

1. (Decoders) Given three four-input Boolean functions

$$f_1(a, b, c, d) = \sum m(1, 3, 5, 7, 9) + \sum d(12, 13, 14, 15),$$

$$f_2(a, b, c, d) = \sum m(1, 5, 7, 12, 15) + \sum d(3, 8, 14),$$

$$f_3(a, b, c, d) = \sum m(2, 3, 9, 15) + \sum d(0, 11, 12).$$

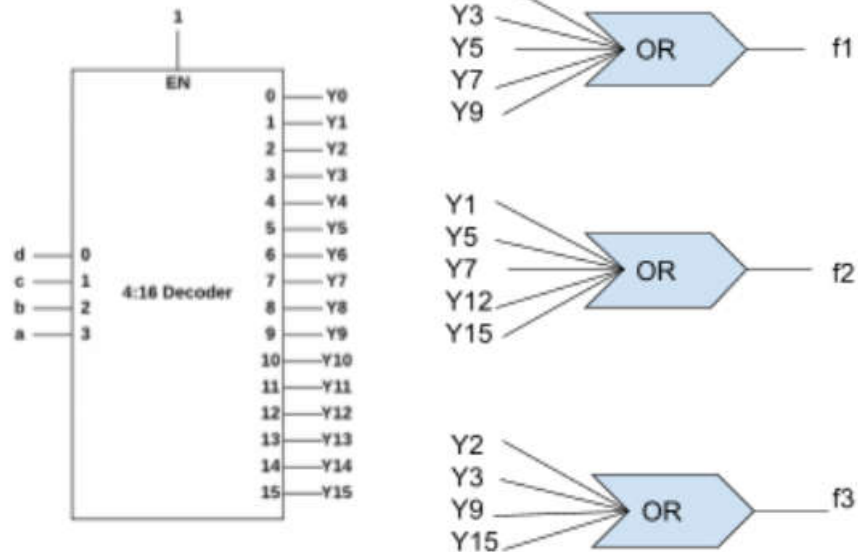
1.1. Implement the functions using a minimal network of 4:16 decoders and OR gates.

1.2. Implement the functions using a minimal network of 3:8 decoders and OR gates.

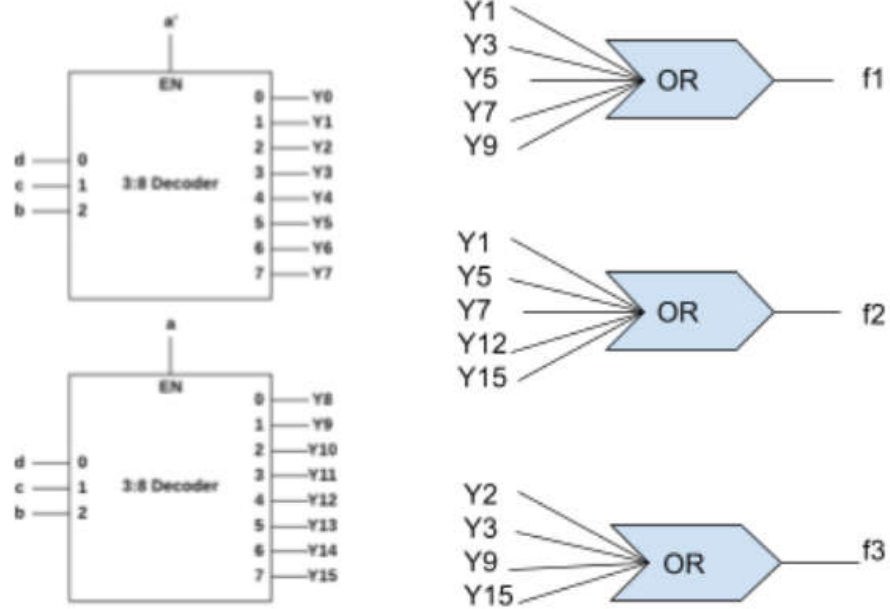
1.3. Implement the functions using a minimal network of 2:4 decoders and OR gates.

