

# Vehicle Dynamics

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CSE291: Physics Simulation  
UCSD  
Spring 2019

# Applications

- Vehicle design
- Vehicle performance optimization
- Interactive simulation & training
- Interactive video games
- Autonomous simulation & safety validation
- Accident reconstruction
- Traffic simulation

# Vehicle Categories

- Wheeled Vehicles
  - Car, truck, trailer, bus, motorcycle, bicycle
- Treaded Vehicles
  - Tank, construction vehicles
- Aircraft
  - Propeller plane, jet plane, helicopter, quad-rotor, glider, blimp, balloon
- Watercraft
  - Sail boat, power boat, hovercraft, hydroplane, submarine
- Rail Vehicles
  - Train, monorail, subway
- Spacecraft
  - Rocket, satellite, lander
- Robot
- Hybrid

# Ground Vehicles

- Main simulation components
  - Chassis
    - Suspension
    - Steering system
    - Brake system
    - Body aerodynamics
  - Tires
  - Drivetrain
    - Engine (gas, electric, hybrid)
    - Transmission, clutch
    - Differentials
  - Sensors & control systems
    - ABS, ESP, ACC...
- We will talk about chassis, tires, & drivetrain today

# Other Vehicle Components

- Treaded ground vehicles
  - Treads
- Aircraft
  - Wing aerodynamics
  - Control surfaces
  - Jet engines
  - Propellers
  - Rotors
- Watercraft
  - Hull, buoyancy
  - Sails
- Spacecraft
  - Solid & liquid fuel rockets
  - Electrostatic thrusters
  - Gyros
  - Orbital mechanics

# Ground Vehicle Models

- There are various levels of complexity that are used for modeling ground vehicles. Ranging from simple to complex, these include:
  - Single track ‘bicycle model’
  - Twin track model
  - Reduced rigid body model
  - Full articulated rigid body model
  - Finite element model
- We will assume a full articulated body model for most of today's discussion

Chassis

# Chassis

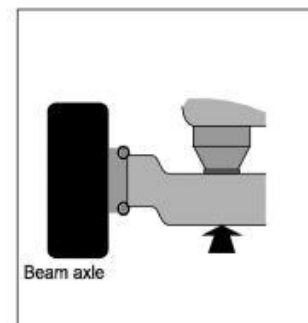
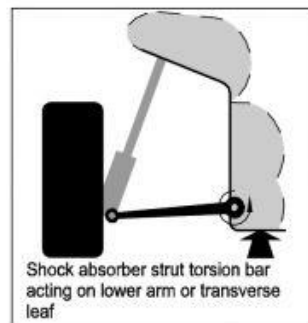
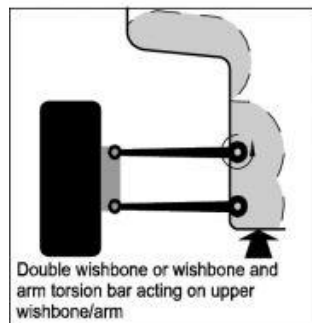
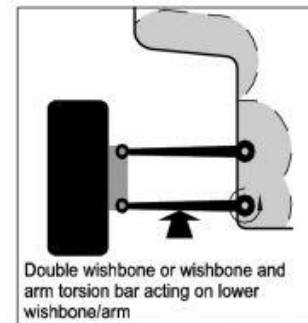
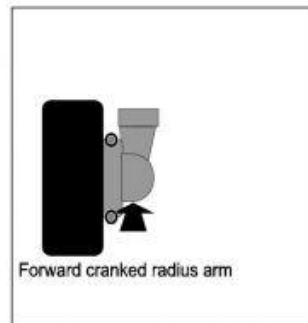
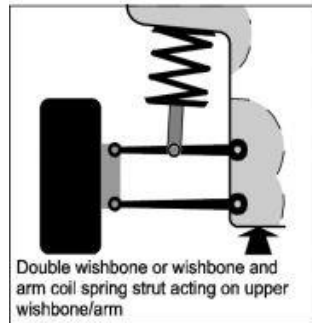
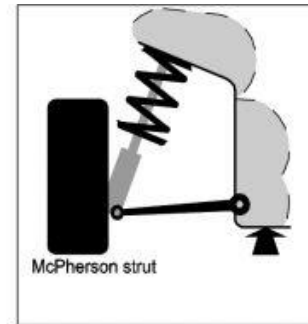
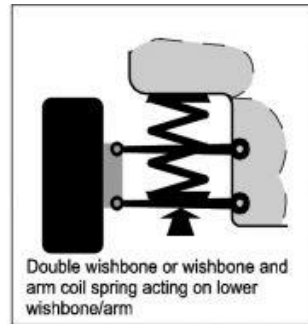
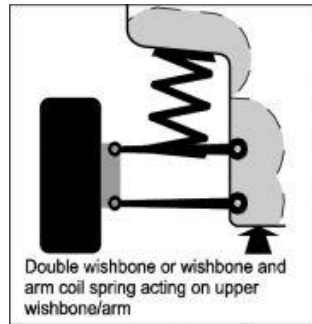
- The chassis refers to the solid frame of the vehicle and usually includes the suspension, steering, and brake systems as well
- The chassis can be modeled as a single rigid body, a set of articulated rigid bodies, or as a set of deformable solids if the effects of the deformation are important



# Suspension

- The purpose of suspension systems is to maintain good contact between the tires and the road over a range of surface types, as well as provide ride comfort for passengers
- A variety of suspension designs are used in modern cars such as:
  - Swing axle
  - Trailing arm
  - MacPherson
  - Double wishbone
  - Multi-link (typically 4-link, or 5-link)

# Suspension Examples



# Suspension Modeling

- Modeling detailed suspension systems requires modeling articulated jointed bodies
- In simpler situations, one can ignore the mass properties of many of the smaller connecting components, and just use them to perform a kinematic analysis of the range of motion
- In more complex simulations, we can model each component as a full rigid body
- Suspension systems have many bodies and very few degrees of freedom making them well suited for reduced DOF methods such as Lagrangian formulations
- They also have multiple kinematic loops requiring some special handling

