Image Acquisition, Geometric Transformations, and Image Interpolation

Image Processing

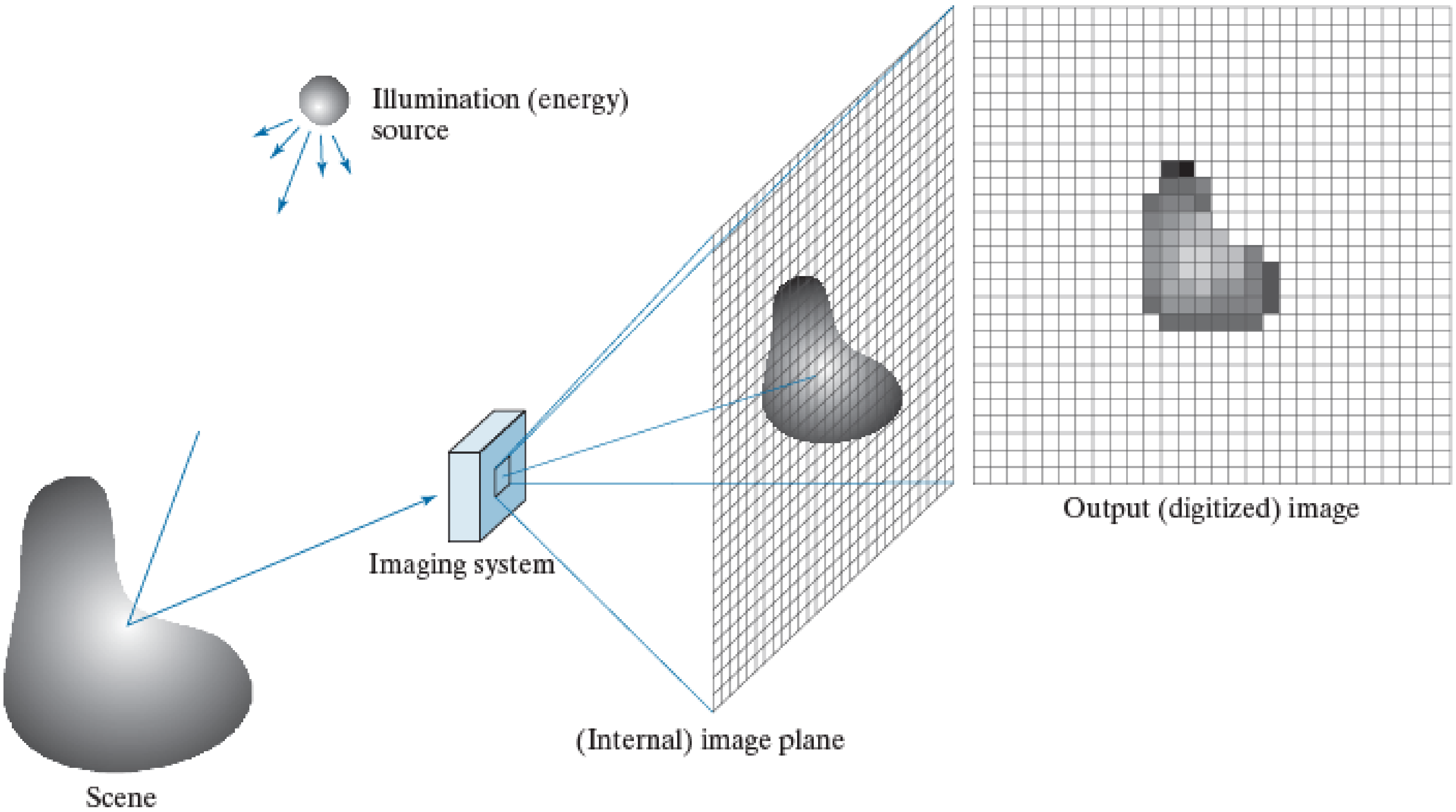
CSE 166

Lecture 2

