

CSE140 HW1, Due Mon. 4/12/2021 by 11:59PM

Introduction

The purpose of this assignment is to practice CMOS logic design style and Boolean algebra. For CMOS logic design style, we implement a Boolean function and show the design with a schematic diagram. For Boolean algebra, first the exercise is to help you practice the application of Boolean algebra theorems to transform and reduce Boolean expressions. The second goal is to help you learn how to go from the world of Boolean expressions to the world of digital circuits. The final goal is to help you translate a problem described in Boolean algebraic expression to digital logic. We hope you can think of why each of these exercises is useful when designing digital circuits.

1 Implementation of Switching Function using CMOS Logic

- 20 points, graded by correctness

Given a circuit with three binary inputs (a, b, c) and a binary output function $y(a, b, c) = (ab + bc)'$, implement the function using CMOS logic technology. Draw the design with a schematic diagram.

The schematic diagram is illustrated in Figure 1.

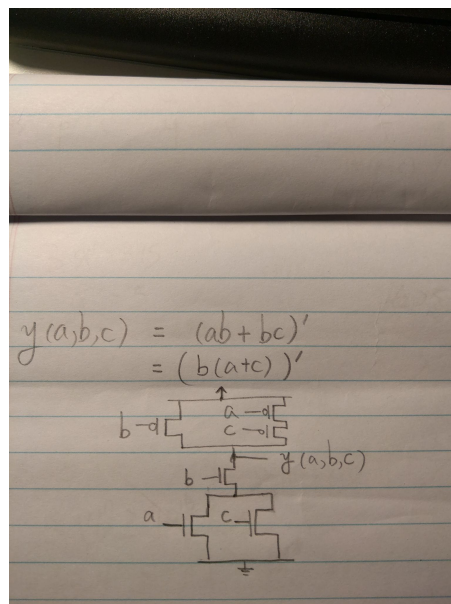


Figure 1: Schematic Diagram

2 Application of Boolean Algebra and Shannon Expansion

- 20 points, graded by completeness

2.1 Prove the following using Boolean algebra.

A. $a + ab = a$.

$$\begin{aligned} a + ab &= a \cdot (1 + b) && \text{Distributive Law} \\ &= a \cdot 1 && \text{Annulment Theorem} \\ &= a && \text{Identity Law} \end{aligned}$$

B. $a \cdot (a + b) = a$.

$$\begin{aligned} a \cdot (a + b) &= a \cdot a + ab && \text{Distributive Law} \\ &= a + ab && \text{Idempotent Law} \\ &= a \cdot (1 + b) && \text{Distributive Law} \\ &= a \cdot 1 && \text{Annulment Theorem} \\ &= a && \text{Identity Law} \end{aligned}$$

C. $(be + d)(a + c + d')(a + be + c) = (be + d)(a + c + d')$

$$(be + d)(a + c + d')(a + be + c) = (be + d)(a + c + d') \quad \text{Consensus Theorem}$$

2.2 Prove the above three equations using Shannon's expansion.

A. $a + ab = a$.

Pick variable a and partition the switching function into two cases: $a = 0$ and $a = 1$

$$\begin{aligned} a + ab &= a(1 + b) + a'(0 + 0 \cdot b) && \text{Shannon's Expansion} \\ &= a \cdot 1 + a'(0 + 0) && \text{Annulment Theorem} \\ &= a + a' \cdot 0 && \text{Identity Law} \\ &= a + 0 && \text{Annulment Theorem} \\ &= a && \text{Identity Law} \end{aligned}$$

B. $a \cdot (a + b) = a$.

Pick variable a and partition the switching function into two cases: $a = 0$ and $a = 1$

$$\begin{aligned}
a \cdot (a + b) &= a(1 \cdot (1 + b)) + a'(0 \cdot (0 + b)) && \text{Shannon's Expansion} \\
&= a \cdot (1 \cdot 1) + a' \cdot 0 && \text{Annulment Theorem} \\
&= a + a' \cdot 0 && \text{Identity Law} \\
&= a + 0 && \text{Annulment Theorem} \\
&= a && \text{Identity Law}
\end{aligned}$$

C. $(be + d)(a + c + d')(a + be + c) = (be + d)(a + c + d')$
Pick variable a and partition the switching function into two cases: $a = 0$ and $a = 1$

$$\begin{aligned}
&(be + d)(a + c + d')(a + be + c) \\
&= a((be + d)(1 + c + d')(1 + be + c)) + a'((be + d)(0 + c + d')(0 + be + c)) \quad \text{Shannon's Expansion} \\
&= a \cdot (be + d) + a' \cdot (be + d)(c + d')(be + c) \quad \text{Annulment Theorem, Identity Law} \\
&= a \cdot (be + d) + a' \cdot (be + d)(c + d') \quad \text{Consensus Theorem} \\
&= (be + d)(a + a'c + a'd) \quad \text{Distributive Law} \\
&= (be + d)(a + c + d) \quad \text{Consensus Theorem}
\end{aligned}$$

3 Prove or disprove the following statements in Boolean expressions

- 30 points, graded by correctness

A. If $a = b + c$ and $a = c$, then $b = 0$.

Disprove with a counterexample: with $a = 1, b = 1, c = 1$, we still have $a = b + c$ being true.