# Springs

- We can model a suspension spring force as:

$$f_{spring} = -k_s x$$

- Where  $x$  is the displacement and  $k_s$  is the spring constant
- We can also estimate the spring constant from the geometric properties of the coil and the elastic properties of the material

$$k_s \approx \frac{Gd^4}{8nD^3}$$

- Where  $d$  is the wire diameter,  $D$  is the mean coil diameter,  $n$  is the number of 'active' coils (which can be less than the actual number of coils), and  $G$  is the shear modulus, computed from the Young's modulus  $E$  and Poisson ratio  $\nu$  as:

$$G = \frac{E}{2(1 + \nu)}$$

# Dampers

- Linear dampers produce a force proportional to the velocity along the axis of the damper

$$f_{damp} = -k_d v_{close}$$

- The force is based on the closing velocity  $v_{close}$  between the two mounting points of the damper  $\mathbf{p}_1$  and  $\mathbf{p}_2$  with velocities  $\mathbf{v}_1$  and  $\mathbf{v}_2$

$$\mathbf{e} = \frac{\mathbf{p}_1 - \mathbf{p}_2}{|\mathbf{p}_1 - \mathbf{p}_2|}$$

$$v_{close} = (\mathbf{v}_2 - \mathbf{v}_1) \cdot \mathbf{e}$$

# Elastic Bushings

- Elastic bushings, elastic couplings, and compliant joints in the suspension can be modeled essentially as tight springs
- Joints can be modeled as tight spring equations on both linear and angular motion for as many DOFs as desired
- These can lead to stiff equations requiring small integration time steps or implicit integration methods, but we have to deal with this anyway, as there will be various stiff equations in the full vehicle system

# Anti-Roll Bars

- Anti-roll bars couple the left and right sides of the suspension to reduce roll
- The torque on a twisted bar is:

$$\tau = \frac{\theta GJ}{L}$$

- Where  $\theta$  is the angle of twist,  $L$  is the length of the bar,  $G$  is the shear modulus of the material, and  $J$  is the *polar moment of inertia* computed as:

$$J = \frac{\pi}{2} (r^4 - r_{in}^4)$$

- Where  $r$  is the radius of the bar and  $r_{in}$  is the inner radius (0 for a solid bar, non-zero for a hollow bar)

# Anti-Roll Bar Model

- We assume the roll bar is held by the chassis, free to rotate around the axis
- Each end of the bar has an angled offset that is attached to the left or right suspension motion
- A difference in the height of the left and right wheels will cause a torque on the bar
- We can measure the angle the bar is twisted and compute the torque needed to twist it based on the previous slide
- We then solve for the force that produces the torque assuming  $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{f}$  where  $\mathbf{r}$  is the offset from the connection point to the bar axis



# Brakes

- The brake torque on a rotating disc brake is going to be proportional to the braking pressure  $p_B$ , the contact area  $a_B$ , the offset radius of the brake pad  $r_B$ , and the sliding friction coefficient  $\mu_B$

$$\tau_{brake} = p_B a_B r_B \mu_B$$

- It is also common to model changes in  $\mu_B$  due to brake temperature and account for the dynamic heating and cooling of brake components

# Force Components

- Remember that all internal force components like springs, dampers, anti-roll bars, and brakes must obey Newton's Third Law and apply equal and opposite forces/torques to two different bodies within the vehicle model
- The brakes, for example, will apply equal and opposite forces on the brake rotor connected to the rotating wheel hub, and the brake caliper connected to the suspension upright

# Aerodynamic Forces

- A basic aerodynamic drag force can be modeled based on the relative velocity of the car to the airflow

$$\mathbf{v}_{rel} = \mathbf{v}_{car} - \mathbf{v}_{air}$$

$$f_{drag} = \frac{1}{2} \rho |\mathbf{v}_{rel}|^2 c_d a$$

- $\rho$  is the density of the air (1.225 kg/m<sup>3</sup> is a reasonable standard)
- $c_d$  is the drag coefficient (Tesla Model 3 has a  $c_d$  of 0.23 and a Jeep Wrangler has a  $c_d$  of 0.58)
- $a$  is the cross sectional area exposed to the airflow
- $f_{drag}$  acts in the direction opposing the velocity

# Aerodynamic Forces

- The force acts on the entire surface of the car, but averages out to a single force and torque on the frame
- Often, it is modeled as a force vector  $\mathbf{f}_{aero}$  and a *center of pressure*  $\mathbf{r}_{aero}$ , which can be used to compute the torque as  $\mathbf{r}_{aero} \times \mathbf{f}_{aero}$
- Technically, some of the constants ( $c_d$ ,  $a$ ,  $\mathbf{r}_{aero}$ ) should vary based on the direction of the airflow with respect to the vehicle, but often, this is ignored, as one is mainly interested in aerodynamic drag force acting directly along the front-back axis of the car
- We can also include lift and downforce effects that have the same formula as the drag force except they act in the vehicle's local vertical axis

# Advanced Aerodynamic Modeling

- For more advanced aerodynamic modeling, we can do full 3D fluid flow simulations, which are often used for drag reduction and studies on downforce, cooling, and noise
- For real time simulation, we can precompute the total aerodynamic force & torque at various vehicle speeds and interpolate them based on the relative airflow velocity transformed into vehicle local space
- This way we can quickly compute accurate aerodynamic forces, depending on the size and accuracy of the precomputed table



*Image: sportskeeda.com*

Tires

# Tires

- Modern tires are quite complex and consist of multiple layers of materials including rubber, steel, nylon, woven fabrics, and more
- Tire forces are ultimately responsible for almost all of the relevant motion of cars (other than aerodynamic forces, which are only really relevant at higher speeds)
- A car has 4 postcard sized contact patches that are responsible for transmitting all of the engine, braking, and handling forces

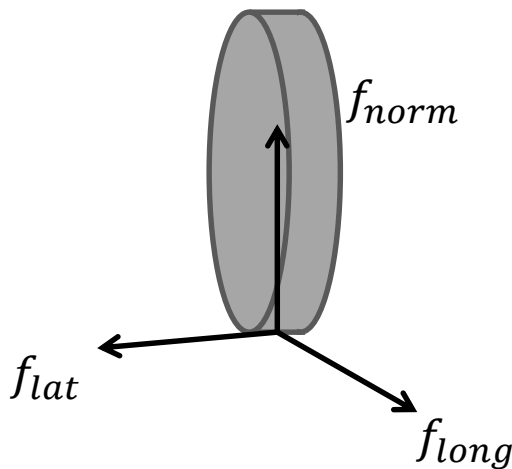
# Tire Deformation

- Tires are deformable bodies that produce a force as a result of displacement
- Understanding the forces that tires produce requires an understanding of the deformations a tire is subjected to