1.

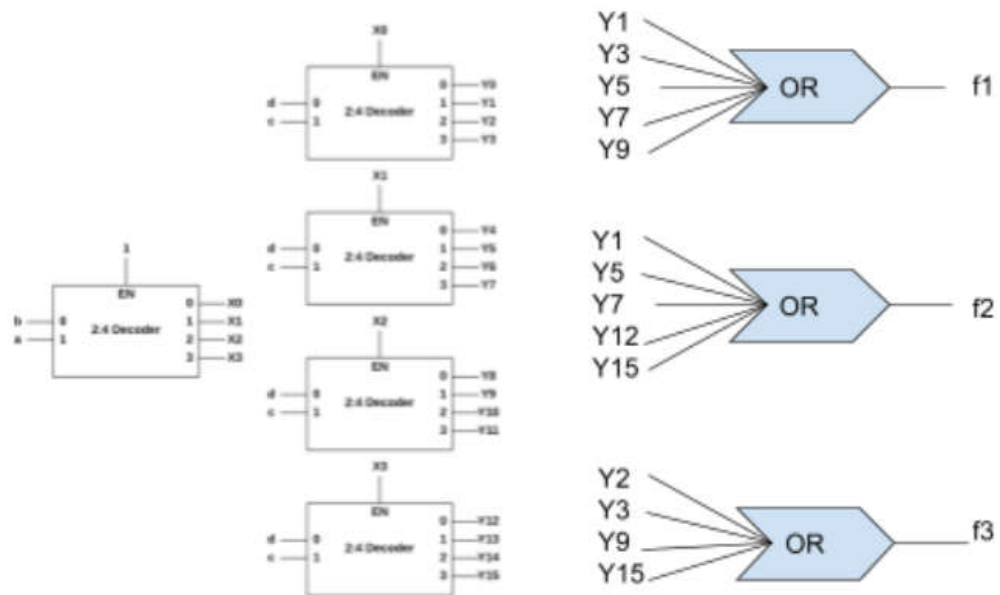
Q1-3: 15 POINTS
EACH



b.



c.



2. (Multiplexers) Assume a dual-railed system, where you have access to any variable and its complement. Implement the following four-input Boolean function as indicated in each of the following subproblems.

$$f(a, b, c, d) = \sum m(0, 2, 7, 12, 13) + \sum d(3, 5, 6, 8, 15).$$

2.1. Implement the function using a minimal network of 8:1 multiplexers.

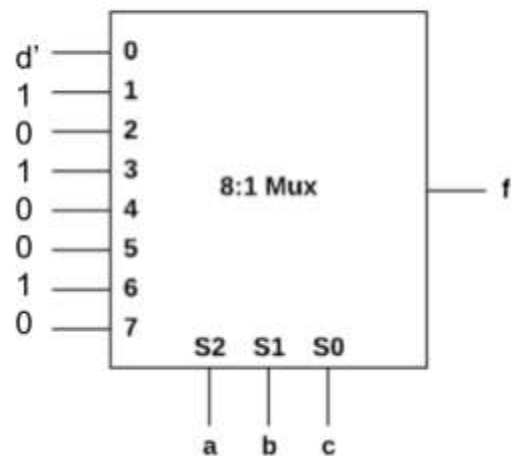
2.2. Implement the function using a minimal network of 4:1 multiplexers.

2.3. Implement the function using a minimal network of 2:1 multiplexers.

2.

