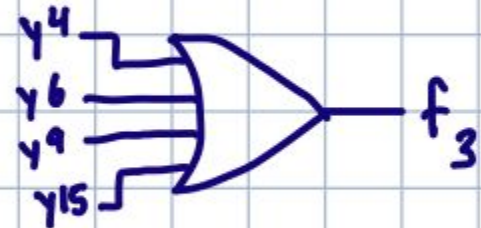
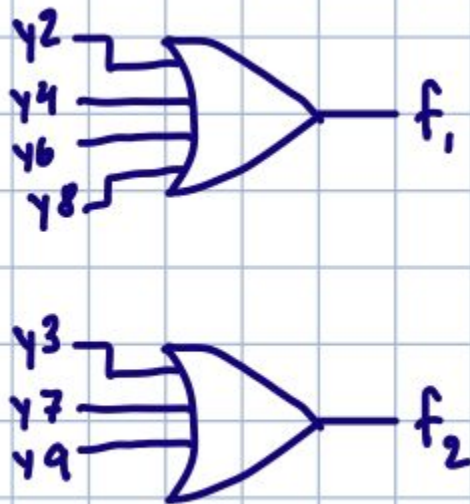
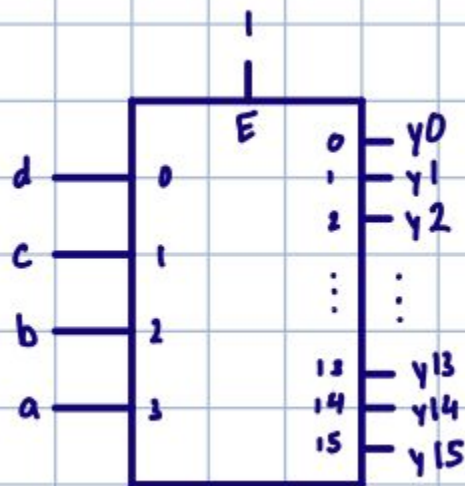


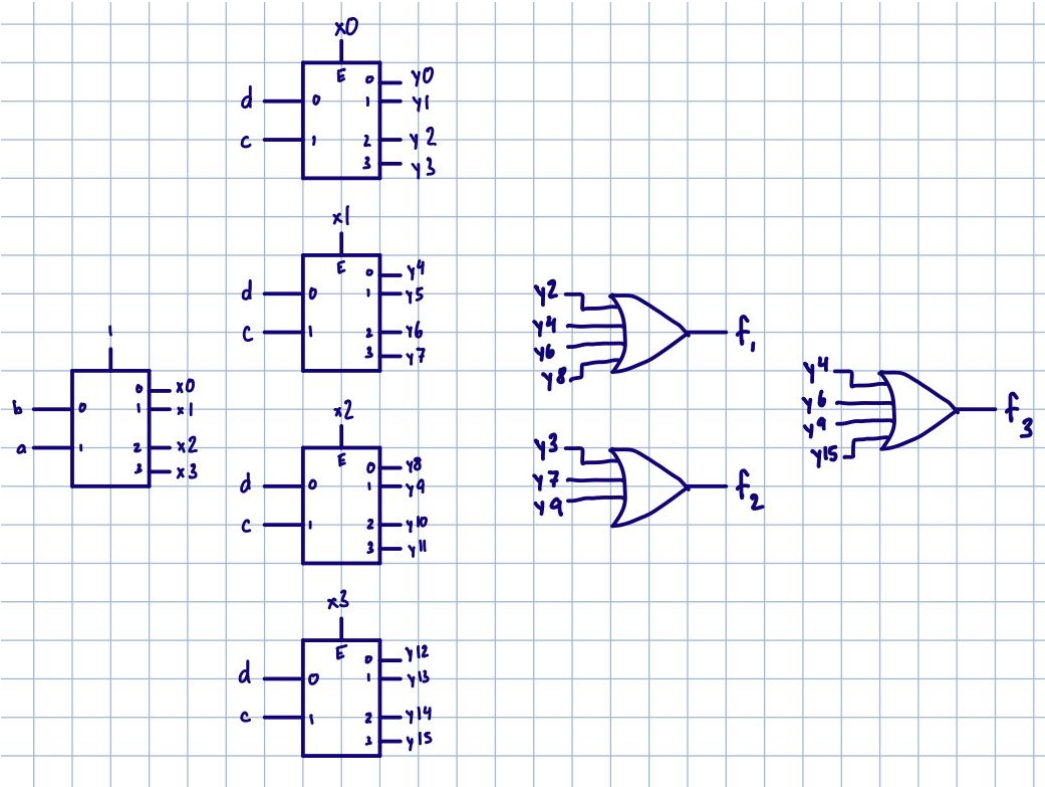
CSE 140 Midterm III

Solutions

Q1.1



Q1.2



Q2.1

$$2. f(a,b,c,d) = \sum m(1,3,6,11,12) + \sum d(2,4,7,9,14)$$

2.1

Id	abc	d=0	d=1	D(d)
0	000	0	1	d
1	001	X	1	1
2	010	X	0	0
3	011	1	X	1
4	100	0	X	0
5	101	0	1	d
6	110	1	0	d'
7	111	X	0	0



Q2.2 With 3 Muxes

2.2

$a \backslash bcd$	000	001	010	011	100	101	110	111	$D(b, c, d)$
0	0	1	X	1	X	0	1	X	D_0
1	0	X	0	1	1	0	X	0	D_1

D_0 :

$b \backslash cd$	00	01	11	10
0	0	1	1	X
1	X	0	X	1

$b=0$:

$c \backslash d$	0	1
0	0	1
1	X	1

$b=1$:

$c \backslash d$	0	1
0	X	0
1	1	X

$c \text{ or } d'$

D_1 :

