

CSE 140 - SP 21

Discussion Hour

Overview

Work-through example that ties together

- Finite state machine
- State Assignment
- Mealy to Moore machine conversion
- Bonus: Implement via D flip flops and combinatorial logic
- Similar to HW 4-5 and Lecture 10

Structure

- Interactive exercise
- Use: Paper/Tablet, Pen/Pencil/Stylus

Problem Statement

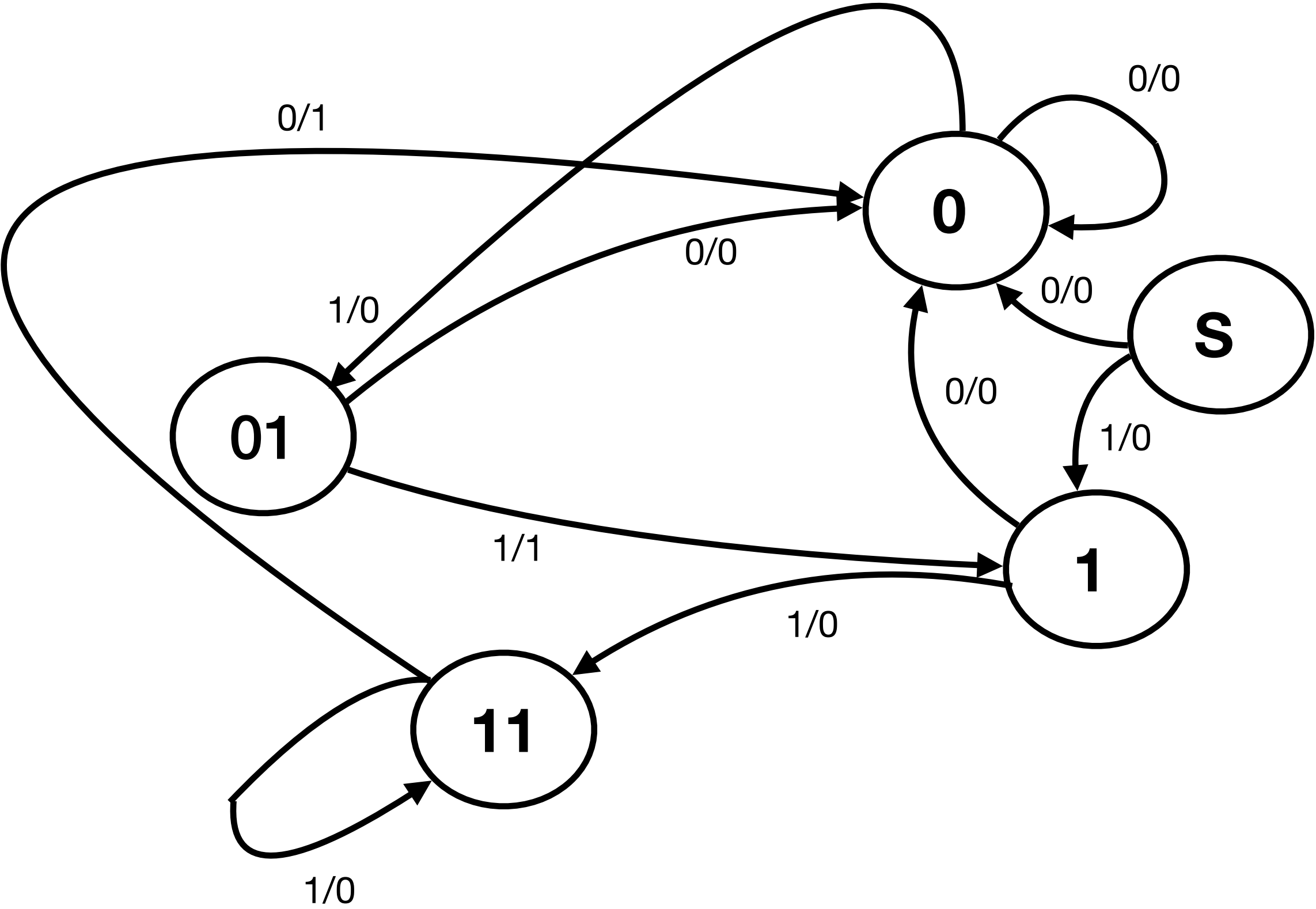
- Construct a system that recognizes and outputs $y(t)=1$ each time the patterns $x(t-2,t) = 110$ or 011 occur in the input
- E.g. $y(3)=1$ if $x(1,3)=110$
- Implement it as a Mealy Machine and convert it to a Moore Machine
- Show the state diagram, state assignment and simplified state equations via KMaps