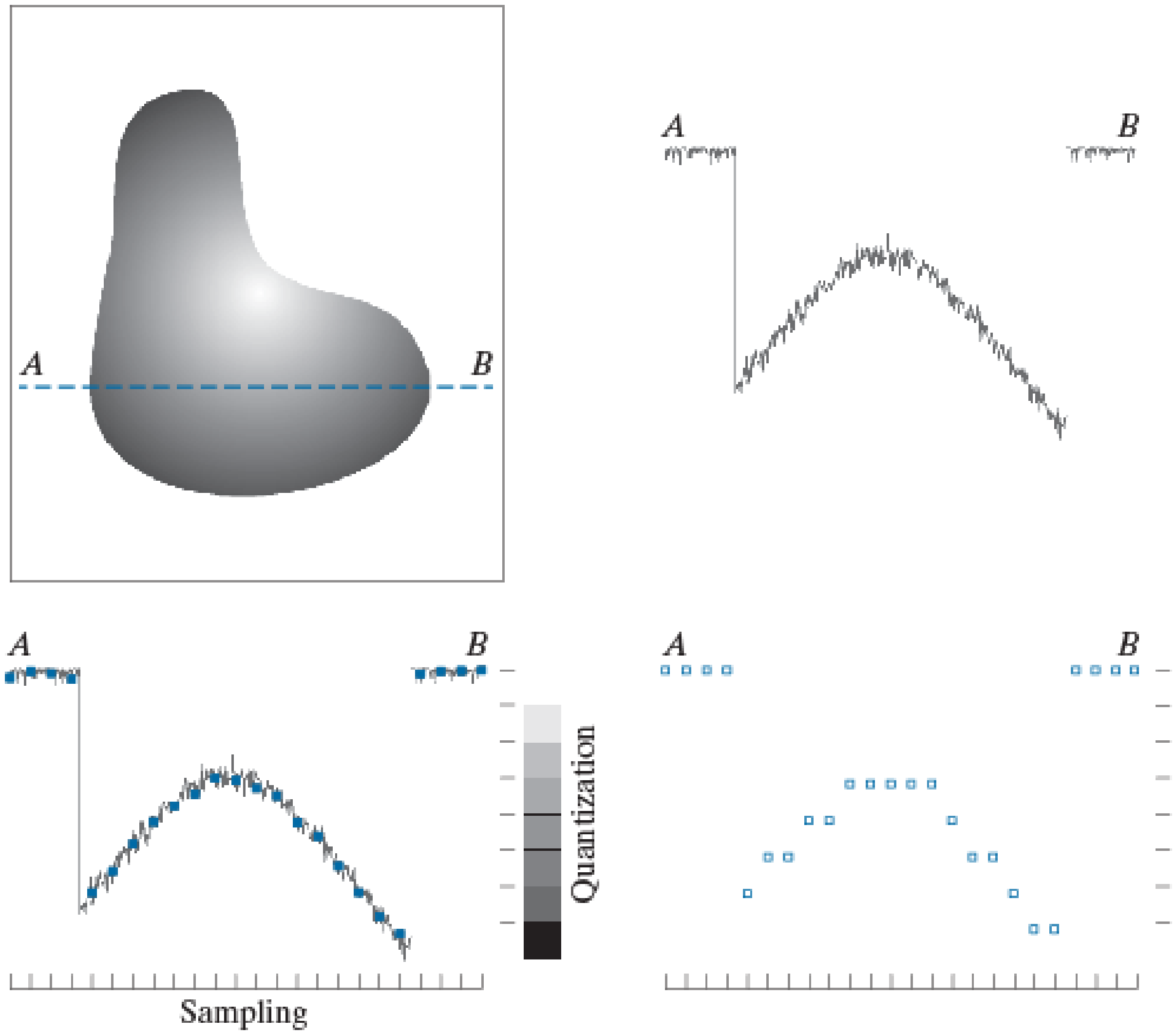
Announcements

- Assignment 1 will be released today
  - Due Apr 8, 11:59 PM
- Reading
  - Chapter 2: Digital Image Fundamentals