$b \backslash cd$	00	01	11	10
0	0	X	1	0
1	1	0	0	X

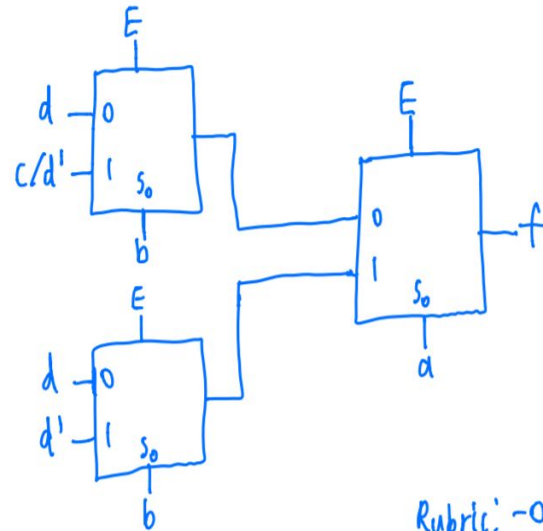
$b=0$:

$c \backslash d$	0	1
0	0	X
1	0	1

$b=1$:

$c \backslash d$	0	1
0	1	0
1	X	0

d'



Rubric: -0 for correct
 -0.5 for each incorrect input
 -4 if structure wrong

Q2.2 With 2 Muxes

2.2

$a \backslash bcd$	000	001	010	011	100	101	110	111	$D(b, c, d)$
0	0	1	X	1	X	0	1	X	D_0
1	0	X	0	1	1	0	X	0	D_1

D_0 :

$b \backslash cd$	00	01	11	10
0	0	1	1	X
1	X	0	X	1

$b=0$:

$c \backslash d$	0	1
0	0	1
1	X	1

d

$b=1$:

$c \backslash d$	0	1
0	X	0
1	1	X

c or d'

D_1 :

$b \backslash cd$	00	01	11	10
0	0	X	1	0
1	1	0	0	X

$b=0$:

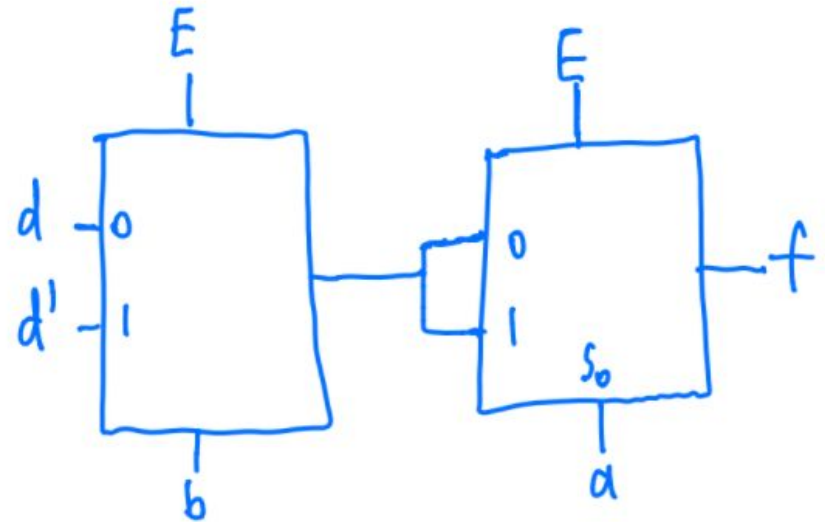
$c \backslash d$	0	1
0	0	X
1	0	1

d

$b=1$:

$c \backslash d$	0	1
0	1	0
1	X	0

d'



Q2.2 With 1 Mux

2.2

$a \backslash abc$	000	001	010	011	100	101	110	111	$D(a,b,c)$
0	0	x	x	1	0	0	1	x	D_0
1	1	1	0	x	x	1	0	0	D_1

D_0 :

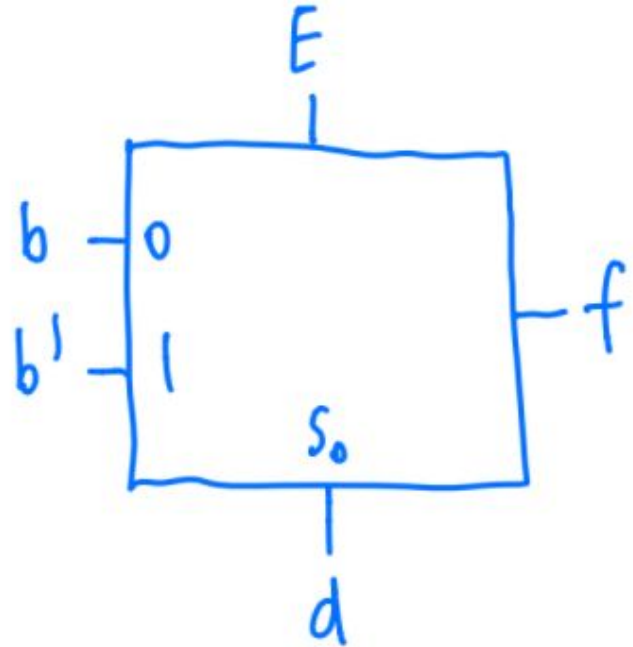
$a \backslash bc$	00	01	11	10
0	0	x	1	x
1	0	0	x	1

$D_0 = b$

D_1 :

$a \backslash bc$	00	01	11	10
0	1	1	x	0
1	x	1	0	0

$D_1 = b'$



Q3

id	a_i	b_i	c_i	Diff	B_0
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	1	1
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	0
6	1	1	0	0	0
7	1	1	1	1	1