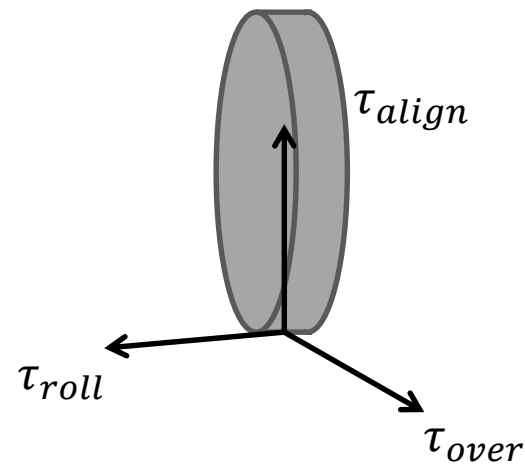


# Tire Forces & Torques

- We can define the wheel frame as the orthogonal frame formed by the contact plane normal, the wheel hub projected onto the plane, and the axle vector projected onto the plane
- The tire produces forces along these axes known as the *normal force*  $f_{norm}$ , *longitudinal force*  $f_{long}$ , and *lateral force*  $f_{lat}$ , as well as torques known as the *overturning torque*  $\tau_{over}$ , the *rolling resistance torque*  $\tau_{roll}$ , and the *self-aligning torque*  $\tau_{align}$



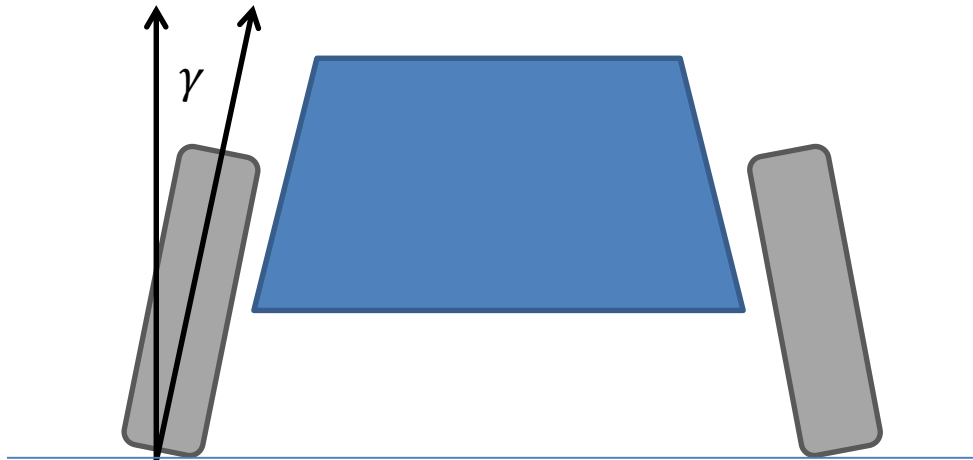
*Tire Forces*



*Tire Torques*

# Camber Angle

- The camber angle  $\gamma$  is the angle the wheel hub is tilted from vertical, and can be positive or negative
- The image below shows a vehicle with negative camber



# \*Camber Forces

- [insert slide here]

# Vertical Forces

- We define  $r_0$  as the *construction radius* of the tire
- We will also define  $\Delta r$  as the difference between the construction radius and the *static tire radius*  $r_{stat}$  which is the distance from the center of the wheel axle to the ground when the vehicle is stationary

$$\Delta r = r_0 - r_{stat}$$

- If we use a linear equation to model the vertical tire force, we get:

$$f_{norm} = \begin{cases} k_1 \Delta r & \text{for } \Delta r \geq 0 \\ 0 & \text{for } \Delta r < 0 \end{cases}$$

- One can also add quadratic and cubic components to the force equation to model cases with larger deformations

# Tire Contact Patch

- The width of the contact patch is approximated as the width of the tire
- The length  $l$  of the contact patch of the tire is approximated as:

$$l \approx 2\sqrt{2r_0\Delta r}$$

# Rolling Resistance

- When the vehicle is sitting still on a flat surface, the tire contact area will be roughly symmetric around
- When the tire is rolling forward, there is a slight compression of the rubber towards the front of the contact patch and a slight stretching towards the back
- This leads to an asymmetric pressure distribution biased towards the front of the contact patch
- We approximate it as the force being applied from the center of the contact patch plus a small offset  $e$  towards the front
- This causes a torque on the wheel axle that acts in the opposite direction of rolling, known as the rolling resistance torque  $\tau_{roll}$

$$\tau_{roll} = e \cdot f_{norm}$$

# Longitudinal Slip

- The *longitudinal slip ratio* is a kinematic quantity that describes the state of motion of a driven, braked, or free-rolling wheel
- We separate the longitudinal slip into  $s_A$  for slip due to acceleration and  $s_B$  for slip due to braking
- We define  $r$  as the wheel radius,  $v$  as the velocity of the wheel center point,  $v_p$  as the velocity of the contact point, and  $\omega$  as the angular velocity of the wheel
- For a free rolling wheel ( $v = \omega r$ ):  $s_A = s_B = 0$
- For a driven wheel ( $v < \omega r$ ):  $s_A = \frac{v_p}{\omega r} = \frac{\omega r - v}{\omega r}$
- For a spinning wheel ( $v < \omega r$ ):  $s_A = 1$
- For a braked wheel ( $v > \omega r$ ):  $s_B = \frac{v_p}{v} = \frac{v - \omega r}{v}$
- For a locked wheel ( $v > \omega r$ ):  $s_B = 1$

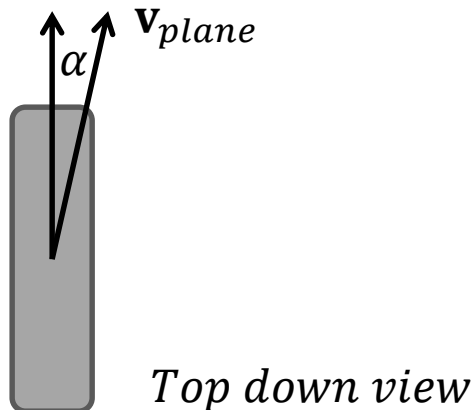
# Force-Slip Relationship

- Longitudinal forces (like all tire forces) are complicated and multiple models exist with a wide range of complexities
- Overall, we have a friction relationship at the contact patch that produces longitudinal forces based on the longitudinal slip and normal force
- For small slip values, we get a linear relationship between the normal and longitudinal force, indicating a static friction type of relationship
- This roughly linear curve continues until we reach the maximum longitudinal force
- At higher slip values, the force drops slightly indicating a transition to more sliding friction



