Lecture 3: Graph representations

G = (V,E) where
V: vertices/nodes
E: edges

Undirected edges: symmetric relationship
directed graphs
(x,y): edge from x to y

V = \{1,2,3,4,5\}
E = \{\{1,2\}, \{2,3\}, \{3,4\}, \{2,5\}, \{4,5\}\}

World wide web
node URL
e.g. edge (u,v) u points to v
Billions of nodes and edges!

How are graphs stored on a computer?

**Adjacency matrix**

V x V matrix A
A(i,j) = 1 if (i,j) is in E
0 otherwise

Symmetric if G undirected

\[
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 \\
\end{pmatrix}
\]

**Adjacency list**

For each node, list of outgoing edges

1 2
2 1 → 3 → 5
3 → 2 → 4
4 → 3 → 5
5 → 2 → 4

**PRO** check for an edge in O(1) time
**CON** uses up O(V^2) space

**PRO** just O(V + E) space
**CON** check for an edge in O(V) time
**PRO** easily iterate through node’s neighbors
Undirected graphs: adjacency list

Directed graphs: adjacency list