$$D = B_0$$

$$Q = C_i$$

Diff

$\begin{matrix} BC \\ A \end{matrix}$	00	01	11	10
0	0	1	0	1
1	1	0	1	0

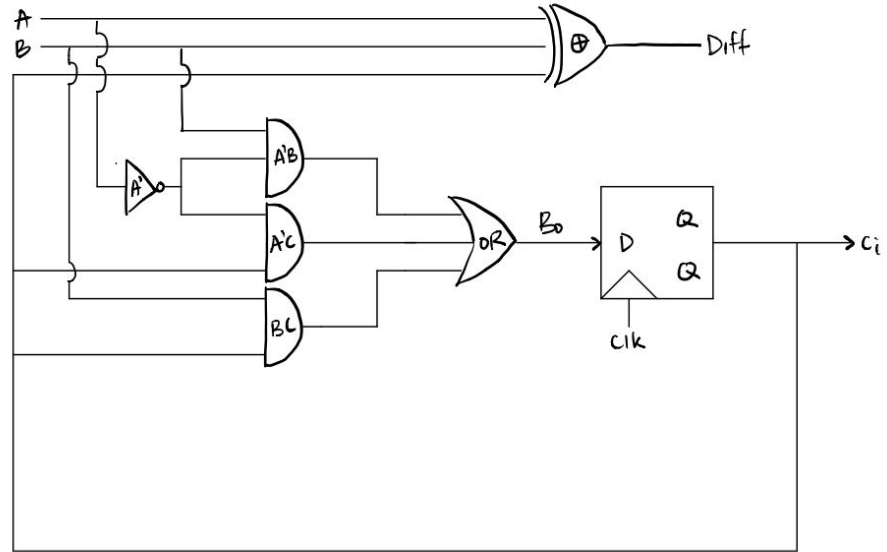
$$\text{Diff} = A \oplus B \oplus C$$

B_0

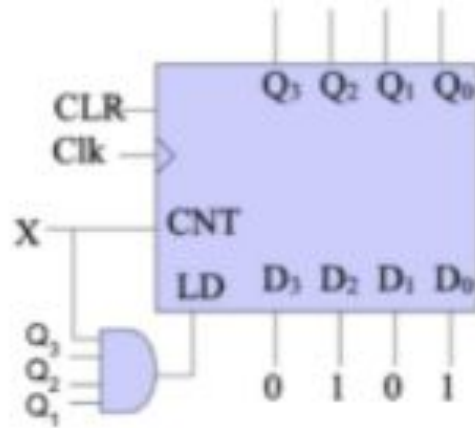
$\begin{matrix} BC \\ A \end{matrix}$	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$$B_0 = BC + A'C + A'B$$

$$\begin{aligned} \text{Diff} &= A \oplus B \oplus C \\ B_0 &= BC + A'C + A'B \end{aligned}$$



