There are 5 questions.

1. Consider the following statements, and determine if they are true or false. Give a brief explanation (a proof or counterexample).
   (a) Let M be an MST of $G$. If M contains an edge of weight $w$, then all MSTs of $G$ must contain an edge of weight $w$.
   (b) Let $G$ be an undirected connected graph with positive edge weights. If $G$ has two edges with the same weight, then it must have more than one MST.
   (c) When using the union-find data structure, any rank $k$ vertex has fewer than $2^k$ vertices in its subtree.

2. Let $G$ be a connected undirected graph with distinct positive edge weights and let $e$ be an edge. Design a linear time algorithm ($O(\vert V \vert + \vert E \vert)$) that determines if $e$ is part of a minimum spanning tree of $G$.
   (Hint: Use the following fact: Edge $e = (u, v)$ does not belong to an MST of $G$ if and only if $v$ and $w$ can be joined by a path consisting entirely of edges that are cheaper than $e$.)

3. Recall the event scheduling problem from class (the event scheduling problem is to find a largest compatible set — a set of non-overlapping events of maximum size). Suppose that instead of always selecting the first activity to finish, we select the last activity to start that is compatible with all previously selected activities. Prove that this strategy yields an optimal solution.

4. Suppose you are running a trucking company that ships packages from San Diego to LA. The amount of packages requires you to send a number of trucks each day from SD to LA. Each truck has a limit of $W$ on the maximum amount of weight they are allowed to carry. Boxes arrive in LA one by one, and each package $i$ has weight $w_i$. The trucking station is quite small so at most one truck can be at the station at any time. Company policy requires that boxes are shipped in the order they arrive; otherwise, a customer might get upset upon seeing a box that arrived after his make it to LA faster.
   The company is using the following greedy algorithm: they pack boxes in the order they arrive and whenever the next box does not fit, they send the truck on its way and repeat on the next truck.
   (a) Determine the
   1. Instance,
   2. Solution format,
   3. Constraints and
   4. Objective function
   for this problem.
   (b) Prove the company’s strategy is optimal.
   (c) Implement this strategy and determine runtime.
5. Suppose that you and your friends are going to hike the John Muir trail this summer\(^1\). You want to hike as much as possible per day, but you do not want to hike after dark. On a map there are a large set of good stopping points for camping and you design your trip based on the following system: Every time you come to a potential stopping point, determine whether you can make it to the next one before nightfall. If you can make it then keep hiking, otherwise stop and camp. You claim that with this strategy you will minimize the number of camping stops you must make.

Suppose the stopping points are located at distances \(x_1, \ldots, x_n\) from the trailhead. Also assume that your group can hike \(d\) distance per day (independent of terrain, weather conditions, etc.)

(a) Determine the
   1. Instance,
   2. Solution format,
   3. Constraints and
   4. Objective function
   for this problem.

(b) Prove this strategy is optimal.

(c) Implement this strategy and determine runtime.

\(^1\)Key word: suppose