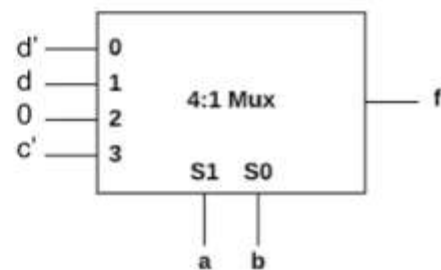
a.

a	b	c	d=0	d=1	D(d)
0	0	0	1	0	d'
0	0	1	1	x	1
0	1	0	0	x	0
0	1	1	x	1	1
1	0	0	x	0	0
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	x	0



b.

a	b	cd=00	cd=01	cd=10	cd=11	D(c,d)
0	0	1	0	1	x	d'
0	1	0	x	x	1	d
1	0	x	0	0	0	0
1	1	1	1	0	x	c'



c or d
(second
row)

c.

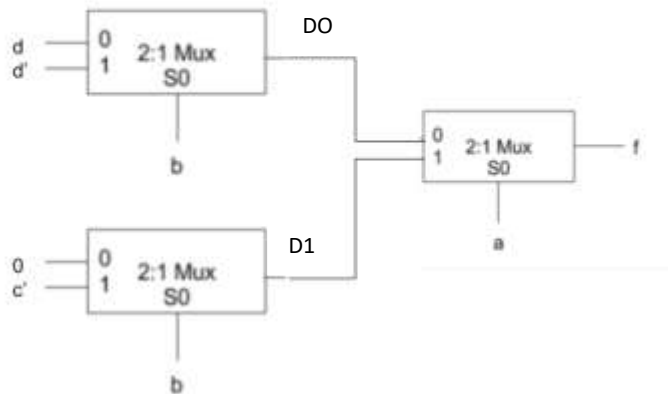
a	bcd=000	bcd=001	bcd=010	bcd=011	bcd=100	bcd=101	bcd=110	bcd=111	D (b,c,d)
0	1	0	1	x	0	x	x	1	$b'd'+bd$
1	x	0	0	0	1	1	x	0	bc'

b	cd=00	cd=01	cd=10	cd=11	D(c,d)
0	1	0	1	x	d'
1	0	x	x	1	d

b	cd=00	cd=01	cd=10	cd=11	D(c,d)
0	x	0	0	0	0
1	1	1	x	0	c'

$$D0 = b'd + bd$$

$$D1 = bc'$$



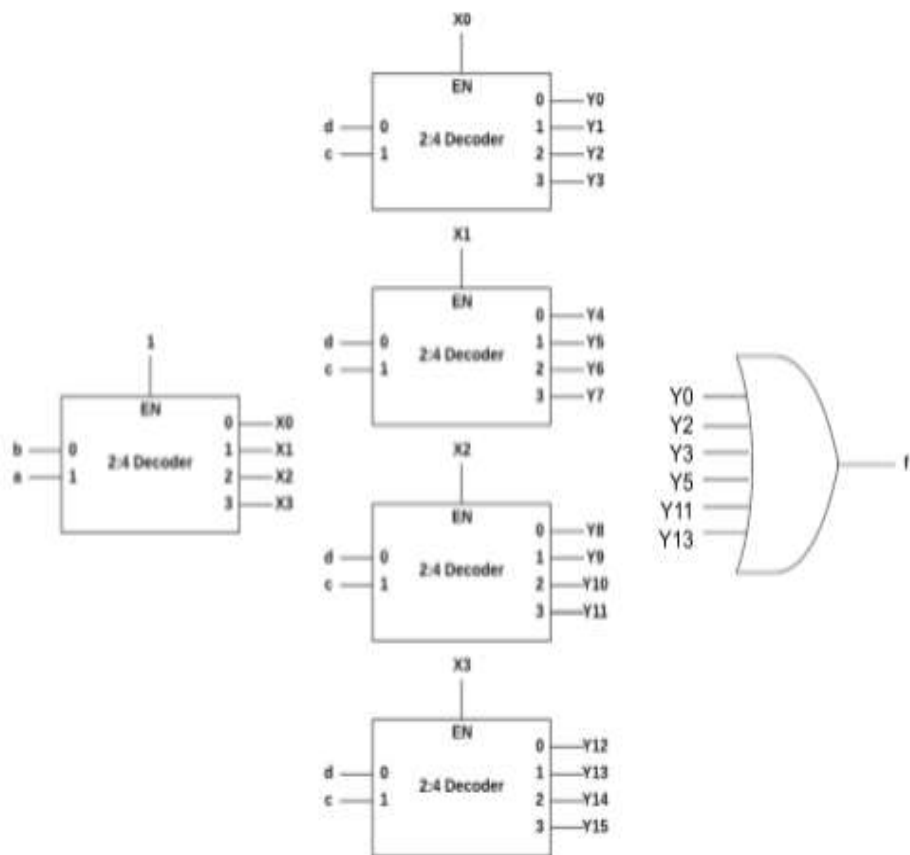
3. Assume a dual-railed system, where you have access to any variable and its complement. Given a four-input Boolean function