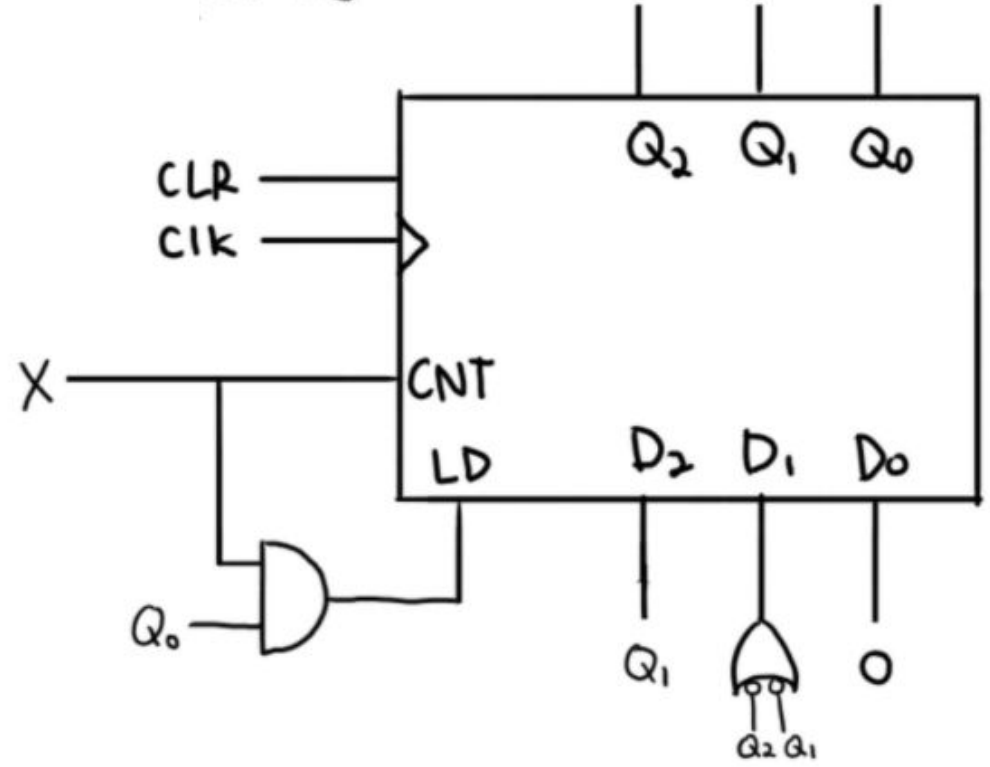
Q 4-a



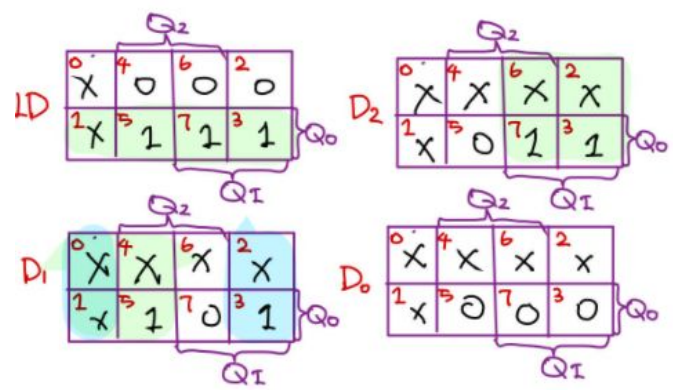
Q4-b

4.2

$LD = Q_0$
 $D_2 = Q_1$
 $D_1 = Q_2' + Q_1'$
 $D_0 = 0$

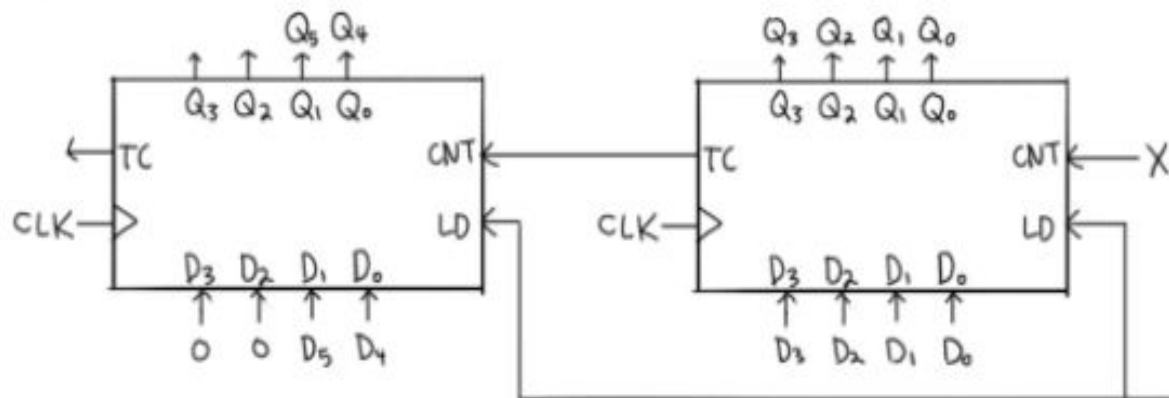


ID	$Q_2Q_1Q_0$	LD	D_2	D_1	D_0
0	000	X	X	X	X
1	001	X	X	X	X
2	010	0	X	X	X
3	011	1	1	1	0
4	100	0	X	X	X
5	101	1	0	1	0
6	110	0	X	X	X
7	111	1	1	0	0



Q4-3

4.3



5.1.1

S1: If start' goto S1 || done <= 1

S2: done <= 0 || A <= X || B <= Y || C <= Z;

S3: If A[7] goto S7 || A <= Add(A,B);

S4: If B[7] goto S7 || B <= Add(B,C);

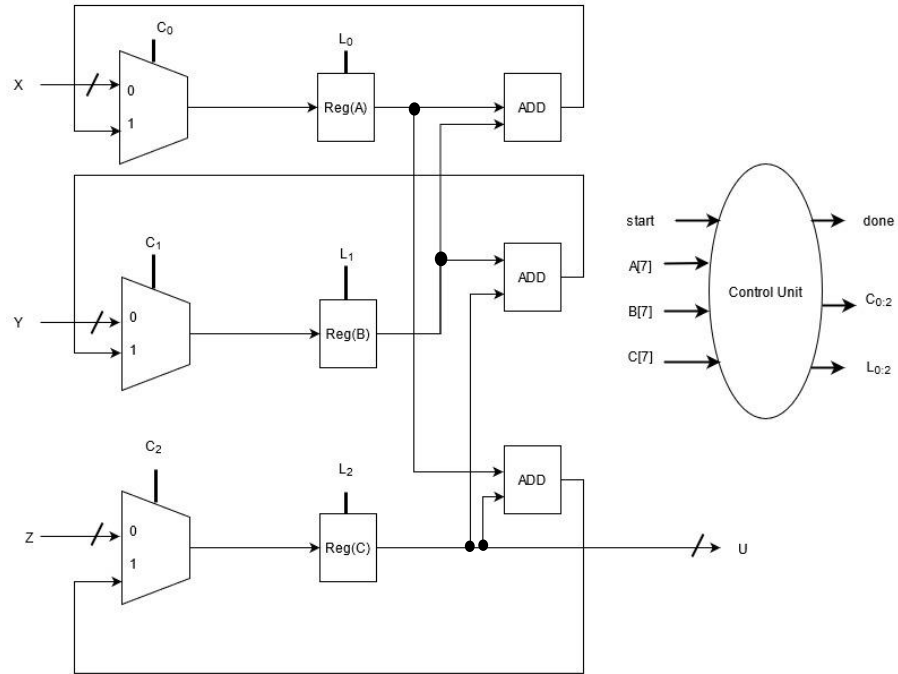
S5: If C[7] goto S7 || C <= Add(A,C);

S6: goto S3 || A <= Add(A,B);

S7: U <= C || goto S1;

State	Instruction
S2	A <= Load(X)
S2	B <= Load(Y)
S2	C <= Load(Z)
S3, S6	A <= Add(A,B)
S4	B <= Add(B,C)
S5	C <= Add(A,C)
S7	U <= C, wires