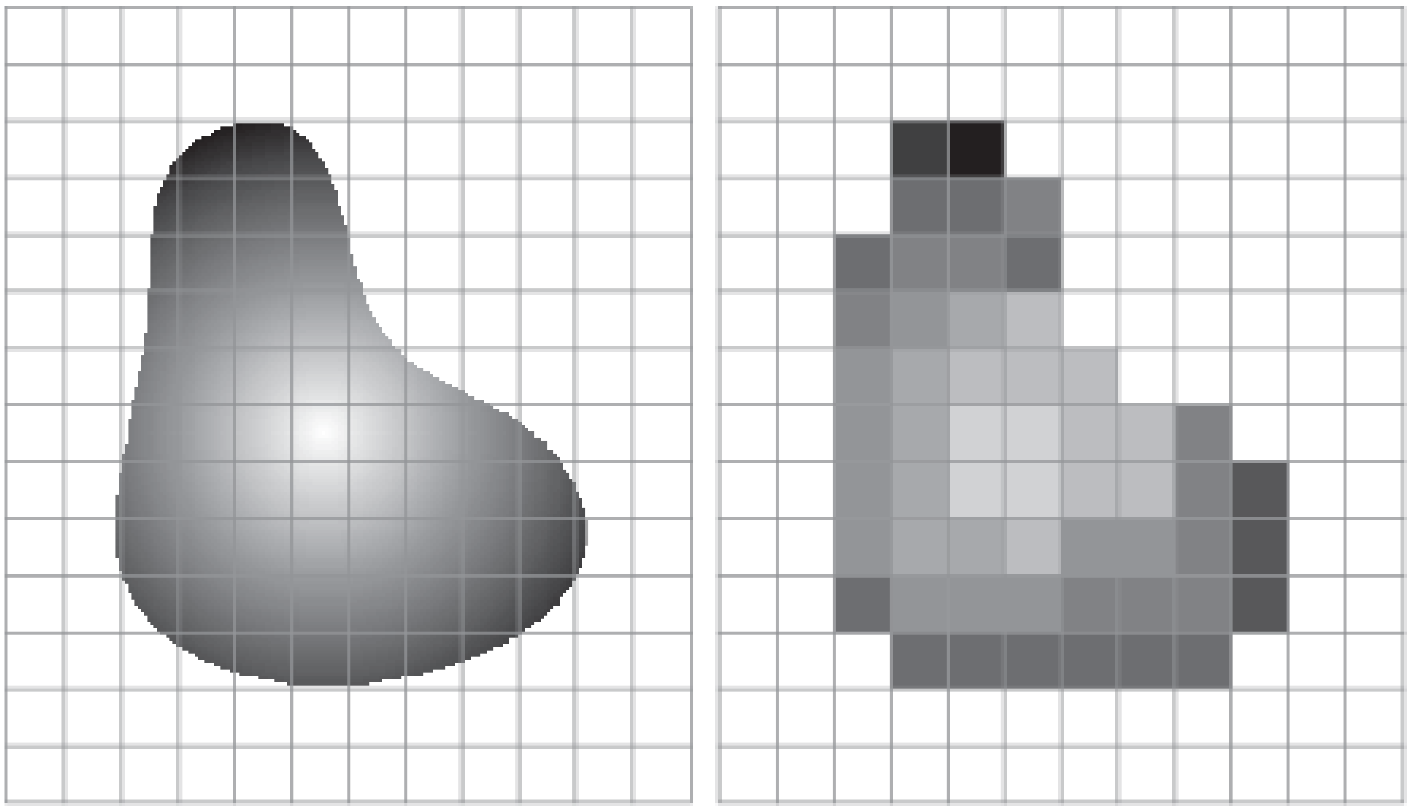
Image acquisition



Digitization, one row of image

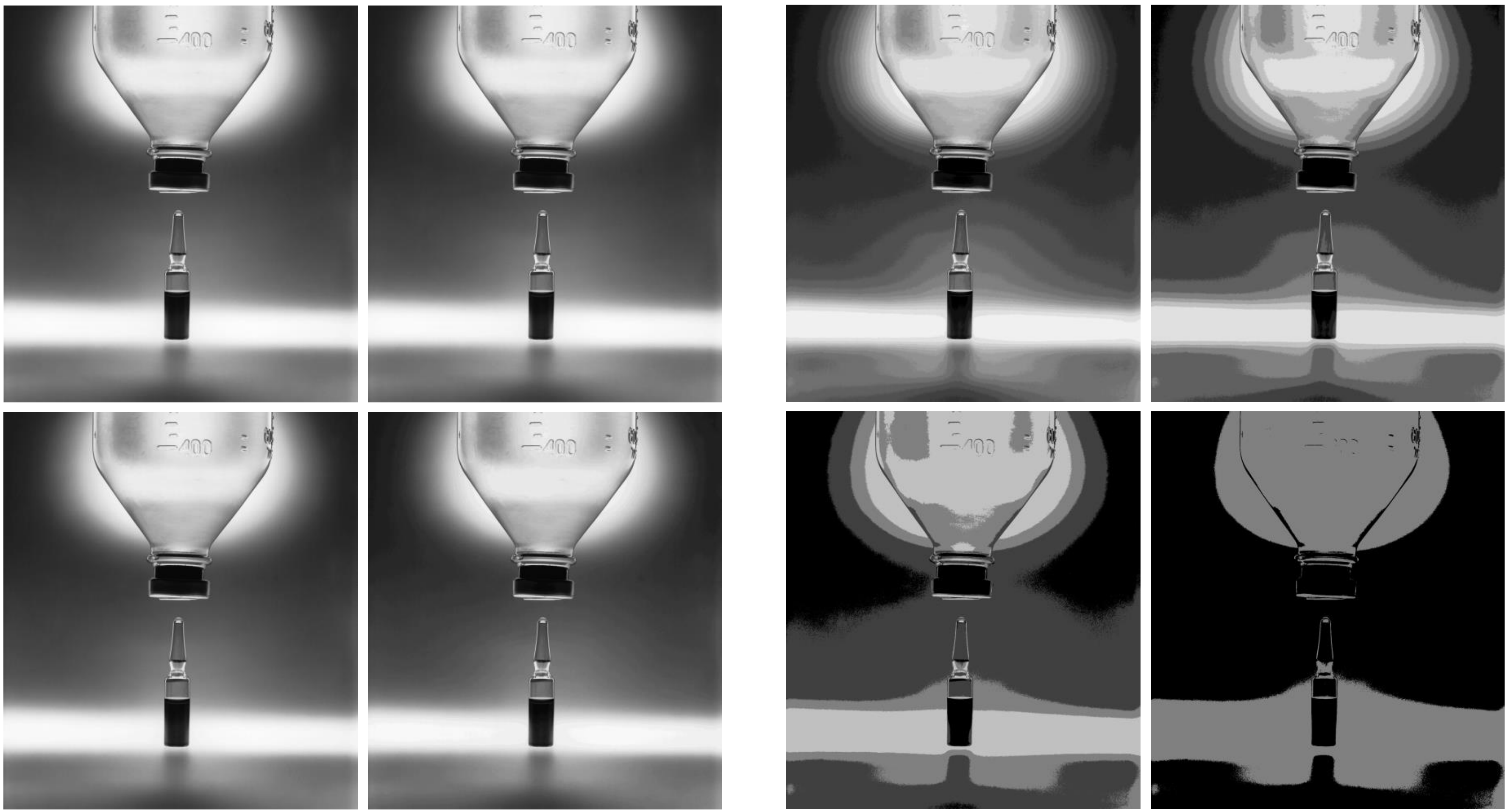


Digitization, whole image



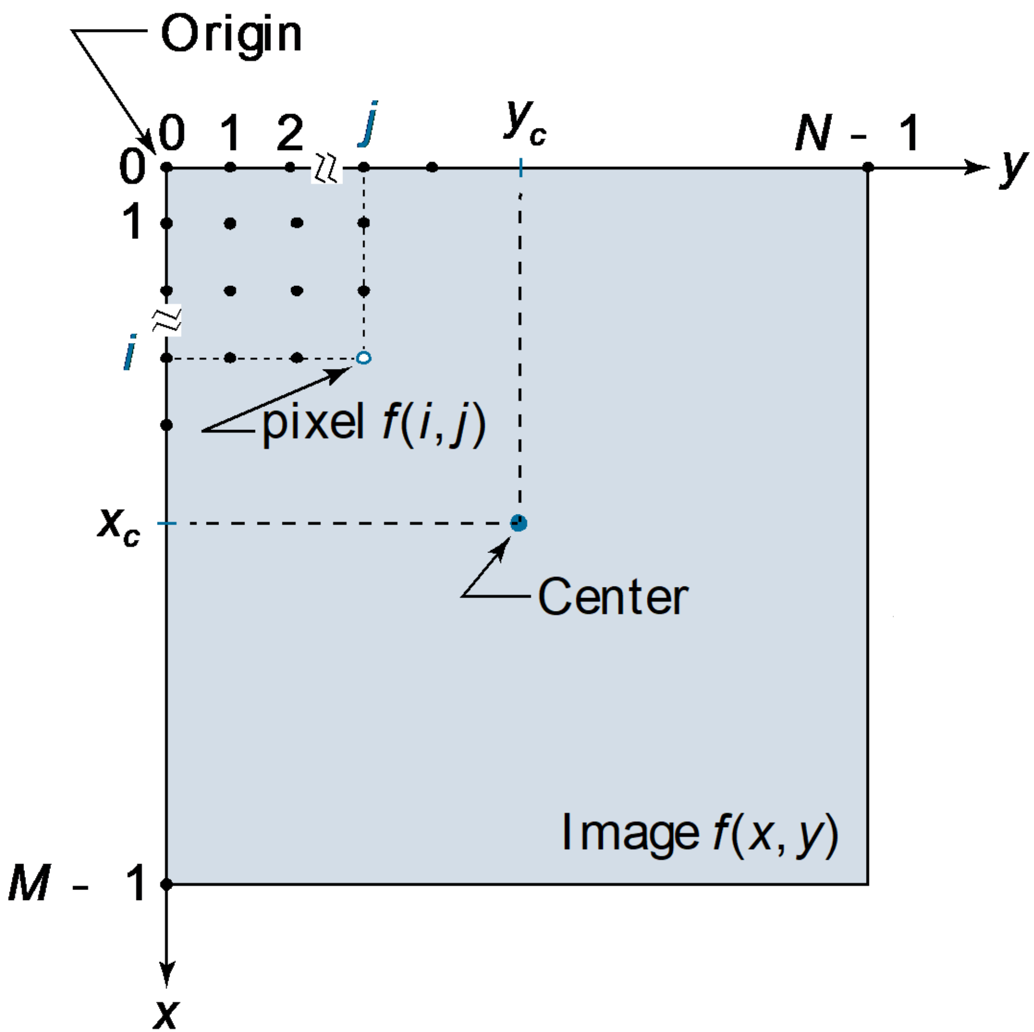
Number of quantization levels

256 128 16 8

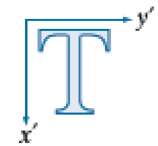
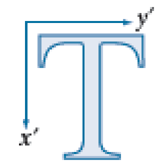
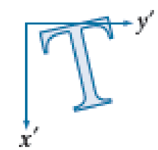
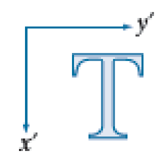
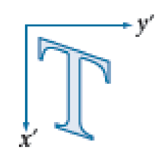
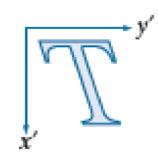


64 32 4 2

Image coordinates



Geometric transformations

Transformation Name	Affine Matrix, A	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = y$	