Finite State Diagram - Attempt 1

Attempt 1

Patterns to recognize

- 011, 110



Finite State Diagram - Identify Issue

Attempt 1

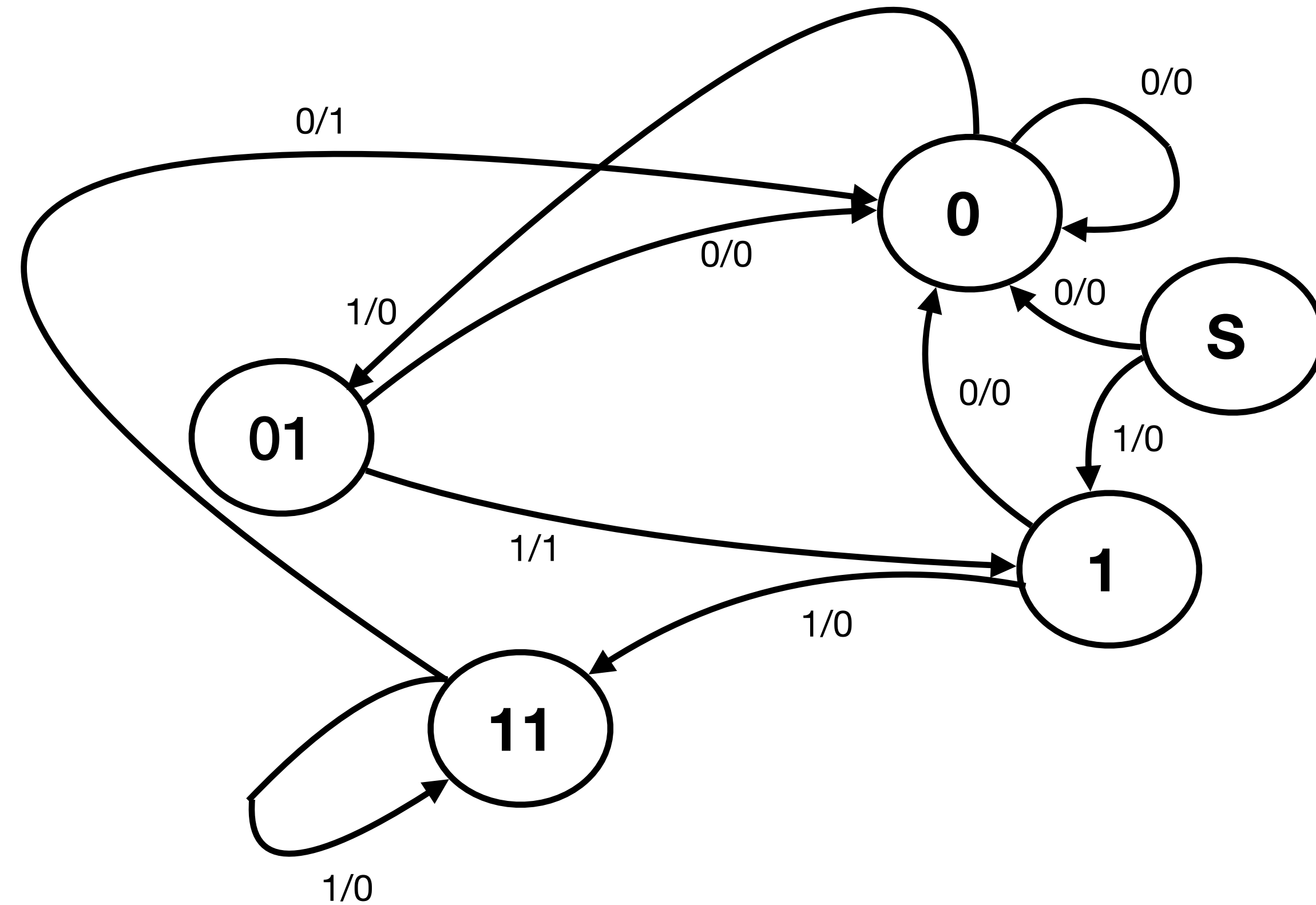
Patterns to recognize:
011, 110

Are there sequences in
which patterns go
undetected?