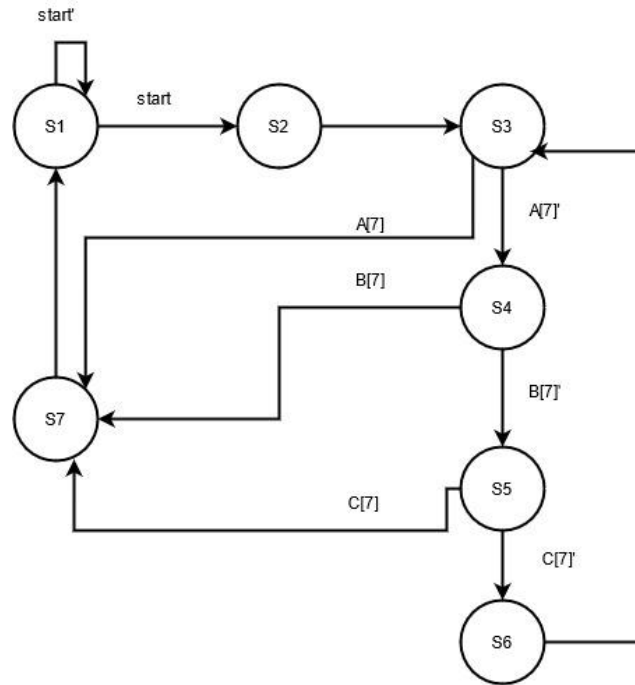
5.1.2



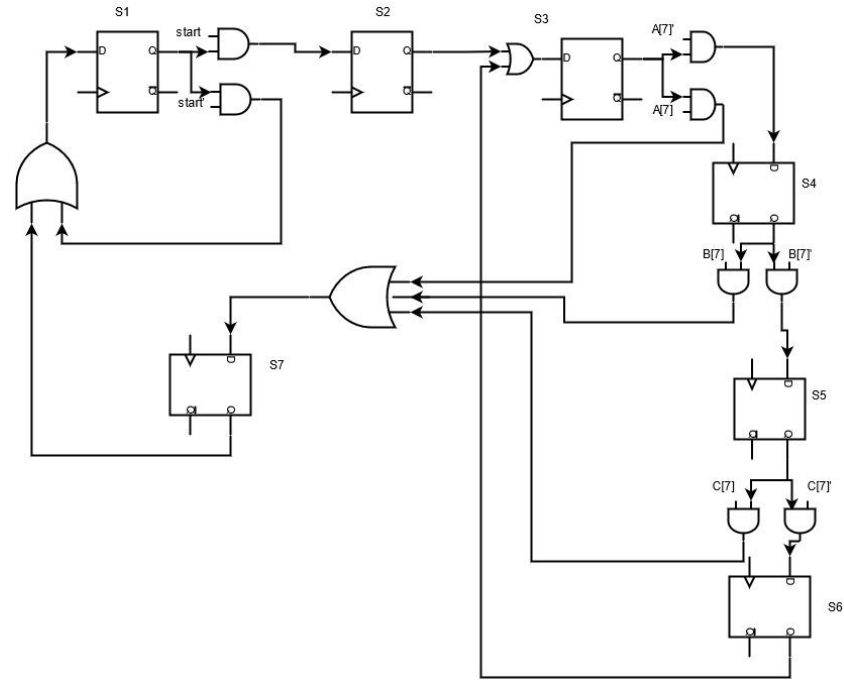
5.2.1

State	L_0	L_1	L_2	C_0	C_1	C_2	done
S1	0	0	0	-	-	-	1
S2	1	1	1	0	0	0	0
S3	1	0	0	1	-	-	0
S4	0	1	0	-	1	-	0
S5	0	0	1	-	-	1	0
S6	0	0	0	1	-	-	0
S7	0	0	0	-	-	-	0

5.2.2



5.2.3



B1

Evaluation of the i th bit of the sum uses the two carry bits c_{i-1} and d_{i-1} that respectively correspond to the 0th and the 1st bits of the $i - 1$ th sum.

One possible evaluation of the sum and carry bits at step i is given by

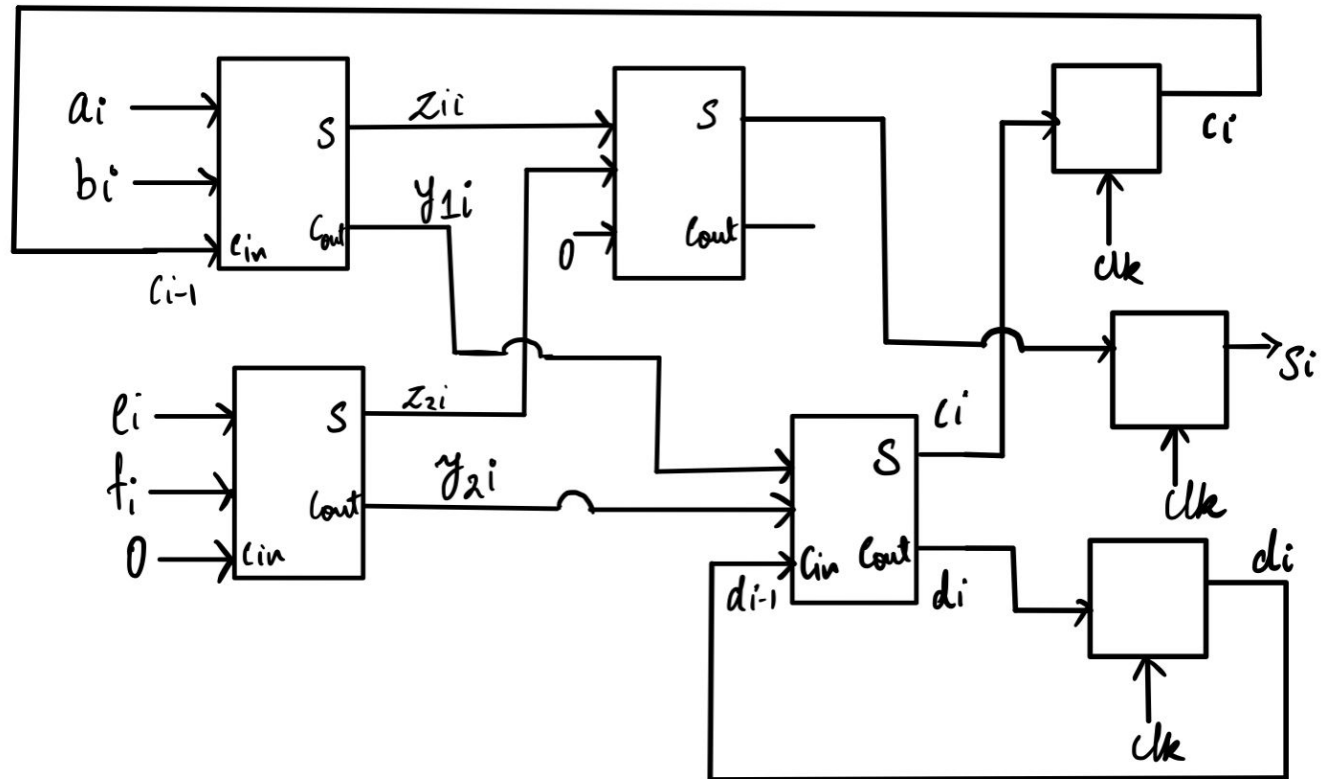
$$\begin{aligned}s_i &= a_i \oplus b_i \oplus c_{i-1} \oplus e_i \oplus f_i \\c_i &= d_{i-1} \oplus y_{i1} \oplus y_{2i} \\d_i &= d_{i-1}y_{i1} + d_{i-1}y_{i2} + y_{i1}y_{i2},\end{aligned}$$

Where the intermediate variables y_{1i} and y_{2i} are defined as

$$\begin{aligned}y_{1i} &= a_ib_i + a_ic_{i-1} + b_ic_{i-1} \text{ and} \\y_{2i} &= e_if_i\end{aligned}$$