Scaling/Reflection (For reflection, set one scaling factor to −1 and the other to 0)	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = c_x x$ $y' = c_y y$	
Rotation (about the origin)	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x\cos\theta - y\sin\theta$ $y' = x\sin\theta + y\cos\theta$	
Translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + t_x$ $y' = y + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + s_v y$ $y' = y$	
Shear (horizontal)	$\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = s_h x + y$	

Geometric transformations

Euclidean transformation

$$x' = x\cos\theta - y\sin\theta + t_x$$
$$y' = x\sin\theta + y\cos\theta + t_y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Similarity transformation

$$x' = sx\cos\theta - sy\sin\theta + t_x$$
$$y' = sx\sin\theta + sy\cos\theta + t_y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine transformation

$$x' = a_{11}x + a_{12}y + a_{13}$$
$$y' = a_{21}x + a_{22}y + a_{23}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Composition and inversion of transformations

First transformation

$$\mathtt{H}_1 : \mathbf{x} \mapsto \mathbf{x}'$$
$$\mathbf{x}' = \mathtt{H}_1\mathbf{x}$$

Second transformation

$$\mathtt{H}_2 : \mathbf{x}' \mapsto \mathbf{x}''$$
$$\mathbf{x}'' = \mathtt{H}_2\mathbf{x}'$$

Third transformation

$$\mathtt{H}_3 : \mathbf{x}'' \mapsto \mathbf{x}'''$$
$$\mathbf{x}''' = \mathtt{H}_3\mathbf{x}''$$

