Lecture 4: Four Input K-Maps

CSE 140: Components and Design Techniques for Digital Systems

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Outlines

- Boolean Algebra vs. Karnaugh Maps
  - Algebra: variables, product terms, minterms, consensus theorem
  - Map: planes, rectangles, cells, adjacency
- Definitions: implicants, prime implicants, essential prime implicants
- Implementation Procedures

An example

id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

4-input K-map

Y: CD \ AB	00	01	11	10
00				
01				
11				
10				

4-input K-map

id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

Y: CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

4-input K-map

- Arrangement of variables
- Adjacency and partition

Y \ AB, CD	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 0	5 1	13 0	9 1
11	3 1	7 1	15 0	11 0
10	2 1	6 1	14 0	10 1

Reading the reduced K-map

Y \ AB / CD	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 0	5: 1	13: 0	9: 1
11	3: 1	7: 1	15: 0	11: 0
10	2: 1	6: 1	14: 0	10: 1

$\sum m(2,3,6,7)$
$\sum m(5,7)$
$\sum m(8,9)$
$\sum m(0,2,8,10)$

$$Y = \overline{A}C + \overline{A}BD + A\overline{B}\,\overline{C} + \overline{B}\,\overline{D}$$

K-Map vs. Boolean Expression

Boolean Expression	**K-Map**
Variable x_i and complement $x_i{}'$	**Half planes** R_i and $R_i{}'$
Product term $P = \Pi_i x_i^*$	**Rectangle** $R_p = \cup_{i \in P} R_i^*$
Each **minterm**	One **element cell**
Two **adjacent** minterms	Two **neighboring** cells
Each minterm has **n adjacent** minterms	Each cell has **n neighbors**
Boolean algebra	Two dimensional **tool**
?	D Don't Care set handling

Procedure for finding the minimal function via K-maps (layman's terms)

1. Truth table => K-map: F, R, D sets
2. Find product terms p: p covers at least one $m_i \in F$ but no intersection with R. (Implicants)
3. Expand implicant p to its limit (Prime Implicants)
4. Select prime implicant p that covers an $m_i \in F$ but m_i is not contained in any other prime implicants. (Essential Primes)
5. Use the essential primes and a minimal set of other primes to cover F. (Challenging task)

Y \ AB / CD	00	01	11	10
00				
01				
11				
10				

Definitions: implicant, prime implicant, essential prime implicant

- Implicant: A product term that has non-empty intersection with on-set F and does not intersect with off-set R .
- Prime Implicant: An implicant that is not covered by any other implicant.
- Essential Prime Implicant: A prime implicant that has an element in on-set F but this element is not covered by any other prime implicants.

Examples of Primes and Essential Primes

Examples of Primes and Essential Primes

Definition: Prime Implicant

1. Implicant: A product term that has non-empty intersection with on-set *F* and does not intersect with off-set *R*.
2. Prime Implicant: An implicant that is not covered by any other implicant.

Y \ AB, CD	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

Q: How about this one? Is it a prime implicant?

A. Yes
B. No

Definition: Essential Prime

- Essential Prime Implicant: A prime implicant that has an element in on-set F but this element is not covered by any other prime implicants.

Y (CD \ AB)	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

Q: Is the blue group an essential prime?

A. Yes
B. No

Definition: Essential Prime

Q: Which of the following product term(s) is (are) not an essential prime for the given K-map ?

cd \ ab	00	01	11	10
00	0 1	4	12 1	8 1
01	1	5 1	13 1	9
11	3	7	15 1	11 1
10	2 1	6	14 1	10 1

A. *bc'd* ∑m(5,13)

B. *d'b'* ∑m(0,2,8,10)

C. *ac* ∑m(10,11,14,15)

D. *ab* ∑m(12,13,14,15)

E. *ad'* ∑m(8,10,12,14)

Procedure for finding the minimal function via K-maps (formal terms)

1. Convert truth table to K-map
2. Include all essential primes
3. Include nonessential primes as needed to completely cover the onset (all cells of value one)

Y: CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

K-maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y: CD \ AB	00	01	11	10
00				
01				
11				
10				

K-maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y: CD \ AB	00	01	11	10
00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X