$$f(a, b, c, d) = \sum m(0, 2, 3, 5, 11, 13) + \sum d(4, 7, 10).$$

- 3.1. Implement the function using a minimal network of 2:4 decoders and OR gates.
- 3.2. Implement the function using a minimal network of 4:1 multiplexers.
- 3.3. Implement the function using a minimal network of 2:1 multiplexers.

3.

a.

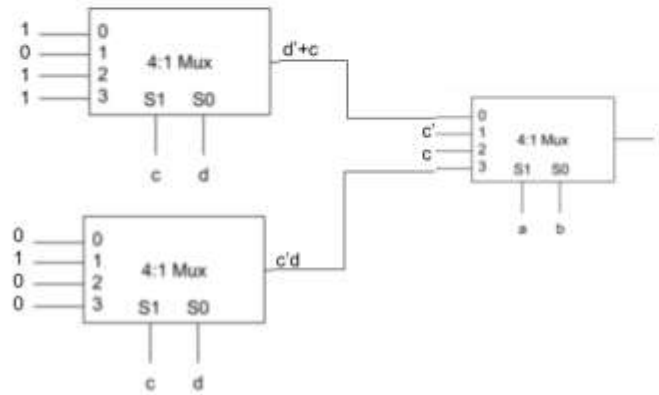


b.

a	b	cd=00	cd=01	cd=10	cd=11	D(c,d)
0	0	1	0	1	1	$d'+c$
0	1	x	1	0	x	c'
1	0	0	0	x	1	c
1	1	0	1	0	0	$c'd$

c	d	D
0	0	1
0	1	0
1	0	1
1	1	1

c	d	D
0	0	0
0	1	1
1	0	0
1	1	0



$$D0 = c + d'$$

$$D1 = c'$$

$$D2 = c$$

$$D3 = c'd$$

D0:

c\d	0	1
0	1	0
1	1	1

D1:

c\d	0	1
0	X	1
1	0	X

D2:

c\d	0	1
0	0	0
1	X	1

D3:

c\d	0	1
0	0	1
1	0	0

$$D0 = c + d'$$

$$D1 = c' \text{ or } D1 = d$$

$$D2 = c$$

$$D3 = c'd$$

c)

	bcd	bcd	bcd	bcd	bcd	bcd	bcd	bcd	
a	000	001	010	011	100	101	110	111	D(b,c,d)
0	1	0	1	1	X	1	0	X	D0
1	0	0	X	1	0	1	0	0	D1

D0:

bcd	0	1
00	1	0
01	1	1
11	0	X
10	X	1

D1:

bcd	0	1
00	0	0
01	X	1
11	0	0
10	0	1

$$D0 = b'd' + b'c + bd$$

$$D1 = b'c + bc'd$$

	cd	cd	cd	cd	
a	00	01	10	11	E(c,d)
0	1	0	1	1	E0
1	X	1	0	X	E1

E0:

c/d	0	1
0	1	0
1	1	1

E1:

c/d	0	1
0	X	1
1	0	X

$$E0 = c + d'$$

$$E1 = c' \text{ or } E1 = d$$

	cd	cd	cd	cd	
a	00	01	10	11	F(c,d)
0	0	0	X	1	F0
1	0	1	0	0	F1

F0:

c\d	0	1
0	0	0
1	X	1

F1:

c\d	0	1
0	0	1
1	0	0

$$F0 = c$$

$$F1 = c'd$$

For E0:

	d	d	
c	0	1	G(c,d)
0	1	0	G0
1	1	1	G1

$$G0 = d'$$

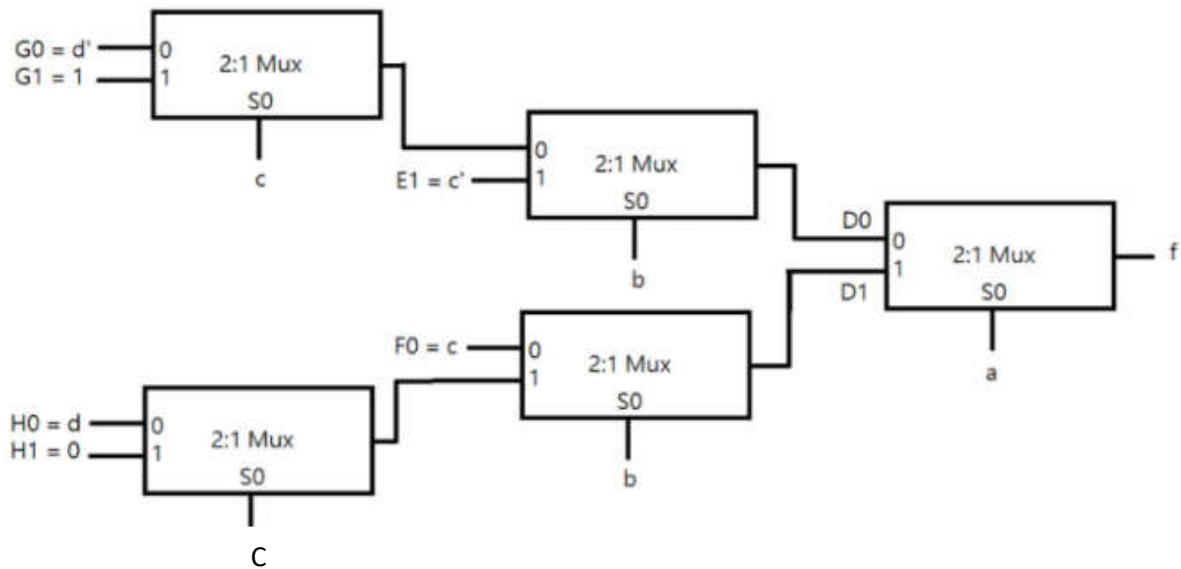
$$G1 = 1$$

For F1:

	d	d	
c	0	1	H(c,d)
0	0	1	H0
1	0	0	H1

$$H0 = d$$

$$H1 = 0$$

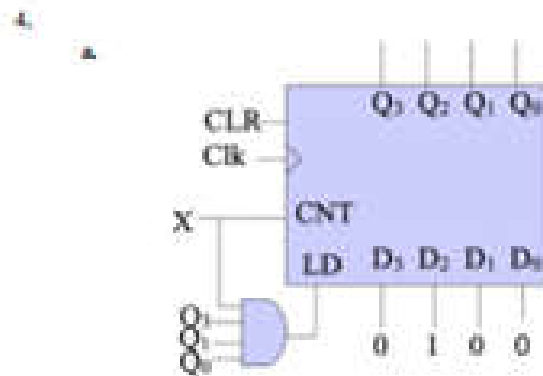


4. Counter: Use standard modulo counters to design various counters.

4.1 Given a modulo 16 counter, construct a 4 to 11 counter.

4.2 Given a modulo 8 counter, construct a counter that counts in a sequence 0, 1, 5, 3, 4, 7, with minimal networks of AND, OR gates.