Which one(s)?

0110

Suggest a fix for 0110?



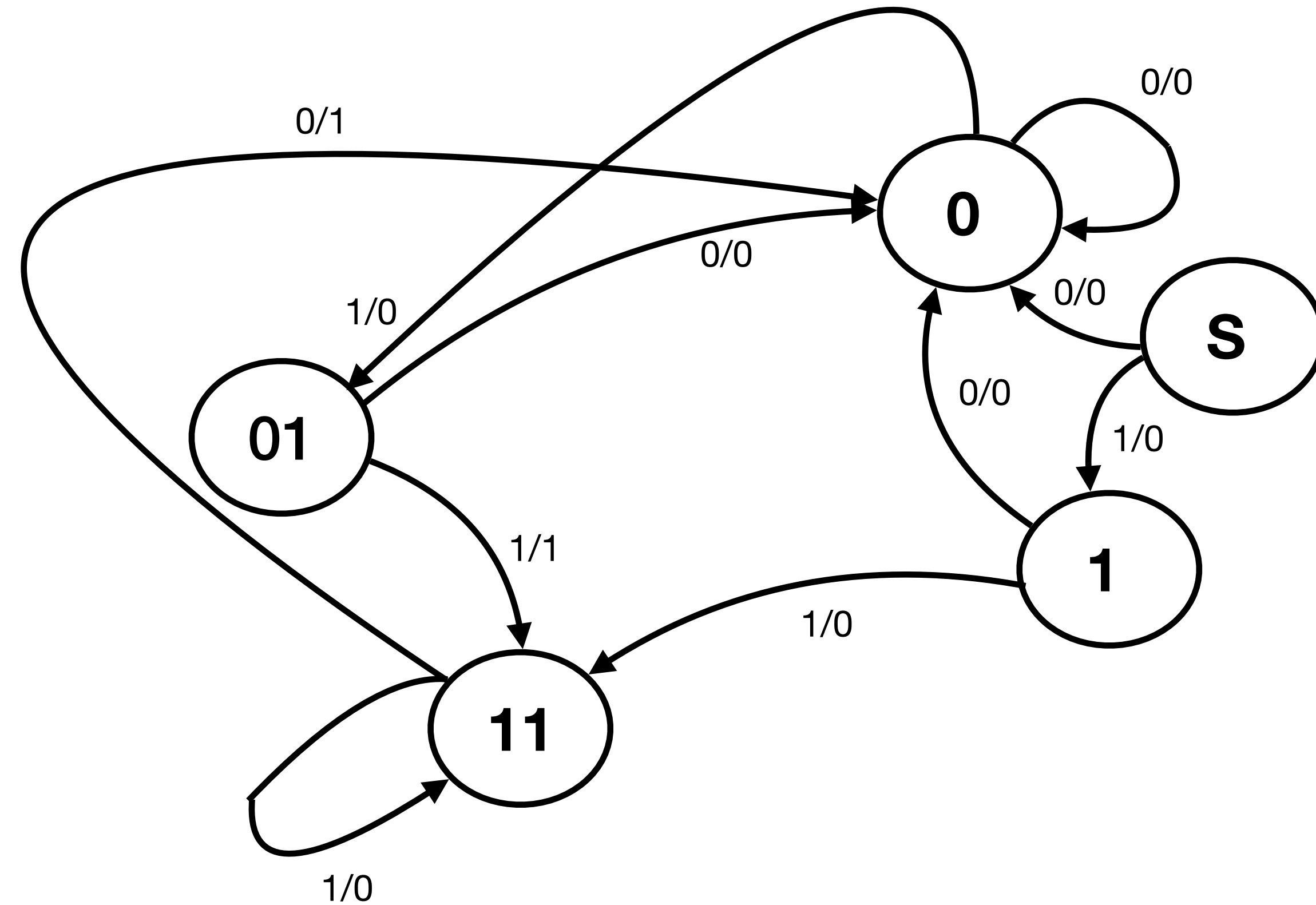
Fixing State Diagram

Patterns to recognize:
011, 110

0110 issue is resolved

Common pitfall: Need to
consider similarity
between patterns outside
of just prefix similarity

