology
 1. Data structure
 2. Algorithms
 3. Software packages
 4. Benchmarks