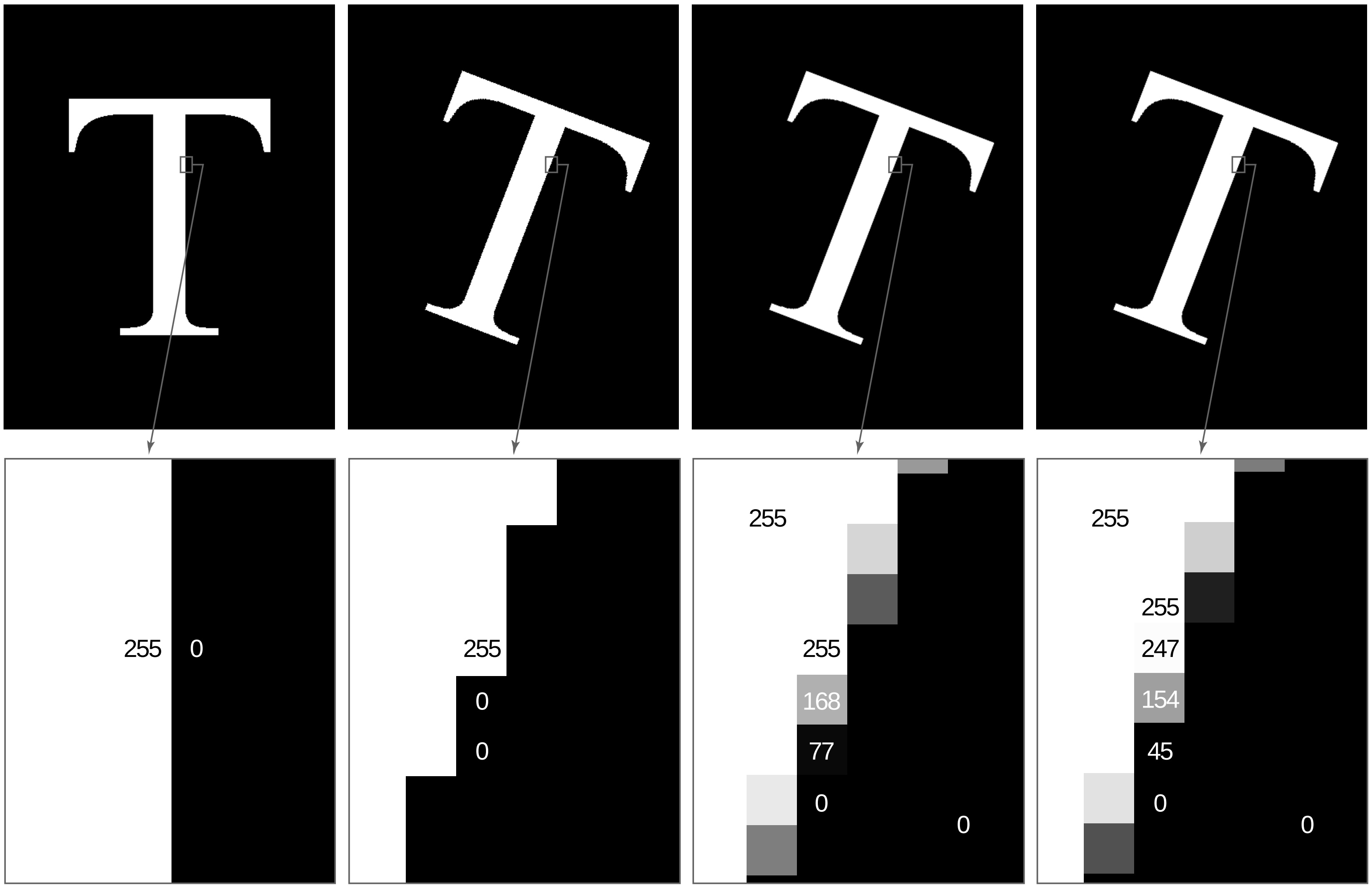
Composition of transformations

$$\mathbf{x}''' = \mathtt{H}_3\mathbf{x}''$$
$$\mathbf{x}''' = \mathtt{H}_3\mathtt{H}_2\mathbf{x}'$$
$$\mathbf{x}''' = \mathtt{H}_3\mathtt{H}_2\mathtt{H}_1\mathbf{x}$$
$$\mathbf{x}''' = \mathtt{H}\mathbf{x} \text{ where } \mathtt{H} = \mathtt{H}_3\mathtt{H}_2\mathtt{H}_1$$