# Lateral Slip

- The angle in the contact plane between the direction the wheel hub is heading and the direction the wheel is actually moving  $\mathbf{v}_{plane}$  is called the *slip angle*  $\alpha$ , and  $\tan \alpha$  is known as the *lateral slip*
- They are approximately equal for normal driving conditions where  $|\alpha| < 10^\circ$



# Lateral Forces

- Lateral forces behave in a similar fashion to longitudinal forces
- The relationship between the lateral slip angle and the lateral force is very similar to the relationship between the longitudinal slip and the longitudinal force
- We have a linear region for small slip angles that hits a peak and then drops off slowly for larger slip angles

# Friction Circle

- As a rule, the friction properties for a tire will be roughly the same for the lateral direction and the longitudinal direction
- It is sometimes helpful to think of a *friction circle* which limits the vector sum of the maximum lateral and maximum longitudinal forces

$$\sqrt{f_{long}^2 + f_{lat}^2} \leq \mu_{max} f_{norm}$$

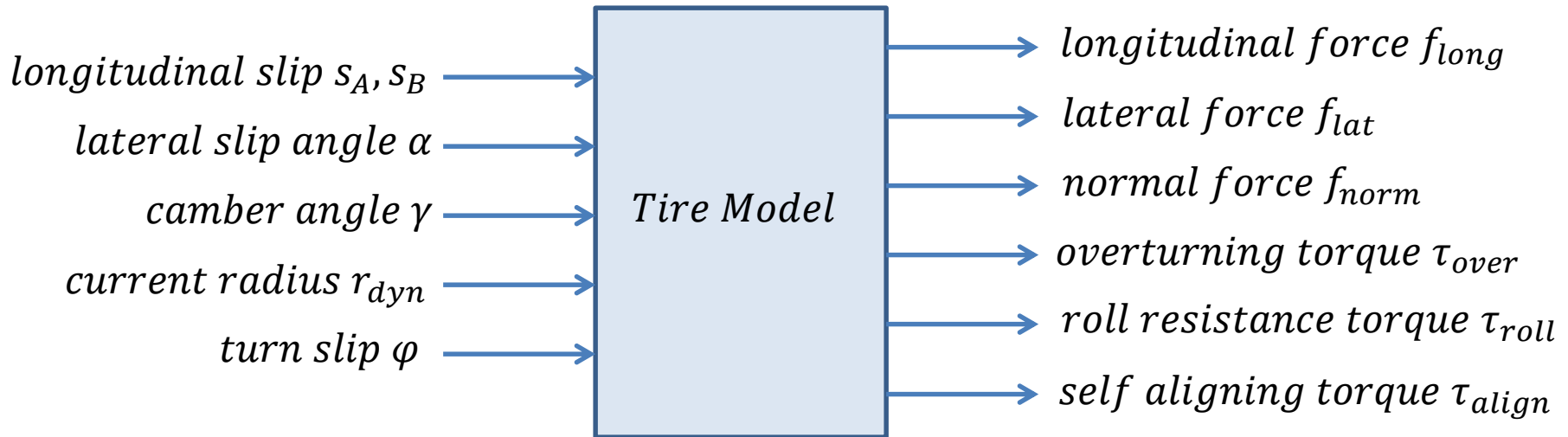
- This also shows that the lateral handling of a car will be compromised under braking or acceleration

# Aligning Torques

- The various tire torques also tend to have similar relationships to the slip angle and longitudinal slip as the forces do
- They tend to be curves that are roughly linear near 0 slip until they peak and then drop off slowly for larger slips
- This isn't surprising as they are all related to the same stick-slip friction process happening in the contact patch

# Tire Models

- A tire model is a nonlinear function with multiple inputs and outputs
- A basic example might work like this:



# Magic Formula

- A variety of heuristic tire models have been used to produce force-slip curves that can fit the behavior of real tires, such as the *Magic Formula* model developed by Hans Pacejka
- The basic model defines a curve of the form:

$$y(x) = D \sin \left( C \arctan \left( Bx - E (Bx - \arctan Bx) \right) \right)$$

- By fitting different values for the constants, one can develop reasonable models for several useful relationships such as:
  - Lateral force vs. slip angle
  - Self-aligning torque vs. slip angle
  - Longitudinal force vs. longitudinal slip