4.3 Construct a modulo 64 counter using modulo 8 counters.



Q4.1&4.3- 5 POINTS

b.

id	$Q_2Q_1Q_0$	LD	D_2	D_1	D_0
0	000	0	-	-	-
1	001	1	1	0	1
2	010	-	-	-	-
3	011	0	-	-	-
4	100	1	1	1	1
5	101	1	0	1	1
6	110	-	-	-	-
7	111	0	-	-	-

4.2 – 20 points

LD:

$Q_2Q_1 \backslash Q_0$	0	1
00	0	1
01	-	0
11	-	0
10	1	1

D2:

$Q_2Q_1 \backslash Q_0$	0	1
00	-	1
01	-	-
11	-	-
10	1	0

D1:

$Q_2Q_1 \backslash Q_0$	0	1
00	-	0
01	-	-
11	-	-
10	1	1

D0:

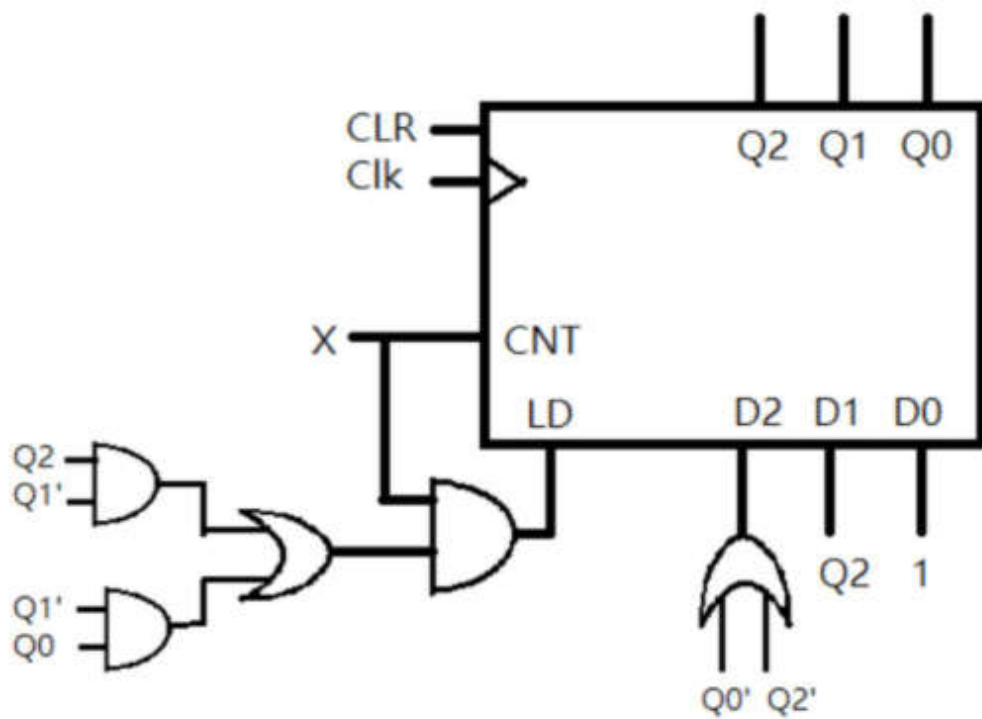
$Q_2Q_1 \backslash Q_0$	0	1
00	-	1
01	-	-
11	-	-
10	1	1