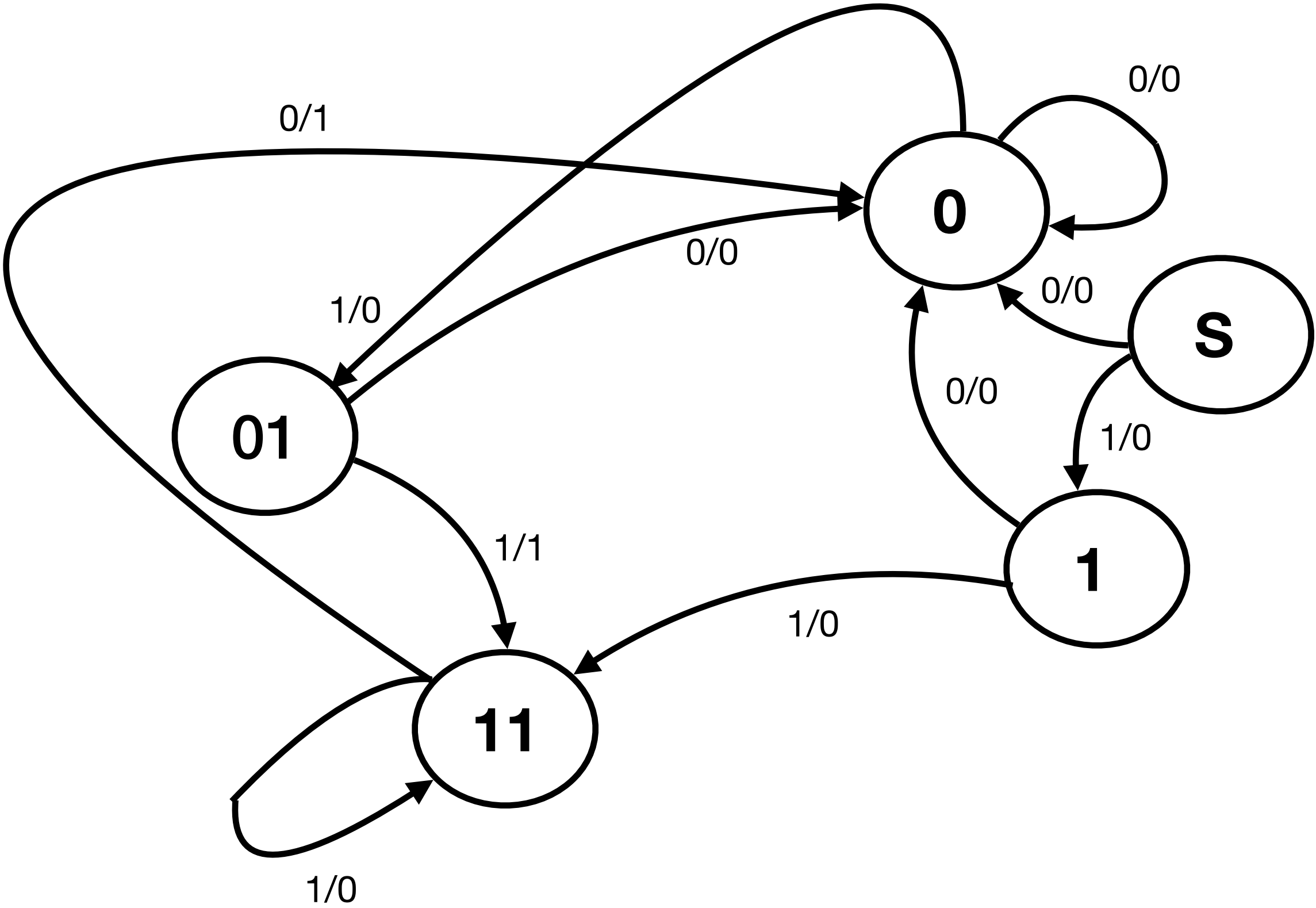
Finite state diagram



State Diagram and Assignment

Current State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
S	0	1	0	0
0	0	01	0	0
1	0	11	0	0
01	0	11	0	1
11	0	11	1	0

Current State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	001	010	0	0
001	001	011	0	0
010	001	100	0	0
011	001	100	0	1
100	001	100	1	0



KMap(s) for Mealy Machine

Represent states as $Q_2Q_1Q_0$

Current State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	001	010	0	0
001	001	011	0	0
010	001	100	0	0
011	001	100	0	1
100	001	100	1	0