B. If $a = bc$ and $a = c$, then $b = 1$.

Disprove with a counterexample: with $a = 0, b = 0, c = 0$, we still have $a = bc$ being true.

C. $(b + de)(a + c + d'e')(a + b + c) = (b + de)(a + c + d'e')$

For left hand side,

$$\begin{aligned}
&(b + de) \cdot (a + c + d'e') \cdot (a + b + c) \\
&= (b + de) \cdot (f + d'e') \cdot (f + b) && \text{let } f = a + c \\
&= (b + de) \cdot (f + bd'e') && \text{Distributive Law} \\
&= bf + bd'e' + def && \text{Distributive, Identity, Idempotent Laws}
\end{aligned}$$

For right hand side,

$$\begin{aligned}
 & (b + de) \cdot (a + c + d'e') \\
 &= (b + de) \cdot (f + d'e') && \text{let } f = a + c \\
 &= bf + bd'e' + def && \text{Distributive, Complement, Null Laws}
 \end{aligned}$$

Since we get the same Boolean expression for the two sides, the statement is true.

4 Translate From a Digital Problem to Boolean Expressions

- 10 points, graded by completeness

A circuit has four binary inputs (a_3, a_2, a_1, a_0) and one binary output y . Output y is true when the following arithmetic operation satisfies the inequality: $4 \times a_3 + 3 \times a_2 + 2 \times a_1 + a_0 \geq 4$.

A. Write the truth table.

a_3	a_2	a_1	a_0	output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

B. Write the function in sum-of-products canonical form.

$$\begin{aligned}
 y(a_3, a_2, a_1, a_0) &= \Sigma m(5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15) \\
 &= a'_3 a_2 a'_1 a_0 + a'_3 a_2 a_1 a'_0 \\
 &\quad + a'_3 a_2 a_1 a_0 + a_3 a'_2 a'_1 a'_0 + a_3 a'_2 a'_1 a_0 \\
 &\quad + a_3 a'_2 a_1 a'_0 + a_3 a'_2 a_1 a_0 + a_3 a_2 a'_1 a'_0 \\
 &\quad + a_3 a_2 a'_1 a_0 + a_3 a_2 a_1 a'_0 + a_3 a_2 a_1 a_0
 \end{aligned}$$

C. Write the function in product-of-sums canonical form.

$$\begin{aligned} y(a_3, a_2, a_1, a_0) &= \Pi M(0, 1, 2, 3, 4) \\ &= (a_3 + a_2 + a_1 + a_0) \cdot (a_3 + a_2 + a_1 + a'_0) \cdot (a_3 + a_2 + a'_1 + a_0) \\ &\quad \cdot (a_3 + a_2 + a'_1 + a'_0) \cdot (a_3 + a'_2 + a_1 + a_0) \end{aligned}$$

5 Boolean Algebra and Implementation

• 20 points, graded by completeness

A. Simplify each of the following two Boolean equations (using Boolean algebra, in particular consensus theorem). Write the results in Boolean expressions.

B. Convert the simplified expressions with logic using *AND*, *OR* gates with bubbles for (True, False) inversion. Draw the logic diagram.

C. List the numbers of literals and operators versus the numbers of gates, nets, and pins in the schematic diagrams

i. $y(a, b, c, d, e) = a + a'b + a'b'c + a'b'c'd + a'b'c'd'e$

The logic diagram is illustrated in Figure 2.