$$LD = Q_2Q_1' + Q_1'Q_0$$

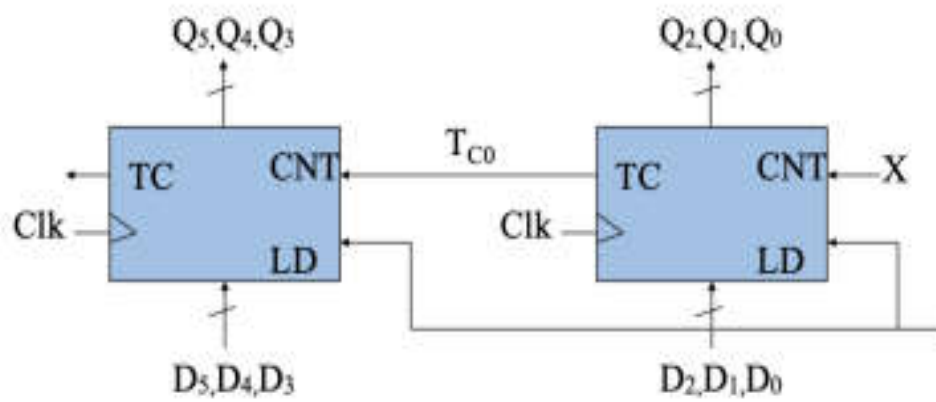
$$D_2 = Q_0' + Q_2'$$

$$D_1 = Q_2$$

$$D_0 = 1$$



c.



5. System Designs: Implement the following algorithm:

```
Alg(X,Y,Z,start,U,done)
Input X[7:0], Y[7:0], Z[7:0], start;
Output U[7:0], done;
Local-object A[7:0], B[7:0], C[7:0];
S1: If start' goto S1;
S2: done  $\leftarrow$  0 || A  $\leftarrow$  X || B  $\leftarrow$  Y || C  $\leftarrow$  Z;
S3: A  $\leftarrow$  Inc(A) || B  $\leftarrow$  Inc(B);
```

```
S4: If A'[7] goto S3 || B  $\leftarrow$  Inc(B);
S5: If B'[7] goto S4 || C  $\leftarrow$  Inc(C);
S6: If C'[7] goto S5 || A  $\leftarrow$  Add(A,B);
S7: U  $\leftarrow$  C || done  $\leftarrow$  1 || goto S1;
End Alg
```

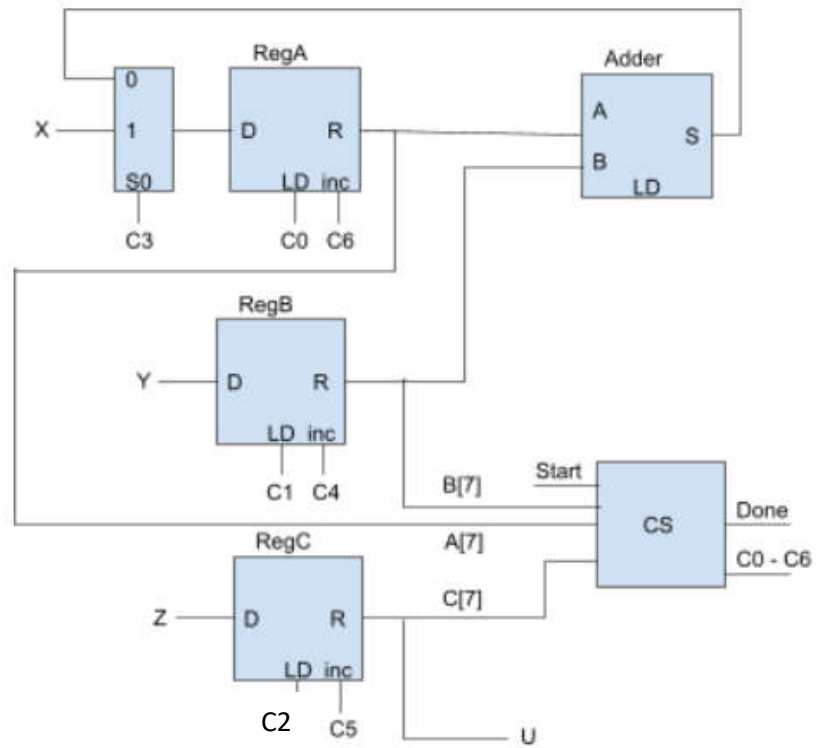
5.1 Design a data subsystem that is adequate to execute the algorithm and draw the schematic diagram.