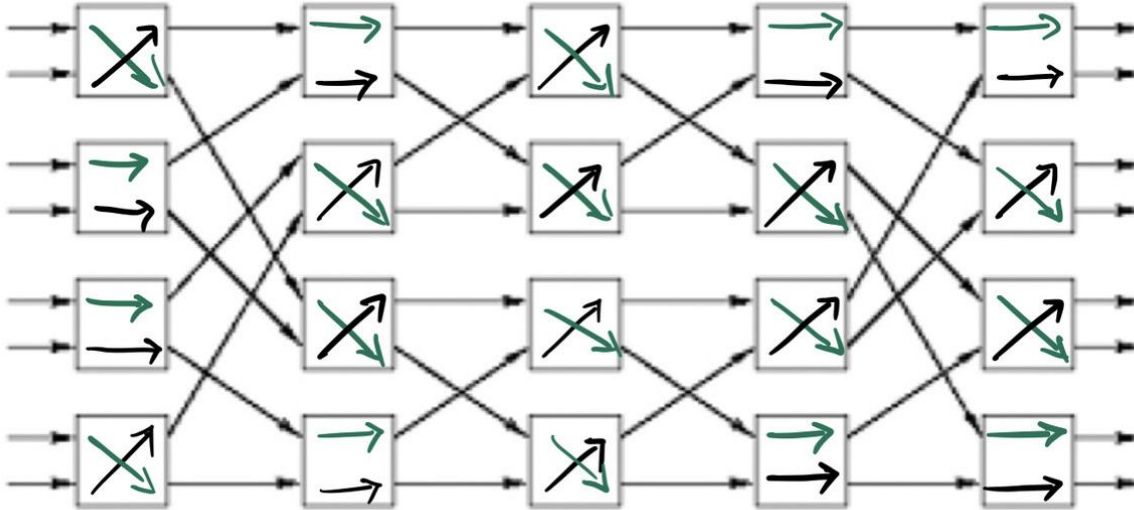
The following schematic implements this using a network of 4 full adders and 3 D flip flops

B1



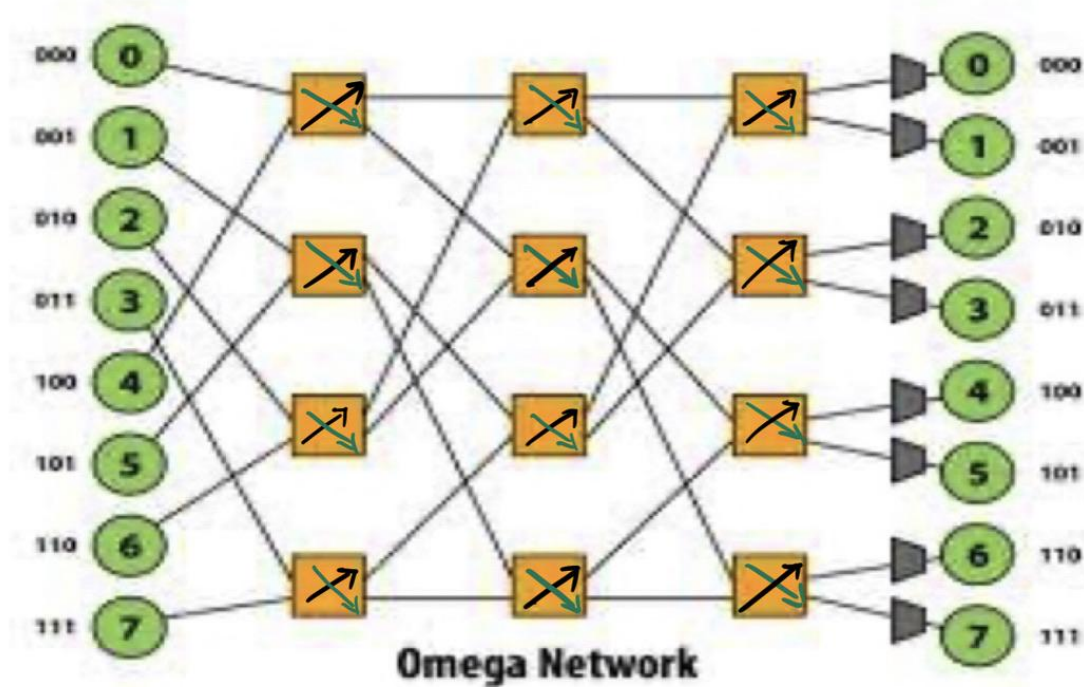
B2

Here is a routing combination that achieves the desired connections via the Benes network



B2

Here is the routing combination that achieves the desired connections via the Omega network



B.4 (way 1)

Multiply₁₆(X, Y, Z , start, done)

Input : $X[31 : 0], V[31 : 0]$

Output : $Z[64 : 0]$

local objects : $A[15 : 0], B[15 : 0], C[15 : 0], D[15 : 0], U[31 : 0], V[31 : 0], W[31 : 0], T[31 : 0], S[13 : 0]$

S_0 : if start' goto S_0 || done $\leftarrow 1$;

S_1 : $A \leftarrow X[15 : 0]$ || $B \leftarrow X[31 : 16]$ || $C \leftarrow Y[15 : 0]$ || $D \leftarrow Y[31 : 16]$ || done $\leftarrow 0$;

S_2 : **Multiply**₁₆(A, C, U , start, done) || **Multiply**₁₆(B, C, V , start, done) || **Multiply**₁₆(A, D, W , start, done) || **Multiply**₁₆(B, D, T , start, done)