KMap(s) for Mealy Machine

Represent states as $Q_2Q_1Q_0$

Current State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	001	010	0	0
001	001	011	0	0
010	001	100	0	0
011	001	100	0	1
100	001	100	1	0

Q_0	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$	1	1	1	1
$\overline{Q_2}X$		1		
Q_2X		X	X	X
$Q_2\overline{X}$	1	X	X	X

KMap(s) for Mealy Machine

Represent states as $Q_2Q_1Q_0$

Current State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	001	010	0	0
001	001	011	0	0
010	001	100	0	0
011	001	100	0	1
100	001	100	1	0

Q_0	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$	1	1	1	1
$\bar{Q}_2X$		1		
Q_2X		X	X	X
$Q_2\bar{X}$	1	X	X	X

Q_1	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$				
$\bar{Q}_2X$	1	1		
Q_2X		X	X	X
$Q_2\bar{X}$		X	X	X

KMap(s) for Mealy Machine

Represent states as $Q_2Q_1Q_0$

Current State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	001	010	0	0
001	001	011	0	0
010	001	100	0	0
011	001	100	0	1
100	001	100	1	0

Q_0	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$	1	1	1	1
$\bar{Q}_2X$		1		
Q_2X		X	X	X
$Q_2\bar{X}$	1	X	X	X

Q_1	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$				
$\bar{Q}_2X$	1	1		
Q_2X		X	X	X
$Q_2\bar{X}$		X	X	X

Q_2	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$				
$\bar{Q}_2X$			1	1
Q_2X		X	X	X
$Q_2\bar{X}$	1	X	X	X

KMap(s) for Mealy Machine

Represent states as $Q_2Q_1Q_0$

Current State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	001	010	0	0
001	001	011	0	0
010	001	100	0	0
011	001	100	0	1
100	001	100	1	0

Y	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$			1	
Q_2X		X	X	X
$Q_2\overline{X}$	1	X	X	X

Q_0	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$	1	1	1	1
$\overline{Q_2}X$		1		
Q_2X		X	X	X
$Q_2\overline{X}$	1	X	X	X

Q_1	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$	1	1		
Q_2X		X	X	X
$Q_2\overline{X}$		X	X	X

Q_2	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$			1	1
Q_2X		X	X	X
$Q_2\overline{X}$	1	X	X	X

State Equations for Mealy Machine

Q_0	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$	1	1	1	1
$\bar{Q}_2X$		1		
Q_2X		X	X	X
$Q_2\bar{X}$	1	X	X	X

Q_1	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$				
$\bar{Q}_2X$	1	1		
Q_2X		X	X	X
$Q_2\bar{X}$		X	X	X

Q_2	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$				
$\bar{Q}_2X$			1	1
Q_2X		X	X	X
$Q_2\bar{X}$	1	X	X	X

$$Q_0(t+1) = \bar{Q}_1(t)Q_0(t) + \bar{Q}_2(t)\bar{X}(t) + Q_2(t)\bar{X}(t)$$

$$Q_1(t+1) = \bar{Q}_2(t)\bar{Q}_1(t)X(t)$$

$$Q_2(t+1) = Q_1(t)X(t) + Q_2(t)\bar{Q}_1(t)\bar{X}(t)$$

$$Y(t+1) = Q_1(t)Q_0(t)X(t) + Q_2(t)\bar{Q}_1(t)\bar{X}(t)$$

Y	$\bar{Q}_1\bar{Q}_0$	$\bar{Q}_1Q_0$	Q_1Q_0	$Q_1\bar{Q}_0$
$\bar{Q}_2\bar{X}$				
$\bar{Q}_2X$			1	
Q_2X		X	X	X
$Q_2\bar{X}$	1	X	X	X

Converting to a Moore Machine

Current State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
000	001	010	0	0
001	001	011	0	0
010	001	100	0	0
011	001	100	0	1
100	001	100	1	0

NState/ Output	Moore State	Next Moore State		Output
		$x = 0$	$x = 1$	
000,0	000	001	010	0
001,0	001	001	011	0
010,0	010	001	100	0
011,0	011	001	101	0
100,0	100	110	001	0
100,1	101	110	001	1
001,1	110	001	010	1

- Map next state/output pair to new state
- Is there an upper bound on how many more states a Moore Machine requires over Mealy for the binary output case?
- 2x
- How many states do we need here?
- $3+2*2=7$