5.2 Design a control subsystem and draw the state diagram.

5.3 Implement the control subsystem with a one-hot encoding design. Draw the logic diagram.

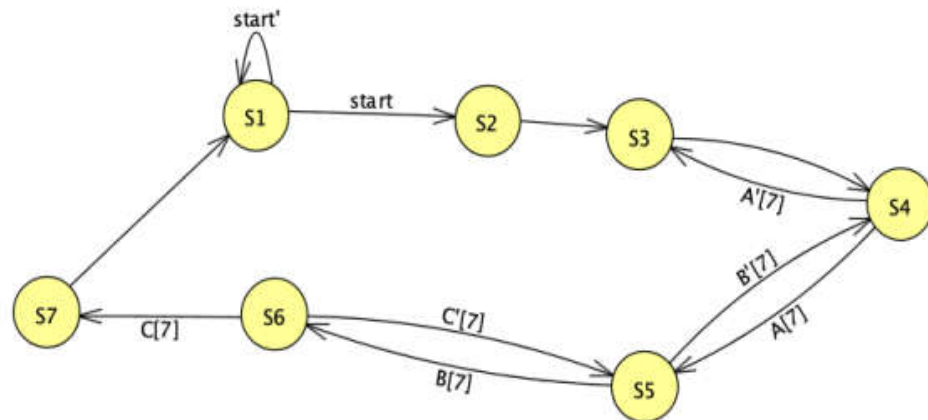
5.

a.



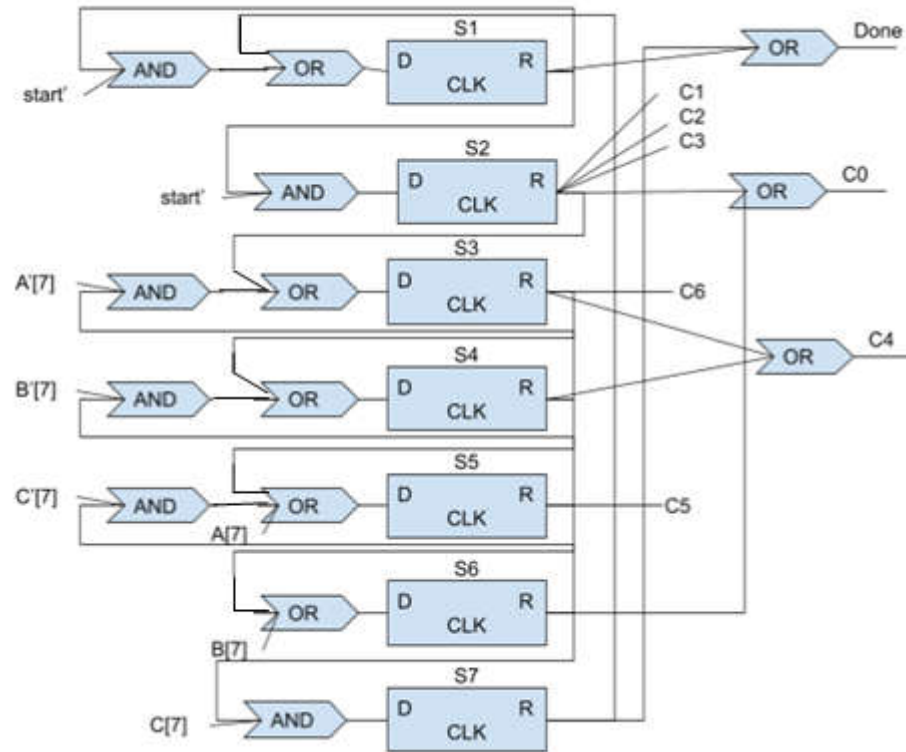
5.1 & 5.2 - 10 points

b.



C.

c.



5.3- 20 points