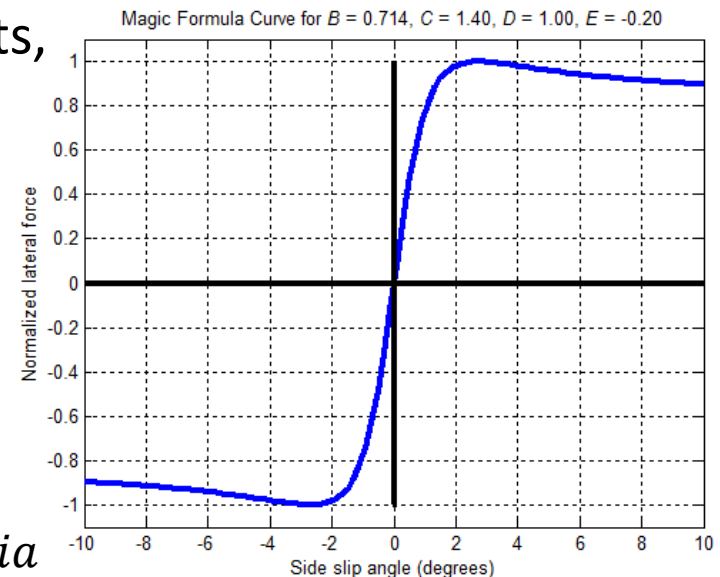


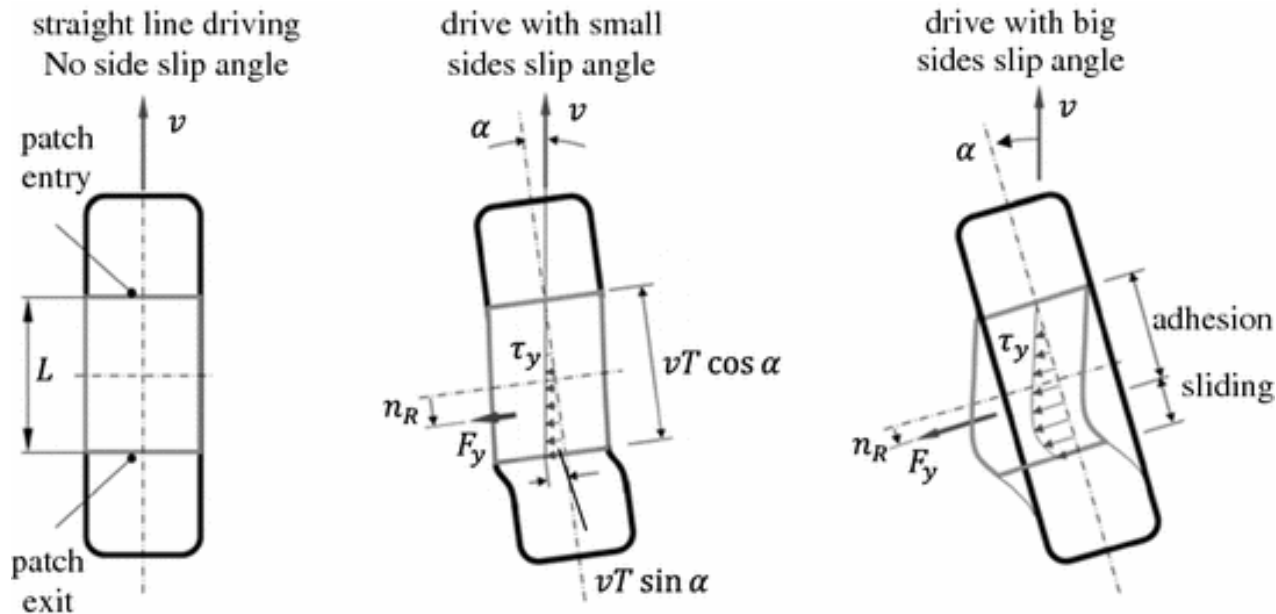
Image: Wikipedia

# Measuring and Fitting Models

- There are actual physical test rigs for measuring tire forces and torques under different rolling and slipping conditions
- These systems can measure and table real world tire forces and then fit Magic Formula or similar curves to the data
- Various standards exist along with standardized test rigs and computational models

# Contact Patch Models

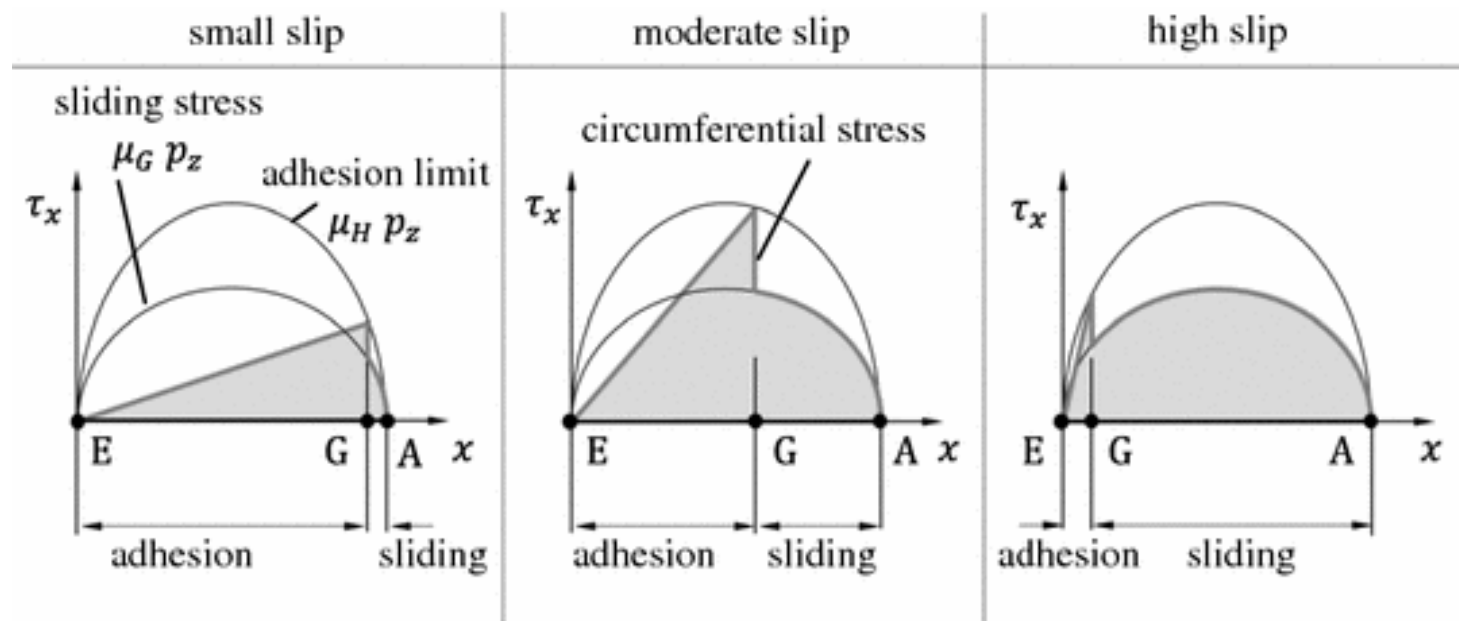
- Some tire models model the accumulation of deformations along the longitudinal direction of the contact patch and integrate the static and dynamic friction forces along the patch