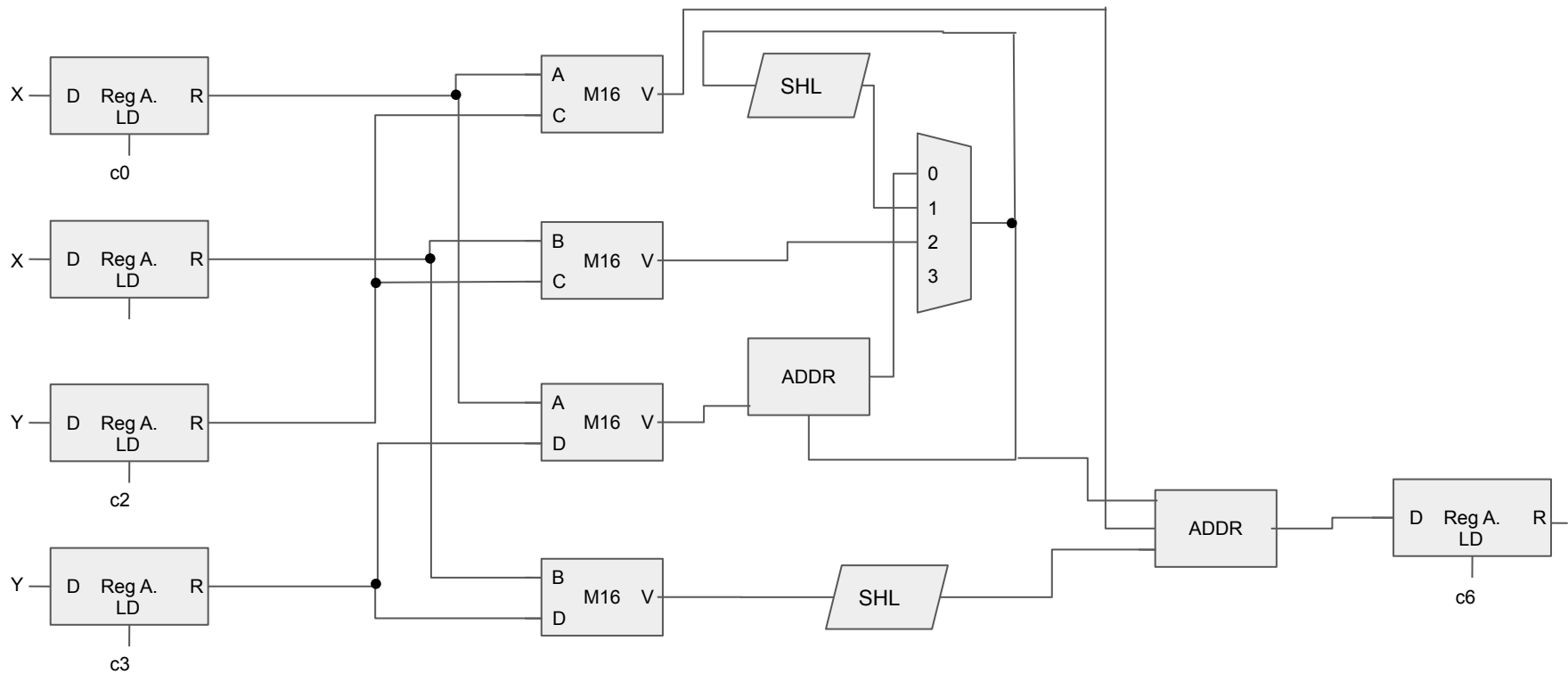
S_3 : $V \leftarrow \text{Add}(V, W)$;

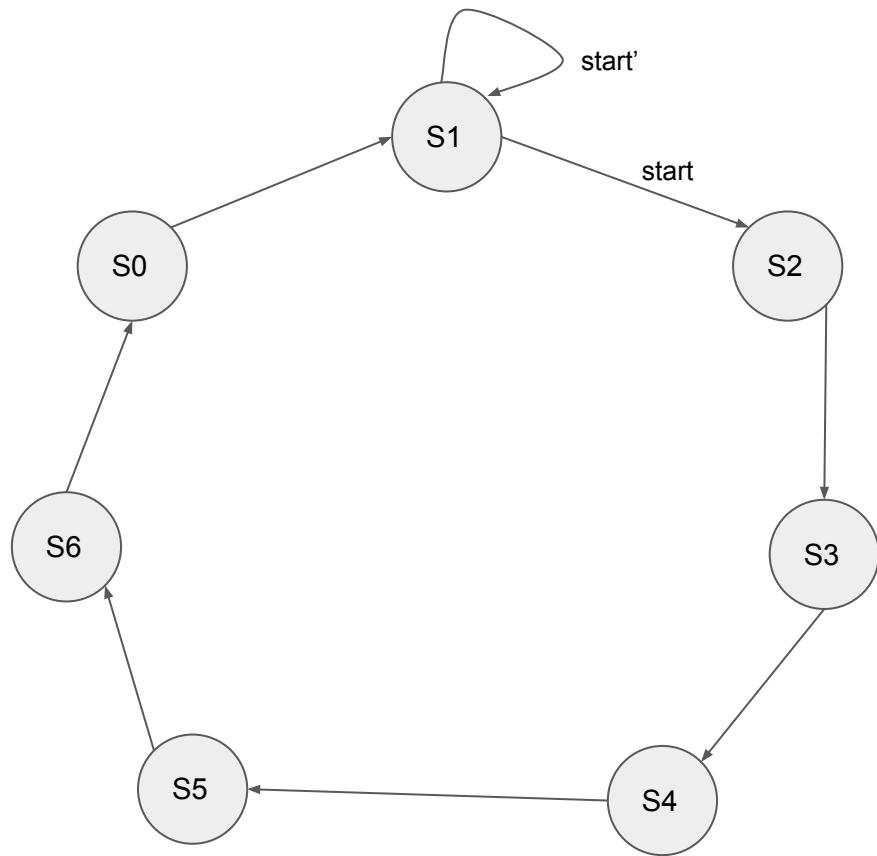
S_4 : $V \leftarrow V \ll 16$ || $T \leftarrow T \ll 32$;

S_5 : $\leftarrow \text{Add}(U, V, T)$;

S_6 : $Z \leftarrow S$ || done $\leftarrow 1$ || goto S_1 ;







	C_0	C_1	C_2	C_3	C_4	C_5	C_6	Done
S_1	1	1	1	1	x	x	0	1
S_2	0	0	0	0	x	x	0	0
S_3	0	0	0	0	1	0	0	0
S_4	0	0	0	0	0	0	0	0
S_5	0	0	0	0	0	0	0	0
S_6	0	0	0	0	x	x	1	0
S_7	0	0	0	0	x	x	0	1

B.4 (way 2)

Multiply₁₆(X, Y, Z , start, done)

Input : $X[31 : 0], V[31 : 0]$

Output : $Z[64 : 0]$

local objects : $A[15 : 0], B[15 : 0], C[31 : 0], M[63 : 0], i, \text{mstart}, \text{mdone}$

S_0 : if start' goto S_0 || done $\leftarrow 1$;

S_1 : $A \leftarrow X[31 : 16] || B \leftarrow Y[31 : 16] || i \leftarrow 0 || C \leftarrow 0$

|| $M \leftarrow 0 || \text{mstart} \leftarrow 1 || \text{done} \leftarrow 0 || \text{goto } S_5$;

S_2 : $M \leftarrow \text{shift}(M, L, 16) || A \leftarrow X[15 : 0] || i \leftarrow i + 1$

|| $C \leftarrow 0 || \text{goto } S_5$;