KMap(s) for Moore Machine

Represent states as $Q_2Q_1Q_0$

Moore State	Next Moore State		Output
	$x = 0$	$x = 1$	
000	001	010	0
001	001	011	0
010	001	100	0
011	001	101	0
100	110	001	0
101	110	001	1
110	001	010	1

Q_0	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$	1	1	1	1
$\overline{Q_2}X$		1	1	
Q_2X	1	1	X	
$Q_2\overline{X}$			X	1

KMap(s) for Moore Machine

Represent states as $Q_2Q_1Q_0$

Moore State	Next Moore State		Output
	$x = 0$	$x = 1$	
000	001	010	0
001	001	011	0
010	001	100	0
011	001	101	0
100	110	001	0
101	110	001	1
110	001	010	1

Q_0	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$	1	1	1	1
$\overline{Q_2}X$		1	1	
Q_2X	1	1	X	
$Q_2\overline{X}$			X	1

Q_1	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$	1	1		
Q_2X			X	1
$Q_2\overline{X}$	1	1	X	

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	$x = 0$	$x = 1$	
000	001	010	0
001	001	011	0
010	001	100	0
011	001	101	0
100	110	001	0
101	110	001	1
110	001	010	1

Q_0	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$	1	1	1	1
$\overline{Q_2}X$		1	1	
Q_2X	1	1	X	
$Q_2\overline{X}$			X	1

Q_2	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$			1	1
Q_2X			X	
$Q_2\overline{X}$	1	1	X	

Q_1	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$	1	1		
Q_2X			X	1
$Q_2\overline{X}$	1	1	X	

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Moore State	Next Moore State		Output
	$x = 0$	$x = 1$	
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001	001	011	0
010	001	100	0
011	001	101	0
100	110	001	0
101	110	001	1
110	001	010	1

Q_0	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$	1	1	1	1
$\overline{Q_2}X$		1	1	
Q_2X	1	1	X	
$Q_2\overline{X}$			X	1

Q_1	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$	1	1		
Q_2X			X	1
$Q_2\overline{X}$	1	1	X	

Q_2	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$			1	1
Q_2X			X	
$Q_2\overline{X}$	1	1	X	

Y	$\overline{Q_1}\overline{Q_0}$	$\overline{Q_1}Q_0$	Q_1Q_0	$Q_1\overline{Q_0}$
$\overline{Q_2}$				
Q_2		1	X	1

State Equations for Moore Machine

$Q_0(t + 1) = \overline{Q_2}(t)\overline{X}(t) + \overline{Q_2}(t)Q_0(t) + Q_2(t)\overline{Q_1}(t)X(t) + Q_2(t)Q_1(t)\overline{X}(t)$

$Q_1(t + 1) = \overline{Q_2}(t)\overline{Q_1}(t)X(t) + Q_2(t)\overline{Q_1}(t)\overline{X}(t) + Q_2(t)Q_1(t)X(t)$

$Q_2(t + 1) = Q_2(t)\overline{Q_1}(t)\overline{X}(t) + \overline{Q_2}(t)Q_1(t)X(t)$

$Y(t + 1) = Q_2(t)Q_0(t) + q_2(t)Q_1(t)$

Q_0	$\overline{q_1}\overline{q_0}$	$\overline{q_1}q_0$	q_1q_0	$q_1\overline{q_0}$
$\overline{Q_2}\overline{X}$	1	1	1	1
$\overline{Q_2}X$		1	1	
Q_2X	1	1	X	
$Q_2\overline{X}$			X	1

Q_2	$\overline{q_1}\overline{q_0}$	$\overline{q_1}q_0$	q_1q_0	$q_1\overline{q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$			1	1
Q_2X			X	
$Q_2\overline{X}$	1	1	X	

Q_1	$\overline{q_1}\overline{q_0}$	$\overline{q_1}q_0$	q_1q_0	$q_1\overline{q_0}$
$\overline{Q_2}\overline{X}$				
$\overline{Q_2}X$	1	1		
Q_2X			X	1
$Q_2\overline{X}$	1	1	X	

Y	$\overline{q_1}\overline{q_0}$	$\overline{q_1}q_0$	q_1q_0	$q_1\overline{q_0}$
$\overline{Q_2}$				
Q_2		1	X	1