*Image: Vehicle Dynamics, Schramm, Hiller, Bardini, 2017*



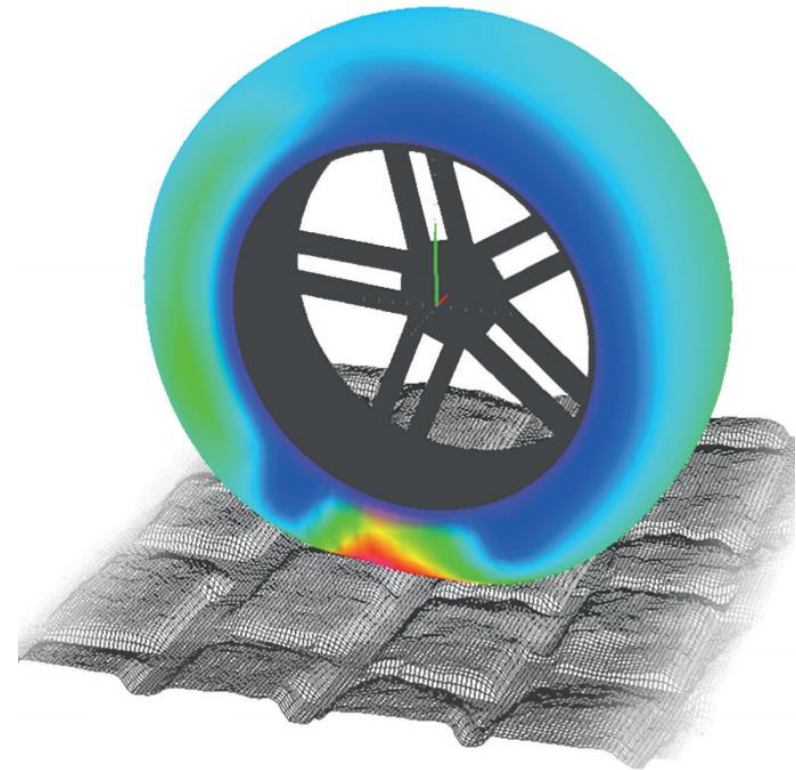
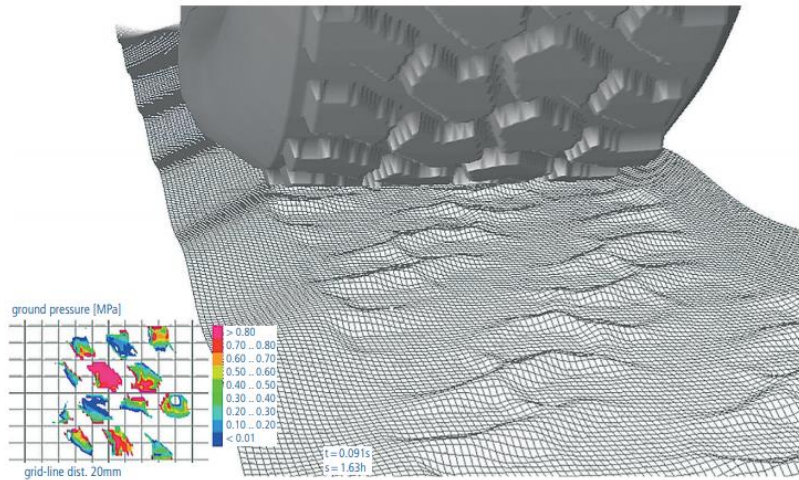
# Contact Patch Models



# Advanced Tire Models

- There are many advanced tire models that account for tire dimensions, rubber compounds, air pressure, tread patterns, belt patterns, heating, tread wear, and more
- Modern methods also include use of finite element models as well as detailed tread pattern models that account for fluid channeling and soil deformation

# Advanced Tire Models



Images: FTire – [www.cosin.eu](http://www.cosin.eu)

# Tire References

- For more information about tire modeling:
- “Tire Modeling for Multibody Dynamics Applications”, Schmid, 2011
- “Tire and Vehicle Dynamics, 3<sup>rd</sup> Edition”, Pacejka, 2012
- “Vehicle Dynamics, Modeling and Simulation, 2<sup>nd</sup> Edition”, Schramm, Hiller, Bardini, 2018

Drivetrain

# Drivetrain

- The drivetrain consists of all of the power delivery components from the engine to the wheels
- This typically consists of:
  - Engine
  - Clutch
  - Transmission
  - Driveshaft
  - Universal joint
  - Differential
  - Axles & couplings
  - Wheels
- Cars with the engine mounted by the drive wheels would omit the driveshaft and universal joint
- Four wheel drive cars may add more differentials and axles
- Some electric cars omit the transmission

# Rotational Inertia

- In some cases, we wish to model individual pistons in an engine as an articulated rigid body system
- In full vehicle simulations however, it is usually sufficient to model it as a single rotating shaft with a constant scalar rotational inertia  $I_E$ , angular momentum  $L_E$ , angular velocity  $\omega_E$ , with the scalar relationship:

$$L_E = I_E \omega_E$$

- Engines typically have a flywheel to add additional rotational inertia to smooth engine performance, improve idling, and smooth gear shifting. As it is rigidly connected to the output shaft, it can be included in  $I_E$
- Other driveshaft components can have similar values

# Engine Torque

- For cars, we mainly have internal combustion or electrical engines (or hybrids of the two)
- Both types produce torque  $\tau_E$  on the rotating output shaft that is typically connected to the transmission through a clutch
- The output shaft of the engine rotates with an angular velocity of  $\omega_E$  radians/second or  $\frac{60}{2\pi} \omega_E$  RPM (rotations per minute)
- The torque that the engine produces will vary based on the current engine speed and the throttle  $T_E$  (ranging from 0 to 1, or sometimes 0% to 100%)

$$\tau_E = f(\omega_E, T_E)$$

- The maximum torque at a particular angular velocity is defined by the torque curve of the engine



# Torque vs. Power

- From the point of view of mechanical simulation, torque is what we are interested in, as it can be used to transmit torques to the bodies in the system
- Power is useful to understand energy production, transmission, and fuel consumption
- For an engine with rotational speed  $\omega_E$  in radians/second and torque  $\tau_E$  in Newton·meters, power in Watts is:

$$power = \omega_E \cdot \tau_E$$

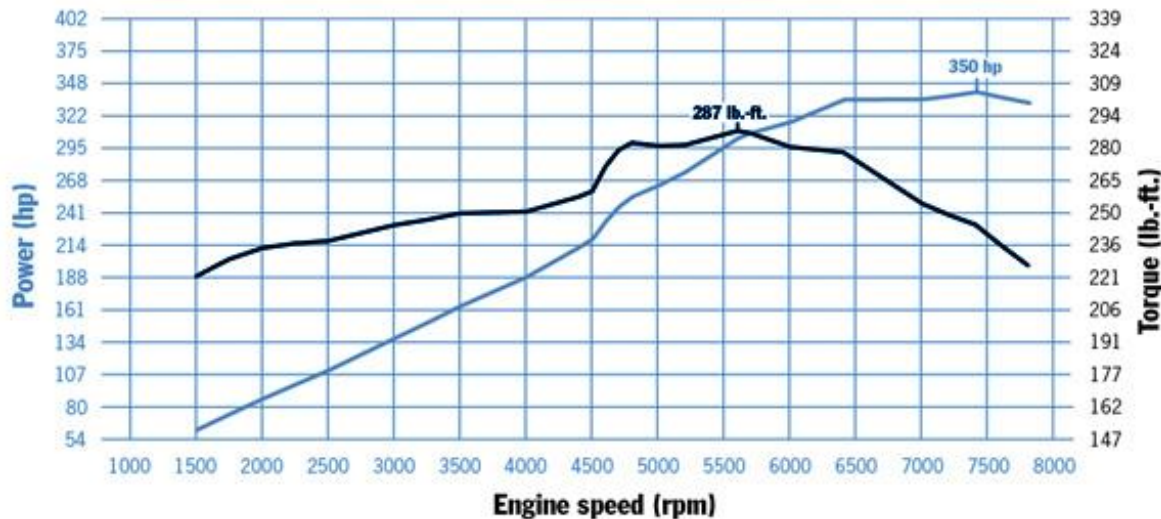
- Also, in US units:

$$horsepower = \frac{torque(lb \cdot ft) \cdot rpm}{5252}$$

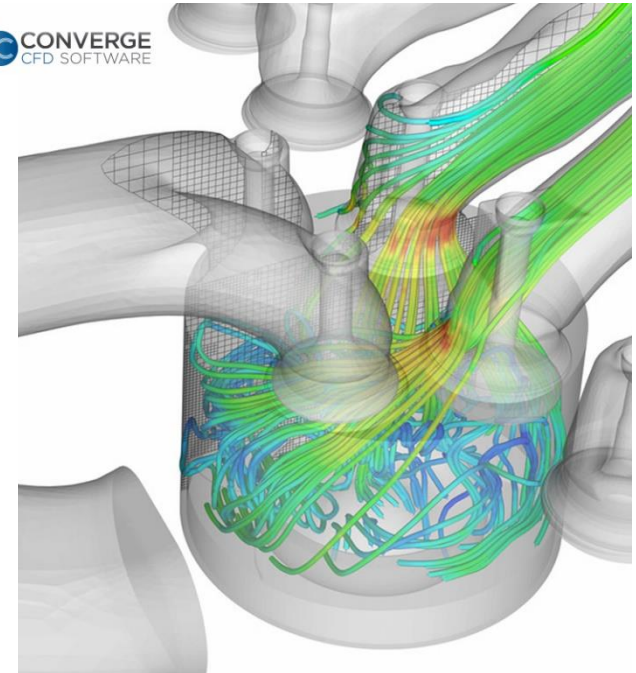
where 1 *horsepower* = 33,000 *ft · lb/min*

# Combustion Engines

- Combustion engines produce effectively zero torque at zero rpm, and increase torque in the shape of a downward parabola, usually hitting a peak at around 75% of the maximum rpm, and then dropping down



CONVERGE  
CFD SOFTWARE



# Combustion Engine Modeling

- The shape of a torque curve for a combustion engine is irregular due to complex valve timing and exhaust pressure effects, among other things
- One can model all of this as well as the detailed combustion effects taking place within the pistons
- For real-time models however, one would most likely just build a table that has engine torque output as a function of RPM and throttle
- These values are easy to measure with special test equipment

# Electric Motors

- Electric motors are more mechanically simple and tend to provide more uniform performance
- The maximum torque is inversely proportional to the rotation speed, which means that electric motors generate their peak torque at zero rpm and generate decreasing torque as the RPM increases
- There will also be a flat region at low RPM where the torque is capped to prevent exceeding electric current limitations in the battery and other systems

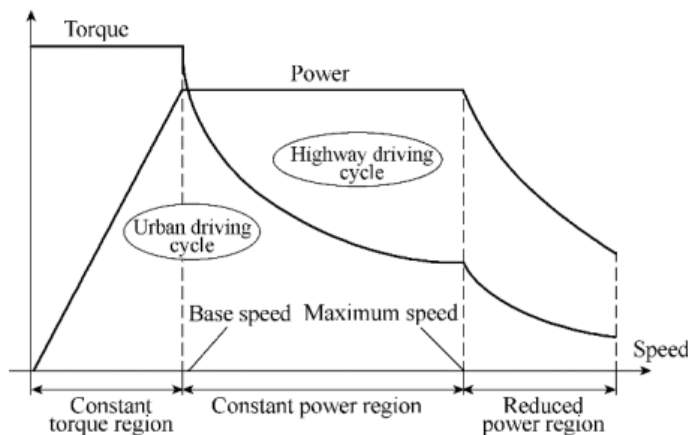
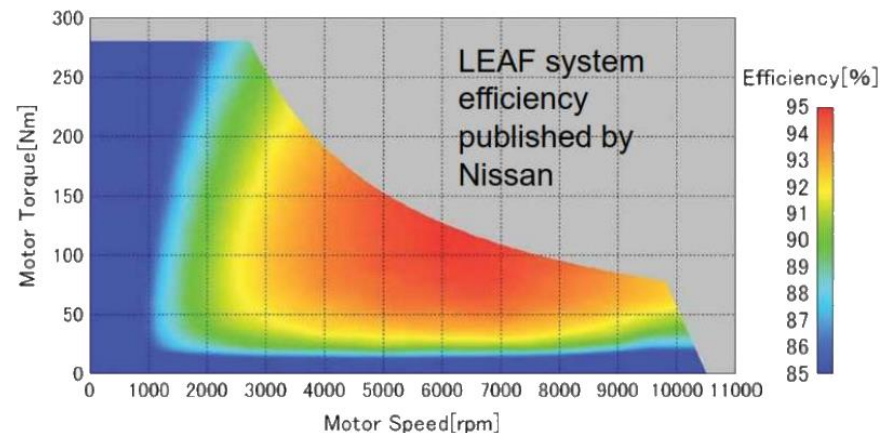


Fig.12 Typical speed-torque characteristics of EVs & HEVs from electric traction motor



# Hybrid Drivetrains

- Hybrid systems combine combustion and electrical power generation
- These can be coupled with clutches to control when each is engaged
- Also, electrical motors can act as energy recovering devices where the unpowered electric motor is engaged with the drivetrain and acts as a brake as well as an electric generator which can re-charge the battery

# Electric Vehicle Types

- HEV: Hybrid Electric Vehicle
  - Uses gasoline powered engine to charge battery and provide current for electric motor
- PHEV: Plug-In Hybrid Electric Vehicle
  - Uses gasoline and/or electricity from grid
  - Typically has smaller gas tank and larger battery than HEV
- BEV: Battery Electric Vehicle
  - 100% electrically powered
  - No gasoline engine at all

# Shaft Friction

- The friction torque on a shaft is due to the linear friction forces operating on the shaft surface
- As with linear sliding friction, these are roughly constant, leading to a constant friction torque for each of the key rotating shaft systems like the engine, transmission, differential, and axles

# Gear Ratios

- For two meshed gears, the gear ratio  $g$  is the ratio between the number of teeth in the output gear  $z_{out}$  and the input gear  $z_{in}$

$$g = \frac{z_{out}}{z_{in}}$$

- The relationship between the two angular speeds is:

$$\omega_{out} = \frac{\omega_{in}}{g}$$

- And the relation between the input and output torque is:

$$\tau_{out} = g\tau_{in}$$

- The power remains constant, minus a small loss to friction (typically around 1-2%)

