

Discussion Session 4/2/2021

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Outline

- Laws of Boolean Algebra
- CMOS Drawing
- Consensus Theorem

Laws of Boolean Algebra

- Associative Law: $(a+b)+c = a+(b+c)$
- Commutative Law: $a+b = b+a$
- Distributive Law: $(a+b)(a+c) = a+bc$
- Identity Law: $a+0 = a$
- Complement Law: $a+a' = 1$

Using Duality, we also have the following statements true

- Associative Law: $(ab)c = a(bc)$
- Commutative Law: $ab = ba$
- Distributive Law: $(ab) + (ac) = a(b+c)$
- Identity Law: $a \cdot 1 = a$
- Complement Law: $a \cdot a' = 0$

Practice Problem: Prove $a + 1 = 1$

Sample Solution:

$$\begin{aligned} a+1 &= (a+1) \cdot 1 \text{ [identity law]} \\ &= (a+1) \cdot (a+a') \text{ [complement law]} \\ &= a + 1 \cdot a' \text{ [distributive law]} \\ &= a + a' \text{ [identity law]} \\ &= 1 \text{ [complement law]} \end{aligned}$$

Practice Problem:

Prove $a'b + a'cd' + b'cd' = a'b + b'cd'$

Sample Solution:

$$a'b + a'cd' + b'cd'$$

$$= a'b + a'cd'(b+b') + b'cd'(a+a') \text{ [complement \& identity laws]}$$

$$= a'b + a'bcd' + a'b'cd' + ab'cd' + a'b'cd' \text{ [distributive law]}$$

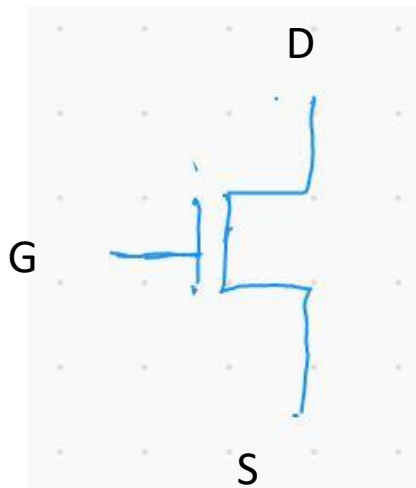
$$= a'b(1+cd') + a'b'cd' + ab'cd' + a'b'cd' \text{ [distributive law]}$$

$$= a'b + (a' + a + a')b'cd' \text{ [using } 1+a = 1 \text{ \& distributive law]}$$

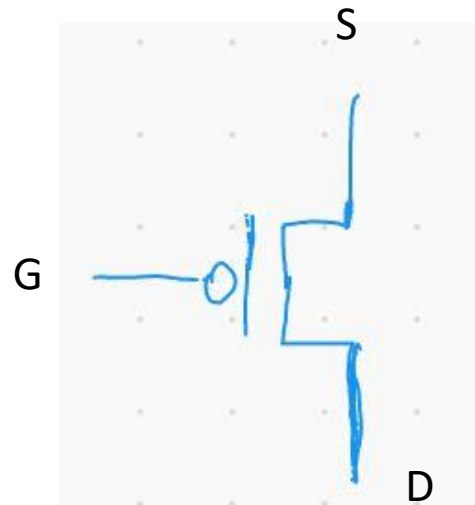
$$= a'b + (1+a')b'cd' \text{ [complement law]}$$

$$= a'b + b'cd' \text{ [using } 1+a=1]$$

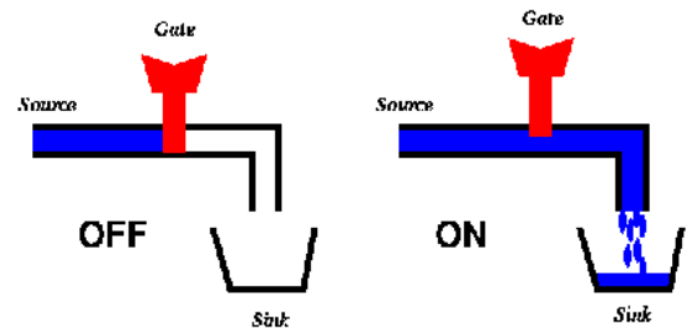
CMOS:



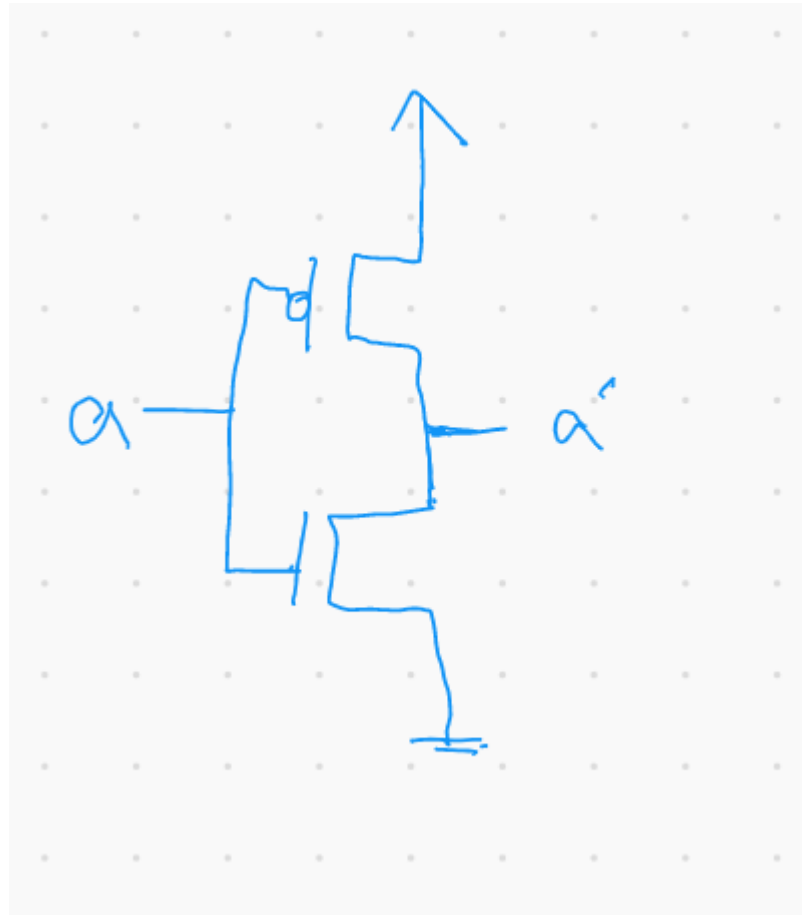
NMOS in digital circuit



PMOS in digital circuit

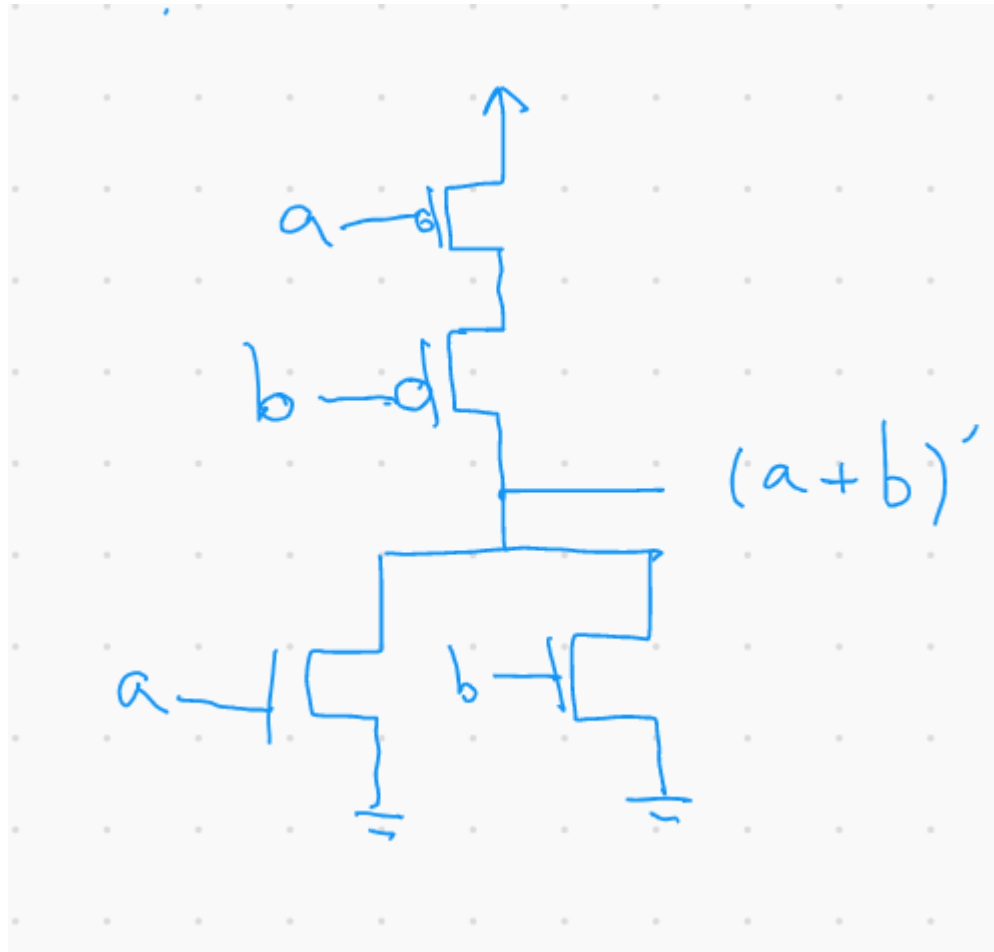


CMOS Design of a'



Problem: CMOS Design of $(A+B)'$

Solution



Consensus Theorem

- Sum of Product Form

$$ab + a'c + bc$$

- Product of Sum Form

$$(a+b)(a'+c)(b+c)$$

Problem: Simplify $a + a'b$

Solution

$$\begin{aligned} a + a'b &= a + a'b + b \\ &= a + b(a'+1) \text{ [distributive law]} \\ &= a + b \end{aligned}$$

Problem: Find all consensus(es) in the expression $(a+b')(b+c)(c'+d)$

Solution

- $a+c$, $b+d$