

Lecture 4: Four Input K-Maps

CSE 140: Components and Design Techniques for Digital Systems

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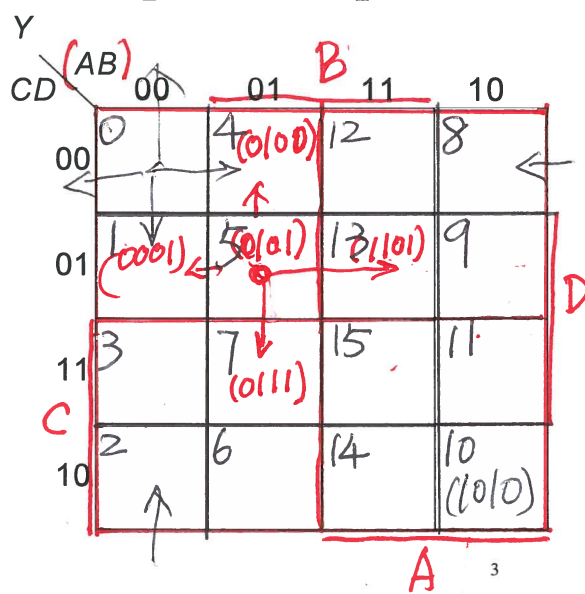
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An example

id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

4-input K-map



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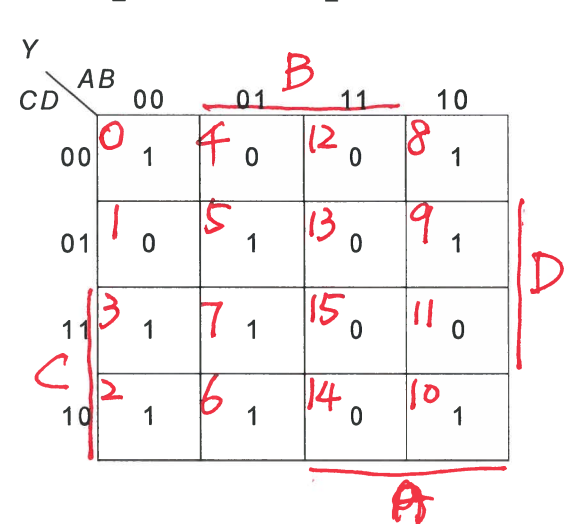
Outlines *2-level Sum of Prods.*

- Boolean Algebra vs. Karnaugh Maps
 - Algebra: variables, product terms, minterms, consensus theorem
 - Map: planes, rectangles, cells, adjacency
- Definitions: implicants, prime implicants, essential prime implicants
 - not a subset of other primes
 - indispensable $m \in F$
 - $m \in P_i \rightarrow \text{Essential}$
 - $m \notin P_j \forall j \neq i$
- Implementation Procedures
 - 1 cell
 - 2 cells
 - 4 cells
 - 8 ..

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id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

4-input K-map



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4-input K-map

- Arrangement of variables
- Adjacency and partition

Y \ CD \ AB	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 0	5 1	13 0	9 1
11	3 1	7 1	15 0	11 0
10	2 1	6 1	14 0	10 1

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Reading the reduced K-map

Y \ CD \ AB	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 0	5 1	13 0	9 1
11	3 1	7 1	15 0	11 0
10	2 1	6 1	14 0	10 1

$$Y = \bar{A}\bar{C} + \bar{A}BD + A\bar{B}\bar{C} + \bar{B}\bar{D}$$

sum of products

$$\Sigma m(5,6,7)$$

$$\begin{matrix} 0101 & 0110 & 0111 \\ \bar{A}\bar{B}\bar{C}D & \bar{A}\bar{B}CD & \bar{A}\bar{B}CD \end{matrix}$$

$$\bar{A}\bar{B}D \quad \bar{A}\bar{B}\bar{C}$$

K-Map vs. Boolean Expression

Boolean Expression	K-Map
Variable x_i and complement x_i'	Half planes R_i and R_i'
Product term $P = \Pi_i x_i^*$	Rectangle $R_p = \cup_{i \in P} R_i^*$
Each minterm	One element cell
Two adjacent minterms	Two neighboring cells
Each minterm has n adjacent minterms	Each cell has n neighbors
Boolean algebra	Two dimensional tool
?	D Don't Care set handling

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Procedure for finding the minimal function via K-maps (layman's terms)