S_3 : $A \leftarrow X[31 : 16] || B \leftarrow Y[15 : 0] || i \leftarrow i + 1 || C \leftarrow 0 || \text{goto } S_5$;

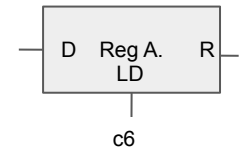
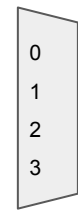
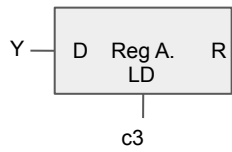
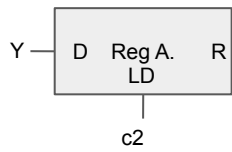
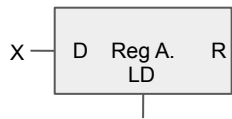
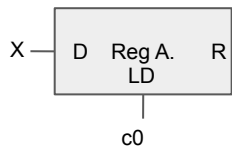
S_4 : $M \leftarrow \text{shift}(M, L, 16) || A \leftarrow X[15 : 0] || i \leftarrow i + 1 || C \leftarrow 0$;

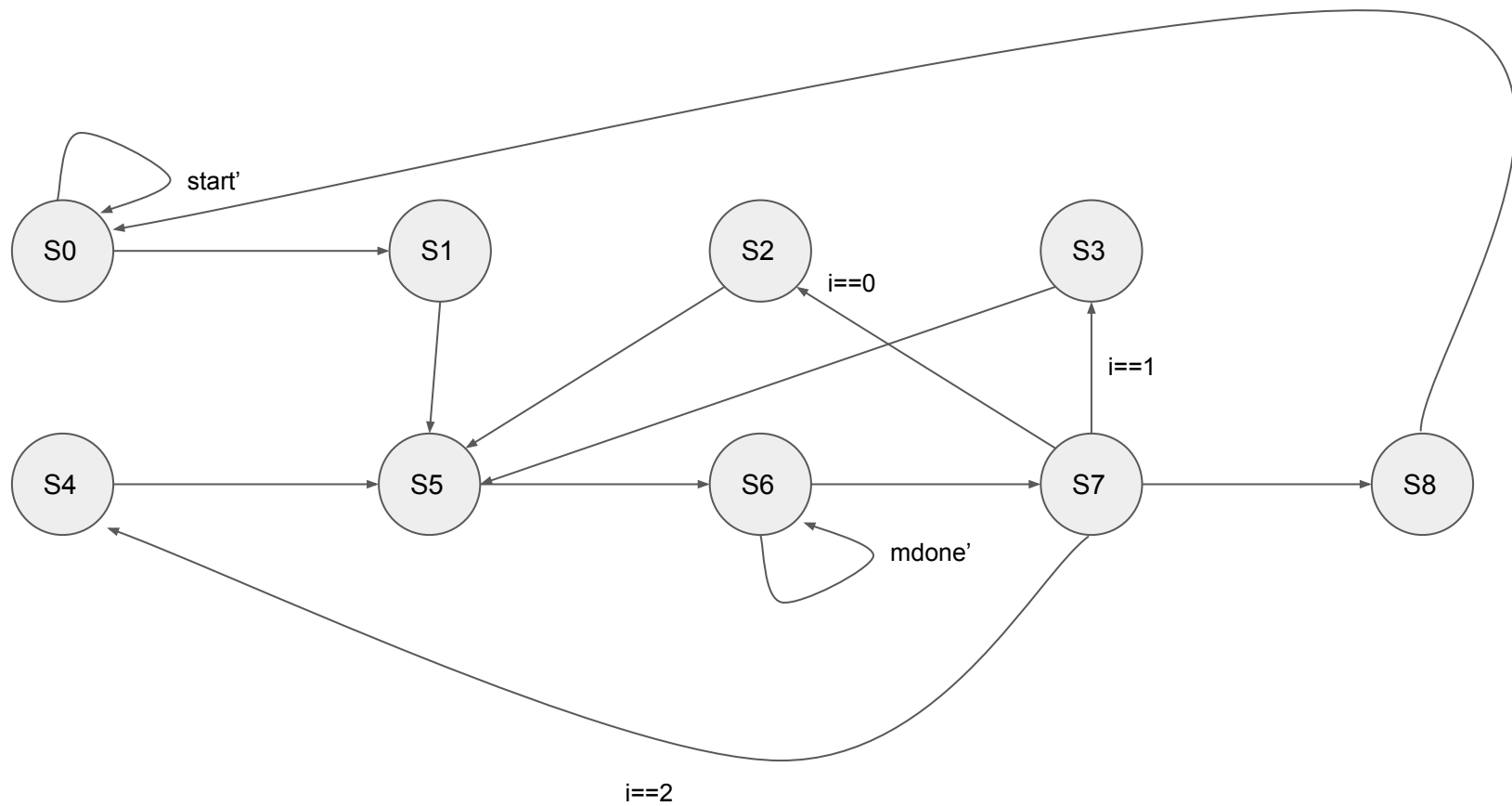
S_5 : **Multiply**₁₆($A, B, C, \text{mstart}, \text{mdone}$);

S_6 : if mdone' goto S_6 || mstart $\leftarrow 1$;

S_7 : if $i == 0$ goto S_2 || $i == 1$ goto S_3 || $i == 2$ goto S_4 || $M[15 : 0] \leftarrow \text{Add}(C, M[15 : 0])$;

S_8 $Z \leftarrow M || \text{goto } S_0$:





	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Done	C_9	C_{10}	C_{11}	C_{12}
S_0	x	x	0	0	0	0	x	0	0	1	0	0	0	0
S_1	1	1	1	1	0	0	x	0	0	0	0	1	1	1
S_2	0	x	1	0	0	0	1	1	0	0	1	0	1	0
S_3	1	0	1	1	0	0	x	0	0	0	1	0	1	0
S_4	0	x	1	0	0	0	0	0	0	0	1	0	1	0
S_5	x	x	0	0	1	1	x	0	0	0	0	0	0	0
S_6	x	x	0	0	0	0	x	0	0	0	0	0	0	0
S_7	x	x	0	0	0	0	1	1	0	0	0	0	0	0
S_8	x	x	0	0	0	0	x	0	1	0	0	0	0	0