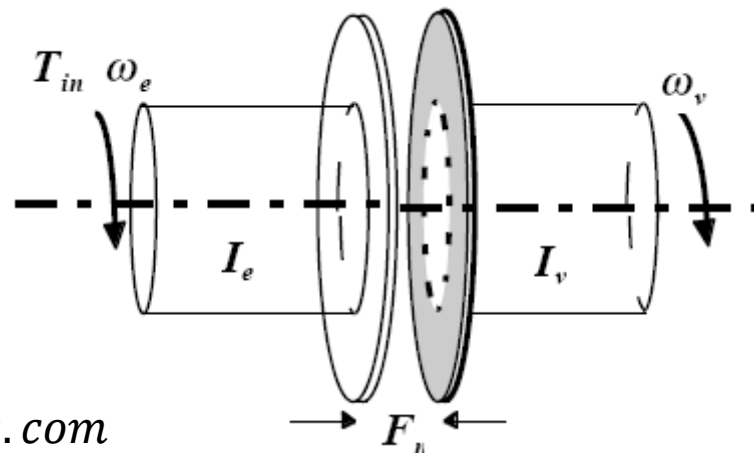


# Transmission & Clutch

- The transmission of a car provides a changeable gear ratio between the engine and eventually to the wheels (typically through a differential first)
- To switch gears requires a clutch to engage and disengage, which can be modeled as a variable friction joint
- There are a few common options for transmission types:
  - Manual clutch
  - Automatic clutch
  - Dual clutch
- In addition, a variety of continuously variable transmissions have been introduced over the years, but are mainly used in low-torque vehicles like scooters or very small cars

# Clutch Models

- A clutch is a type of shaft coupling that can be engaged or disengaged
- Typically, they are based on circular plates coupled with friction
- Springs in the system hold the plates together, engaging the clutch
- To disengage the clutch, requires an externally applied force typically coming from a human pressing a pedal, or an automatic hydraulic actuator
- When engaged, the clutch can be locked with static friction or slipping with dynamic friction. Clutch models often are two-state machines that can alternate between locked and slipping states



# Clutch Model

- Consider a two-state clutch model that has locked and slipping states
- The clutch couples the rotation of the engine output shaft with the rotation of the transmission input shaft, and therefore the remainder of the rigidly coupled drivetrain
- The torque capacity is the maximum torque the clutch can transmit, and is based on the *effective radius*  $R$  of the plates, the static friction  $\mu_s$ , and normal force  $f_n$  holding the plates together

$$\tau_{max} = \frac{2}{3} R f_n \mu_s$$

- The effective radius  $R$  is the equivalent radius of a circular disc, but accounts for the fact that most clutch plates are flattened rings with an inner radius  $r_{in}$  and outer radius  $r_{out}$ :

$$R = \frac{r_{out}^3 - r_{in}^3}{r_{out}^2 - r_{in}^2}$$

# Clutch Model

- When the clutch is in the locked state, the output angular velocity  $\omega_E$  of the engine matches that of the remainder of the vehicle drivetrain  $\omega_V$
- The engine outputs a torque of  $\tau_E$
- The tires apply a longitudinal force at the contact patch, applying a torque on the wheel and axle. This torque is transmitted up through the drivetrain (along with various drivetrain frictions) to the clutch, resulting in a vehicle torque of  $\tau_V$

# Clutch Model

- $\tau_E$  is the engine torque on the input shaft of the clutch and  $\tau_V$  is the torque the drivetrain puts on the output shaft
- If the velocities of the two clutch shafts are equal and the difference of torques is within the maximum torque, then the shafts will be rigidly locked together, otherwise they will slip
- If they slip, then we have two separate rotating systems each with their own torques

# Transmission

- Combustion engine cars typically have around 4-7 forward gears, ranging from first gear (highest ratio) to top gear (lowest ratio)
  - First gear often has a gear ratio  $g_1$  near 3 or 4
  - In some cars, the top gear ratio is 1.0
  - Others have one or more *overdrive gears* with ratios under 1.0
  - Typically, the forward gears are spaced with roughly equal proportions:  $g_{n+1}/g_n \approx \text{const}$
- The reverse gear typically has a ratio near -3 or -4
- Due to the torque properties of electrical motors, many electrical cars do not use transmissions at all, plus they can run the electric motor directly in reverse

# \*Differential

- [insert slide here]

# Engine Mount

- In full vehicle dynamic models, the engine block is often modeled as a rigid body that is coupled to the chassis with a combination of rigid and elastic joints
- This allows a small amount of engine movement which can be used in analysis of vibrations for comfort and noise



# Drivetrain Modeling

- We could model the drivetrain as articulated rigid bodies connected to the chassis with joints
- Or we could take a slightly simpler approach where we model the drivetrain as its own system and ultimately connect it to the vehicle by applying torques on the wheels and the chassis (or engine mount)

# Drivetrain Modeling

- To compute drivetrain forces:
  - We first need to know the longitudinal tire forces on the driven wheels, which need to be computed anyway, and can be computed by kinematic analysis and tire force modeling
  - We compute the torque these forces put on the drivetrain based on their offset from the wheel axle ( $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{f}$ ) and adjust it for the gear ratio to compute the final torque back on the output shaft of the clutch
  - We compute the torque that the engine is currently producing based on the current RPM and throttle settings to get the torque on the input shaft of the clutch
  - We then use the clutch model to determine if the systems are locked or slipping and compute the appropriate torque transmitted through the clutch
  - This is then passed through the transmission and ultimately to the rigid body wheel hubs

# Vehicle Simulation

# Vehicle Simulation

- To solve all forces for the vehicle:
  - Compute known kinematic forces
    - Compute tire forces based on current configuration
    - Compute misc. forces from gravity, springs, dampers, roll bars, elastic bushings, brakes, aerodynamics, etc.
  - Compute drivetrain forces based on tire forces, clutch force, throttle, and engine RPM
  - Solve unknown constraint forces for suspension

# Vehicle Simulation

- Additional topics:
  - Sensors (cameras, LIDAR, IMU, ultrasonic...)
  - Control systems (cruise control, automatic transmission...)
  - Advanced Driver Assistance Systems (ADAS)
  - Driver behavior simulation
  - Traffic simulation
  - Environment modeling
  - Autonomous vehicles