

Discussion 2

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Outline

- HW1 Q4 Practice Problem
- HW1 Q5 Practice Problem
- XOR
- Half/Full adders
- Half subtractors
- Shannon's Expansion

Problem: for a 3-bit input (a,b,c) and 1-bit output y, output 1 when the number of 1's in input is even and output 0 otherwise

Truth Table

a	b	c	y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Minterms & Maxterms

a	b	c	y	minterm	MAXterm
0	0	0	1	$a'b'c'$	$a+b+c$
0	0	1	0	$a'b'c$	$a+b+c'$
0	1	0	0	$a'bc'$	$a+b'+c$
0	1	1	1	$a'bc$	$a+b'+c'$
1	0	0	0	$ab'c'$	$a'+b+c$
1	0	1	1	$ab'c$	$a'+b+c'$
1	1	0	1	abc'	$a'+b'+c$
1	1	1	0	abc	$a'+b'+c'$

Minterms & Maxterms

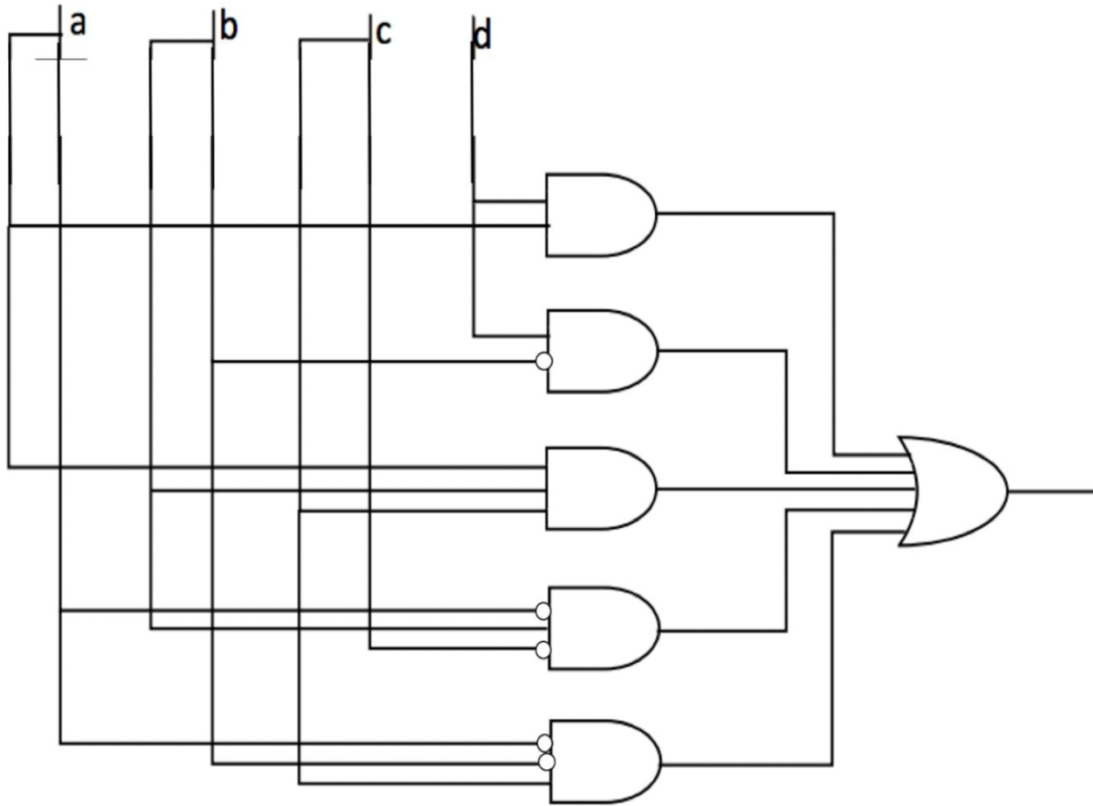
a	b	c	y	minterm	MAXterm
0	0	0	1	$a'b'c'$	$a+b+c$
0	0	1	0	$a'b'c$	$a+b+c'$
0	1	0	0	$a'bc'$	$a+b'+c$
0	1	1	1	$a'bc$	$a+b'+c'$
1	0	0	0	$ab'c'$	$a'+b+c$
1	0	1	1	$ab'c$	$a'+b+c'$
1	1	0	1	abc'	$a'+b'+c$
1	1	1	0	abc	$a'+b'+c'$

$$\text{SOP: } y = a'b'c' + a'bc + ab'c + abc'$$

$$\text{POS: } y = (a+b+c')(a+b'+c)(a'+b+c)(a'+b'+c')$$

Problem: Draw the logic diagram of
the Boolean expression
 $ad + b'd + a'bc' + a'b'c + abc$

$$ad + b'd + a'bc' + a'b'c + abc$$



#nets: #variables + #operators

#pins: #literals + #gates*2 - 1

- #Variables:
 - 4
- #Literals:
 - 13
- #Operators:
 - 6
- #Gates:
 - 6
- #Nets:
 - 10
- #Pins:
 - 24