K-maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y: CD \ AB	00	01	11	10
00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X

$$Y = A + \overline{B}\,\overline{D} + C$$

Reducing Canonical expressions

Given $\mathcal{F}(a,b,c,d) = \Sigma m\ (0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m\ (9, 10)$

1. Draw K-map

cd \ ab	00	01	11	10
00				
01				
11				
10				

Reducing Canonical Expressions

Given $\mathcal{F}(a,b,c,d) = \Sigma m\ (0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m\ (9, 10)$

1. Draw K-map

cd \ ab	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Reducing Canonical Expressions

Given $\mathcal{F}(a,b,c,d) = \Sigma m\ (0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m\ (9, 10)$

1. Draw K-map

cd \ ab	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 0	13: 0	9: X
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 1	10: X

Reducing Canonical Expressions

1. Draw K-map
2. Identify Prime implicants
3. Identify Essential Primes

cd \ ab	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 0	13: 0	9: X
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 1	10: X

Q: How many primes (P) and essential primes (EP) are there?

A. Four (P) and three (EP)
B. Three (P) and two (EP)
C. Three (P) and three (EP)
D. Four (P) and Four (EP)

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
2. Essential Primes: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)

cd \ ab	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 1	5 0	13 0	9 X
11	3 0	7 0	15 0	11 0
10	2 1	6 0	14 1	10 X

Q: Do the E-primes cover the entire on set?

A. Yes

B. No

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
2. Essential Primes: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
3. Min exp: Σ (Essential Primes)=Σm (0, 1, 8, 9) + Σm (0, 2, 8, 10) + Σm (10, 14)

f(a,b,c,d) = ?

cd \ ab	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 0	13: 0	9: X
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 1	10: X

Q: Do the E-primes cover the entire on set?

A. Yes

B. No

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
2. Essential Primes: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
3. Min exp: Σ (Essential Primes)=Σm (0, 1, 8, 9) + Σm (0, 2, 8, 10) + Σm (10, 14)

$f(a,b,c,d) = b'c' + b'd' + acd'$

cd \ ab	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 0	13: 0	9: X
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 1	10: X

Q: Do the E-primes cover the entire on set?

A. Yes

B. No

Another example

Given $\mathcal{F}(a,b,c,d) = \Sigma m\ (0, 3, 4, 14, 15)$

$\mathcal{D}(a,b,c,d) = \Sigma m\ (1, 11, 13)$

1.Draw the K-Map

cd \ ab	00	01	11	10
00				
01				
11				
10				

Another example

Given $\mathcal{F}(a,b,c,d) = \Sigma m\ (0, 3, 4, 14, 15)$

$\mathcal{D}(a,b,c,d) = \Sigma m\ (1, 11, 13)$

cd \ ab	00	01	11	10
00	0: 1	4: 1	12: 0	8: 0
01	1: X	5: 0	13: X	9: 0
11	3: 1	7: 0	15: 1	11: X
10	2: 0	6: 0	14: 1	10: 0

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 4), Σm (0, 1), Σm (1, 3), Σm (3, 11), Σm (14, 15), Σm (11, 15), Σm (13, 15)

2. Essential Primes: Σm (0, 4), Σm (14, 15)

cd \ ab	00	01	11	10
00	0: 1	4: 1	12: 0	8: 0
01	1: X	5: 0	13: X	9: 0
11	3: 1	7: 0	15: 1	11: X
10	2: 0	6: 0	14: 1	10: 0

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 4), Σm (0, 1), Σm (1, 3), Σm (3, 11), Σm (14, 15), Σm (11, 15), Σm (13, 15)
2. Essential Primes: Σm (0, 4), Σm (14, 15)
3. Min exp: Σm (0, 4), Σm (14, 15), (Σm (3, 11) or Σm (1,3))
4. f(a,b,c,d) = a'c'd'+ abc+ b'cd (or a'b'd)

cd \ ab	00	01	11	10
00	0: 1	4: 1	12: 0	8: 0
01	1: X	5: 0	13: X	9: 0
11	3: 1	7: 0	15: 1	11: X
10	2: 0	6: 0	14: 1	10: 0