1. Truth table $\Rightarrow$ K-map: F, R, D sets
2. Find product terms p : p covers at least one $m_i \in F$ but no intersection with R . (Implicants)
3. Expand implicant p to its limit (Prime Implicants)
4. Select prime implicant p that covers an $m_i \in F$ but m_i is not contained in any other prime implicants. (Essential Primes)
5. Use the essential primes and a minimal set of other primes to cover F . (Challenging task)

$$\Sigma m(0, 5, 7)$$

$$\Sigma m(5, 7, 13, 15)$$

$$\Sigma m(14, 15)$$

Y \ CD \ AB	00	01	11	10
00	0 0	4 0	12 0	8 1
01	1 0	5 1	13 0	9 1
11	3 1	7 1	15 0	11 0
10	2 0	6 0	14 1	10 0

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Definitions: implicant, prime implicant, essential prime implicant

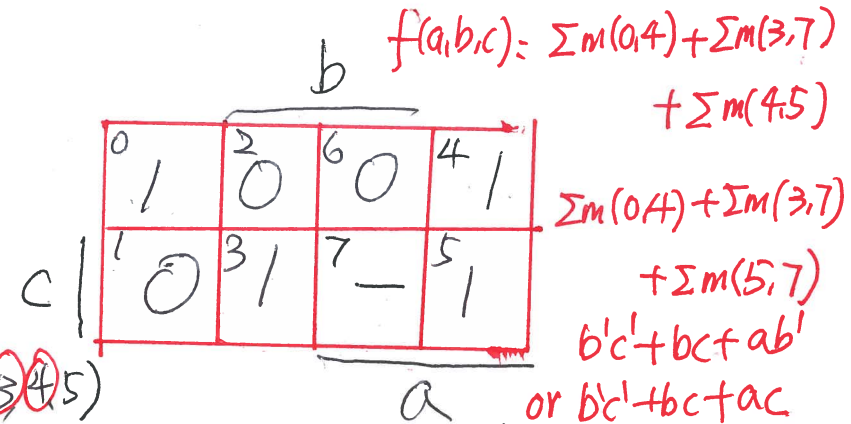
- **Implicant:** A product term that has non-empty intersection with on-set F and does not intersect with off-set R .
- **Prime Implicant:** An implicant that is not covered by any other implicant.
- **Essential Prime Implicant:** A prime implicant that has an element in on-set F but this element is not covered by any other prime implicants.

P s.t. $P \cap F \neq \emptyset$ & $P \cap R = \emptyset$

P_i is not a proper subset of other P_j

$A_n(E)P_i : \exists m \in J, \text{ s.t. } m \in P_i, m \notin P_j \forall j \neq i$

Examples of Primes and Essential Primes



$$F = \sum m(0,3,4,5)$$

$$R = \sum m(1,2,6)$$

$$D = \sum m(7)$$

$$(011) \sum m(3) a'bc$$

$$(111) \sum m(7) abc$$

$$\sum m(0,4) b'c' E$$

$$\sum m(3,7) bc E$$

$$\sum m(4,5) ab' NE$$

$$\sum m(5,7) ac NE$$

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Examples of Primes and Essential Primes

Definition: Prime Implicant

1. **Implicant:** A product term that has non-empty intersection with on-set F and does not intersect with off-set R .
2. **Prime Implicant:** An implicant that is not covered by any other implicant.

Y	AB				
		00	01	11	10
CD	00	1	0	0	1
	01	0	1	0	1
	11	1	0	0	0
	10	1	1	0	1

Q: How about this one? Is it a prime implicant?

- A. Yes
B. No

Definition: Essential Prime

- Essential Prime Implicant: A prime implicant that has an element in on-set F but this element is not covered by any other prime implicants.

Y \ CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

Q: Is the blue group an essential prime?

- A. Yes
B. No

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Procedure for finding the minimal function via K-maps (formal terms)

- Convert truth table to K-map
- Include all essential primes
- Include nonessential primes as needed to completely cover the onset (all cells of value one)

Y \ CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

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Definition: Essential Prime

Q: Which of the following product term(s) is (are) not an essential prime for the given K-map ?

ab \ cd	00	01	11	10
00	1	0	1	1
01	1	1	1	0
11	0	0	1	1
10	1	0	1	1

A. $bc'd$ $\Sigma m(5,13) \rightarrow E$
 B. $d'b'$ $\Sigma m(0,2,8,10) \rightarrow E$
 C. ac $\Sigma m(10,11,14,15) \rightarrow E$
 D. ab $\Sigma m(12,13,14,15) \rightarrow NE$
 E. ad' $\Sigma m(8,10,12,14) \rightarrow NE$

$\Sigma m(5,13) + \Sigma m(0,2,8,10)$
 $+ \Sigma m(10,11,14,15)$
 $+ \Sigma m(12,13,14,15)$
 $\{ \Sigma m(5,13) + \Sigma m(0,2,8,10)$
 $+ \Sigma m(10,11,14,15)$
 $+ \Sigma m(8,10,12,14)$

K-maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y \ CD \ AB	00	01	11	10
00				
01				
11				
10				

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