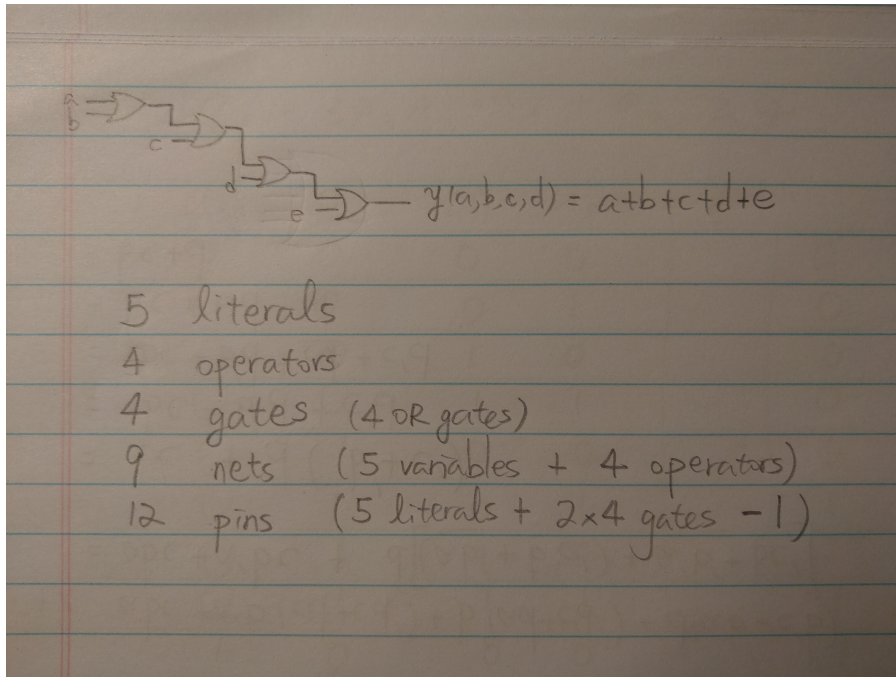


Figure 2: (i) Logic Diagram

ii. $y(a, b, c, d) = abc + a'b(cd + cd') + b'(ad + c'd) + d(a'b' + c'b)$

The logic diagram is illustrated in Figure 3.

Derivation:

(i) $a + a'b + a'b'c + a'b'c'd + a'b'c'd'e$.

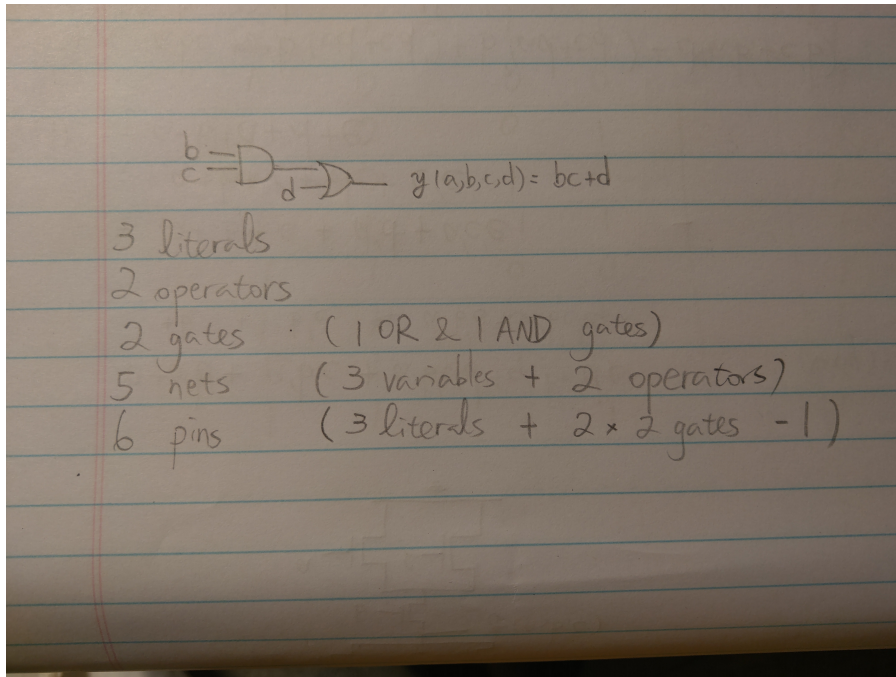


Figure 3: (ii) Logic Diagram

We can use Consensus Theorem, distributive, and null laws to show that $(x+x'y) = (x+y)$. By using $(x+x'y)=x+y$ layer by layer, we can get the simplified Boolean expression.

$$\begin{aligned}
 & a + a'b + a'b'c + a'b'c'd + a'b'c'd'e \\
 &= a + a'b + a'b'c + a'b'c'(d + d'e) \\
 &= a + a'b + a'b'(c + d + e) \\
 &= a + a'(b + c + d + e) \\
 &= a + b + c + d + e
 \end{aligned}$$

$$(ii) \ abc + a'b(cd + cd') + b'(ad + c'd) + d(a'b' + c'b).$$

$$\begin{aligned}
 & abc + a'b(cd + cd') + b'(ad + c'd) + d(a'b' + c'b) \\
 &= abc + a'bc + \mathbf{bc} + ab'd + a'b'd + \mathbf{b'd} + b'c'd + bc'd + \mathbf{c'd} \quad \text{Consensus Theorem} \\
 &= bc + b'd + c'd \quad \text{Annulment Theorem} \\
 &= bc + b'd + c'd + cd + bd \quad \text{Consensus Theorem} \\
 &= bc + d \quad \text{Distributive, Complement Laws}
 \end{aligned}$$

Assignment Turn-in

Once you're finished with the assignment, turn it in as a pdf on *Gradescope*.