XOR

a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

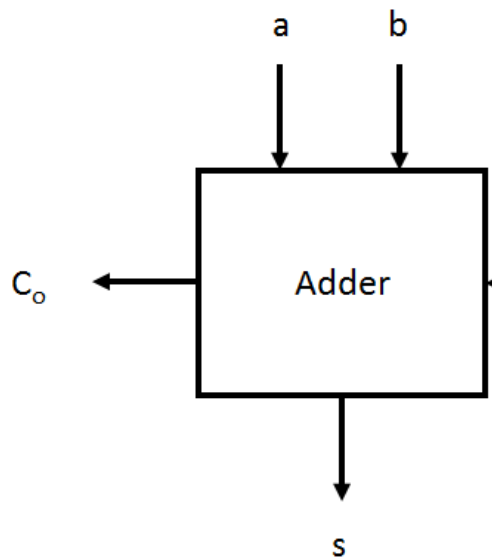
XOR - Properties

1. Commutative: $x \oplus y = y \oplus x$
2. Associative: $(x \oplus y) \oplus z = x \oplus (y \oplus z)$
3. $1 \oplus x = x'$; $0 \oplus x = x$
4. $x \oplus x = 0$; $x \oplus x' = 1$

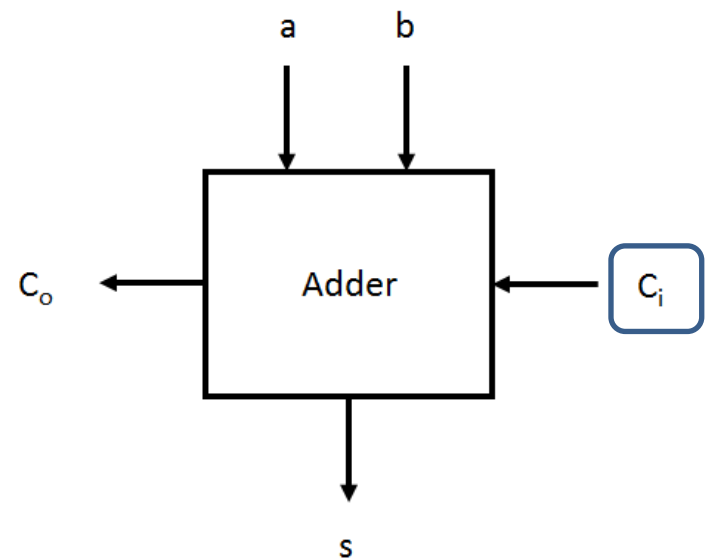
XOR - Problem

- Given a binary string with the numbers of bit 1 and bit 0 being n and m , what is the output of XOR every bit in this binary string?

Half / Full Adders



1-bit half adder



1-bit full adder

Half Adders

a	b	s	c _o
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Full Adder

a	b	c_i	c_o	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Full Adder

a	b	c_i	c_o	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Trick:

Add the three bits a, b, and, c_i and then express the result with base=2

e.g. $a=1$, $b=1$, $c_i=1$
=>

$$a+b+c_i = 3$$

$$3 \text{ (base 2)} = 11$$

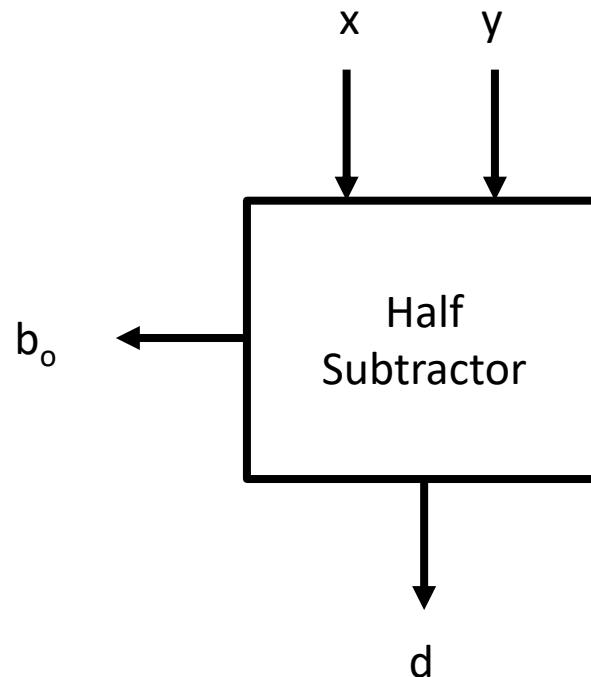
Full Adder – Boolean Expression

a	b	c _i	c _o	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = a \oplus b \oplus c_i$$
$$c_o = c_i(a \oplus b) + ab$$

Half Subtractor

x	y	d	B _o
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



Trick:

Compute $x-y$ and check if you need a borrow bit for negative

Shannon's Expansion

- Sum-of-Product form:

$$f(x, y) = xf(1, y) + x'f(0, y)$$

- Product-of-Sum form:

$$f(x, y) = (x + f(0, y)) \cdot (x' + f(1, y))$$

Shannon's Expansion – simplify the expression

$$f(x, y) = (x' + y) \oplus (x + y') \oplus xy \oplus x'y' \oplus x'$$

XOR - Properties

1. Commutative: $x \oplus y = y \oplus x$
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3. $1 \oplus x = x'$; $0 \oplus x = x$
4. $x \oplus x = 0$; $x \oplus x' = 1$

Shannon's Expansion – simplify the expression

$$\begin{aligned}f(x, y) &= (x' + y) \oplus (x + y') \oplus xy \oplus x'y' \oplus x' \\&= x f(1, y) + x' f(0, y) \\&= x((0 + y) \oplus (1 + y') \oplus 1y \oplus 0y' \oplus 0) + x'((1 + y) \oplus (0 + y') \oplus 0y \oplus 1y' \oplus 1) \\&= x(y \oplus 1 \oplus y \oplus 0 \oplus 0) + x'(1 \oplus y' \oplus 0 \oplus y' \oplus 1) \\&= x\end{aligned}$$