Inverse of transformation

$$\mathbf{x}''' = \mathtt{H}\mathbf{x}$$
$$\mathbf{x} = \mathtt{H}^{-1}\mathbf{x}'''$$

Transformation with interpolation



Nearest neighbor

Bilinear

Bicubic

Transformation with interpolation

- Nearest neighbor interpolation

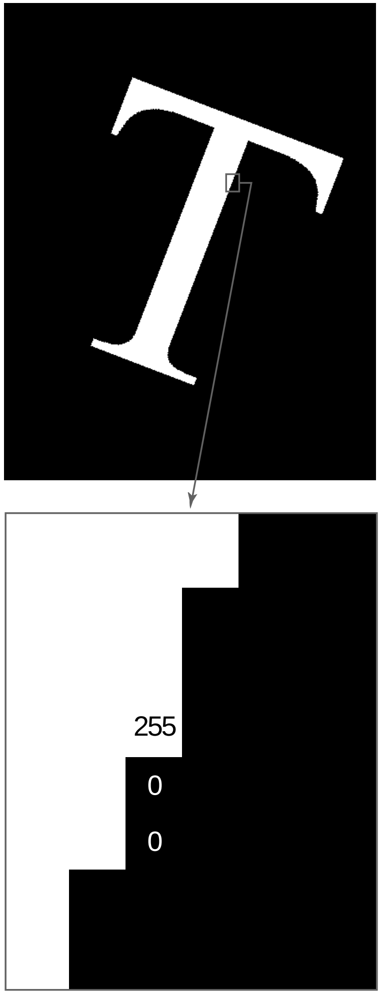
$$\mathtt{H} : I(x, y) \mapsto J(x', y')$$

for each (x', y') in J

$\quad \mathbf{x} = \mathtt{H}^{-1}\mathbf{x}'$

$\quad J(x', y') = I(\text{round}(x), \text{round}(y))$

end



Transformation with interpolation

- Linear interpolation

$\mathtt{H} : I(x, y) \mapsto J(x', y')$

for each (x', y') in J

$\mathbf{x} = \mathtt{H}^{-1}\mathbf{x}'$

$x_0 = \lfloor x \rfloor$

$x_1 = x_0 + 1$

$y_0 = \lfloor y \rfloor$

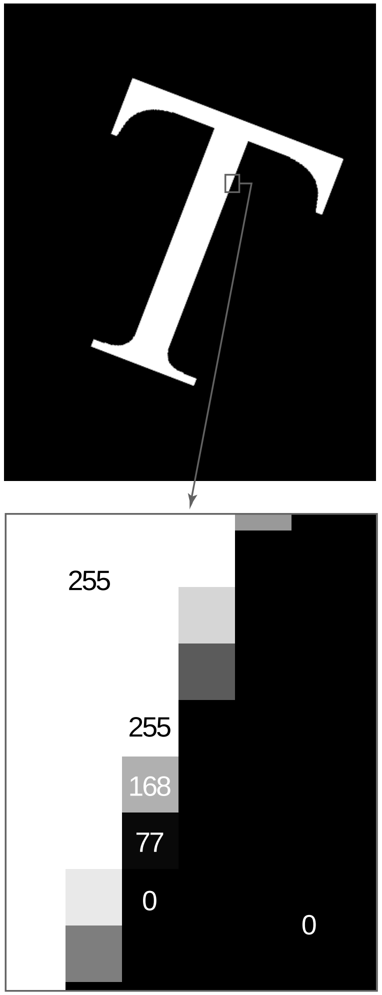
$y_1 = y_0 + 1$

$a = (x_1 - x)I(\lfloor x \rfloor, \lfloor y \rfloor) + (x - x_0)I(\lceil x \rceil, \lfloor y \rfloor)$

$b = (x_1 - x)I(\lfloor x \rfloor, \lceil y \rceil) + (x - x_0)I(\lceil x \rceil, \lceil y \rceil)$

$J(x', y') = (y_1 - y)a + (y - y_0)b$

end



Next Lecture

- Intensity transformations
- Reading
  - Chapter 3: Intensity Transformations and Spatial Filtering