

Discussion 3

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Outline

- Practice Problem for HW2 Q2
- Universal Sets
- K-maps

Problem: A majority function produces output $f=1$ when **more than half** of the inputs are true; otherwise, $f=0$. For example, $f(1,1,0,0) = 0$, $f(1,0,1,1)=1$

- Write out the truth table of the 4-input majority function
- Construct a 3-input majority function using 4-input majority function modules as basic building blocks (no other gates except 0, 1, constants, and inverters).

Truth Table

a	b	c	d	f(a,b,c,d)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Construct a 3-input majority function using 4-input majority function modules as basic building blocks

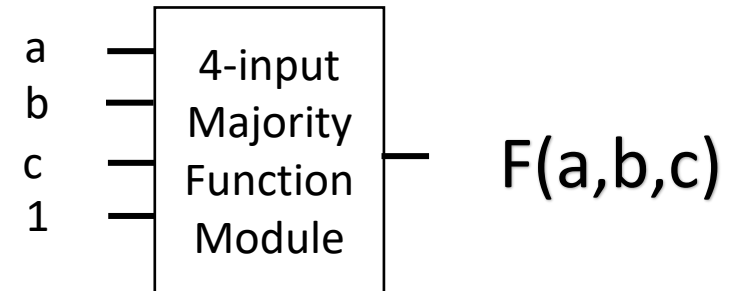
a	b	c	$F(a,b,c)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Truth table of a 3-input majority function

Construct a 3-input majority function using 4-input majority function modules as basic building blocks

a	b	c	$F(a,b,c)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Truth table of a 3-input majority function



Universal Sets

- Definition: A set of gates such that every switching function can be implemented with the gates in this set.
- Criterion: If the set of gates can implement AND, OR, and NOT gates, then the set is universal.
- Examples of universal set:
 - {AND, NOT}
 - {OR, NOT}

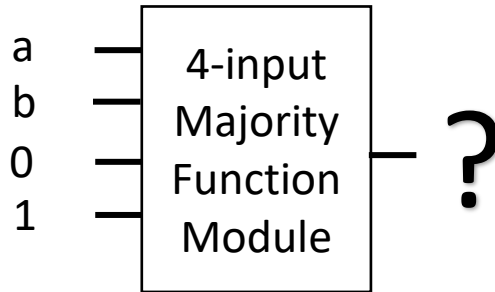
Problem: Is {NAND} universal?

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- Yes.
- We can implement NOT with $\text{NAND}(a,1) = a'$
- We can implement AND with two NAND gates.

Hints of HW2 Q2

- You are given inverters
- See figure below



K-map: all possible minimal two-level sum of products expressions?

		ab			
		00	01	11	10
cd	00	1	0	1	1
	01	1	1	0	0
	11	1	1	1	X
	10	0	X	1	X

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level sum of products expressions?

ab

cd

	00	01	11	10
00	1	0	1	1
01	1	1	0	0
11	1	1	1	X
10	0	X	1	X

Essential Prime Implicants:

$$\sum m(8,10,12,14)$$

$$\sum m(1,3,5,7)$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level sum of products expressions?

ab

cd

	00	01	11	10
00	1 [•]	0	1 [•]	1
01	1	1 [•]	0	0
11	1	1	1 [•]	X
10	0	X	1	X

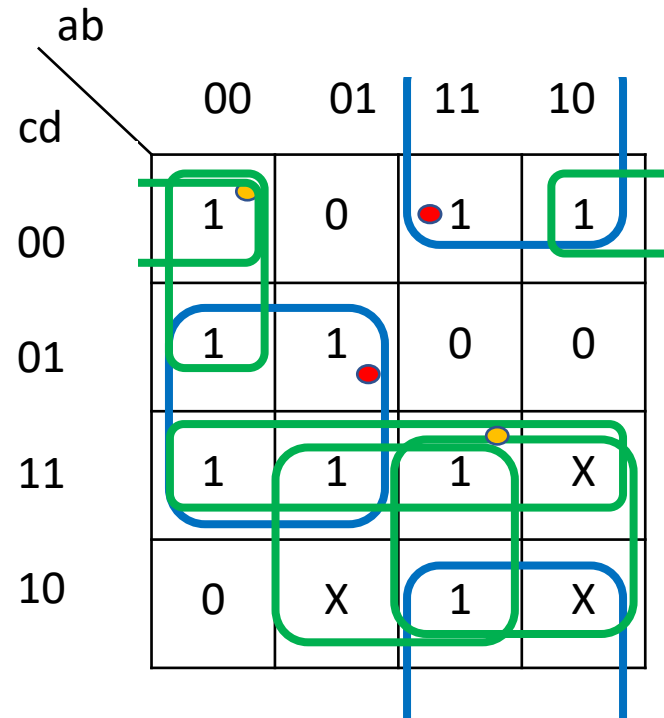
Essential Prime Implicants:

$$\sum m(8,10,12,14)$$

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0	4	12	8
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K-map: all possible minimal two-level sum of products expressions?



Essential Prime Implicants:

$$\sum m(8,10,12,14)$$

$$\sum m(1,3,5,7)$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

Prime Implicants that cover the yellow circled cells:

$$\sum m(3,7,11,15) \text{ or } \sum m(6,7,14,15) \text{ or } \sum m(10,11,14,15)$$

$$\sum m(0,1) \text{ or } \sum m(0,8)$$

K-map: all possible minimal two-level sum of products expressions?

- $F(a,b,c,d) = ad' + a'd + cd + a'b'c'$
- $F(a,b,c,d) = ad' + a'd + cd + b'c'd'$
- $F(a,b,c,d) = ad' + a'd + bc + a'b'c'$
- $F(a,b,c,d) = ad' + a'd + bc + b'c'd'$
- $F(a,b,c,d) = ad' + a'd + ac + a'b'c'$
- $F(a,b,c,d) = ad' + a'd + ac + b'c'd'$

K-map: all possible minimal two-level product of sums expressions?

		ab			
		00	01	11	10
cd	00	1	0	1	1
	01	1	1	0	0
	11	1	1	1	X
	10	0	X	1	X

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level product of sums expressions?

		ab			
		00	01	11	10
cd	00	1	0	1	1
	01	1	1	0	0
	11	1	1	1	X
	10	0	X	1	X

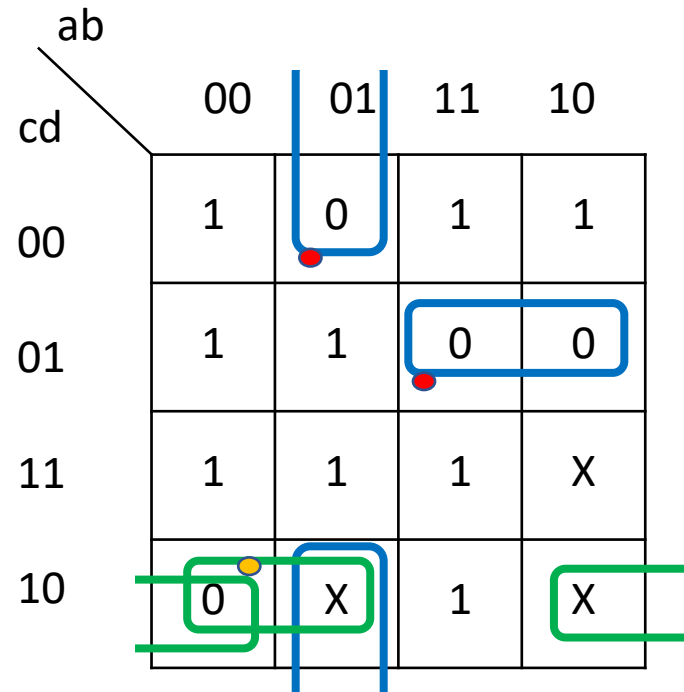
Essential Prime Implicates:

$$\prod_{M(9,13)}$$

$$\prod_{M(4,6)}$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level product of sums expressions?



Essential Prime Implicates:

$$\prod M(9,13)$$

$$\prod M(4,6)$$

Prime Implicates that cover the yellow circled cells:

$$\prod M(2,6) \text{ or } \prod M(2,10)$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level product of sums expressions?

- $F(a,b,c,d) = (a' + c + d')(a + b' + d)(a + c' + d)$
- $F(a,b,c,d) = (a' + c + d')(a + b' + d)(b + c' + d)$