Discussion Session 4/2/2021

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Outline

• Laws of Boolean Algebra
• CMOS Drawing
• Consensus Theorem
Laws of Boolean Algebra

- Associative Law: \((a+b)+c = a+(b+c)\)
- Commutative Law: \(a+b = b+a\)
- Distributive Law: \((a+b) (a+c) = a+bc\)
- Identity Law: \(a+0 = a\)
- Complement Law: \(a+a’ = 1\)
Using Duality, we also have the following statements true

• Associative Law: \((ab)c = a(bc)\)
• Commutative Law: \(ab = ba\)
• Distributive Law: \((ab) + (ac) = a(b+c)\)
• Identity Law: \(a \cdot 1 = a\)
• Complement Law: \(a \cdot a' = 0\)
Practice Problem: Prove $a + 1 = 1$
Sample Solution:

\[ a + 1 = (a + 1) \cdot 1 \ [\text{identity law}] \]
\[ = (a + 1) \cdot (a + a') \ [\text{complement law}] \]
\[ = a + 1 \cdot a' \ [\text{distributive law}] \]
\[ = a + a' \ [\text{identity law}] \]
\[ = 1 \ [\text{complement law}] \]
Practice Problem:
Prove $a'b + a'cd' + b'cd' = a'b + b'cd'$
Sample Solution:

\[ a'b + a'cd' + b'cd' \]
\[ = a'b + a'cd'(b+b') + b'cd'(a+a') \text{ [complement & identity laws]} \]
\[ = a'b + a'bcd' + a'b'cd' + ab'cd' + a'b'cd' \text{ [distributive law]} \]
\[ = a'b (1+cd') + a'b'cd' + ab'cd' + a'b'cd' \text{ [distributive law]} \]
\[ = a'b + (a' + a + a') b'cd' \text{ [using 1+a = 1 & distributive law]} \]
\[ = a'b + (1+a')b'cd' \text{ [complement law]} \]
\[ = a'b + b'cd' \text{ [using 1+a=1]} \]
CMOS:

NMOS in digital circuit

PMOS in digital circuit
CMOS Design of a’
Problem: CMOS Design of \((A+B)\)'
Solution

(a + b)′
Consensus Theorem

- **Sum of Product Form**
  \[ ab + a'c + bc \]

- **Product of Sum Form**
  \[ (a+b) (a'+c) (b+c) \]
Problem: Simplify $a + a'b$
Solution

\[ a + a'b = a + a'b + b \]

\[ = a + b(a' + 1) \text{ [distributive law]} \]

\[ = a + b \]
Problem: Find all consensus(es) in the expression \((a+b')(b+c)(c'+d)\)
Solution

• $a+c, b+d$