Sampling and Aliasing, and The Discrete Fourier Transform

Image Processing

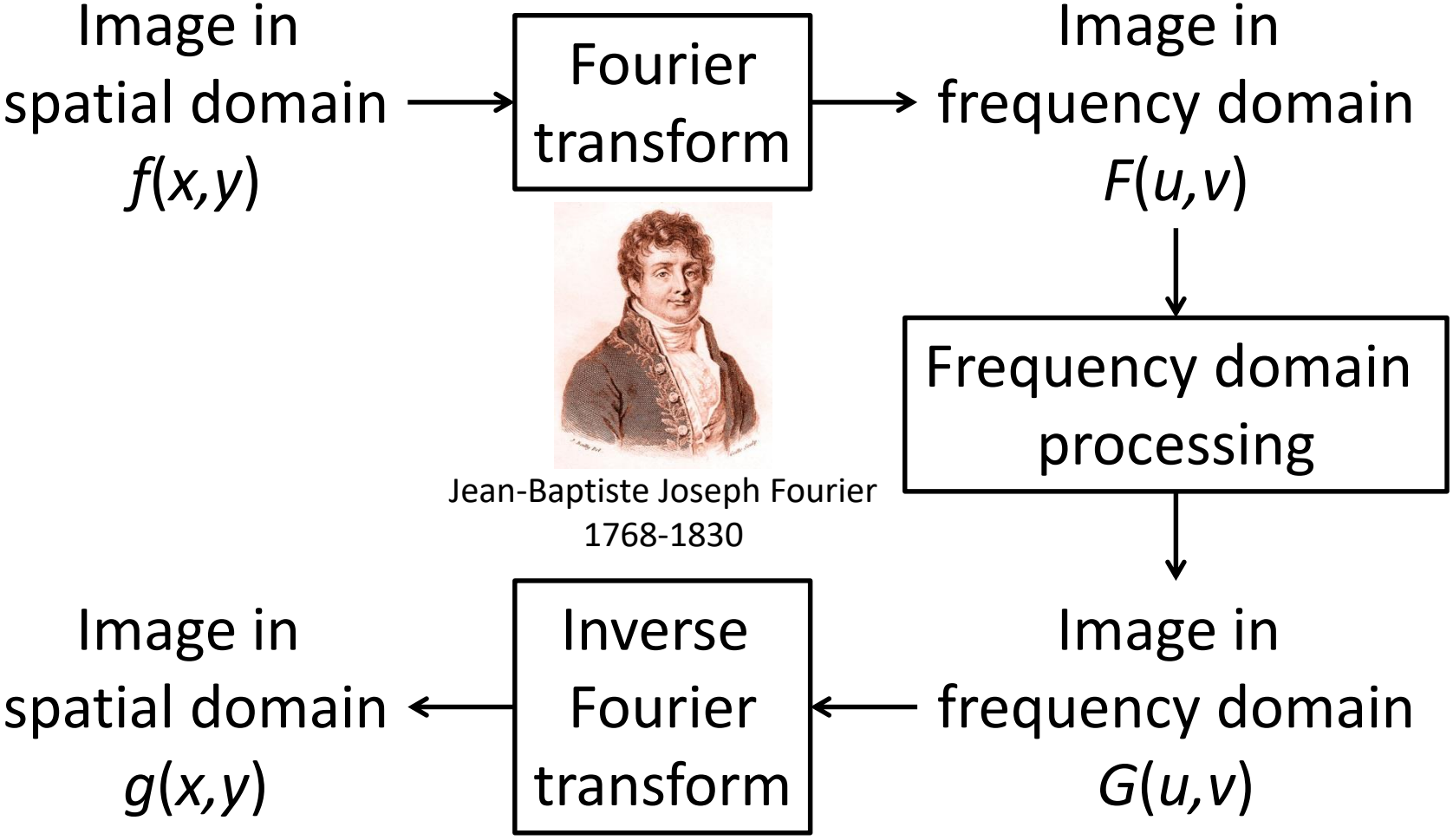
CSE 166

Lecture 6

Announcements

- Assignment 2 is due today, 11:59 PM
- Assignment 3 will be released Apr 22
- Reading
  - Chapter 4: Filtering in the Frequency Domain

Overview: Image processing in the frequency domain



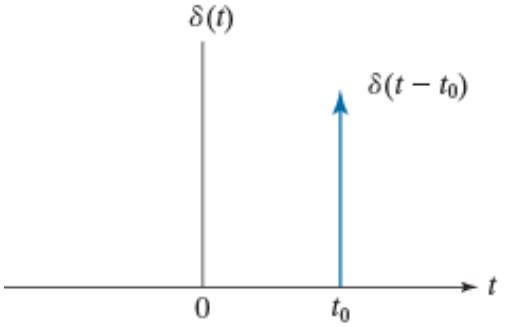
Jean-Baptiste Joseph Fourier
1768-1830

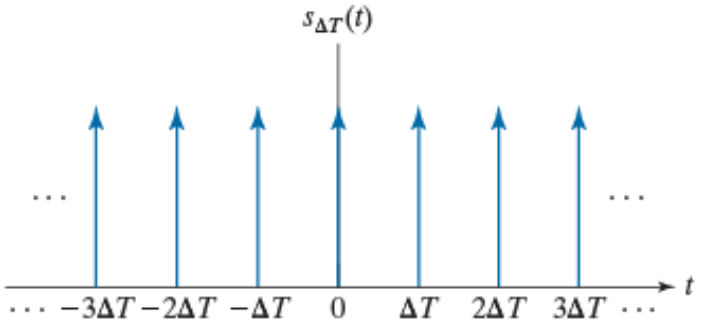
1D impulse function and impulse train

Impulse function

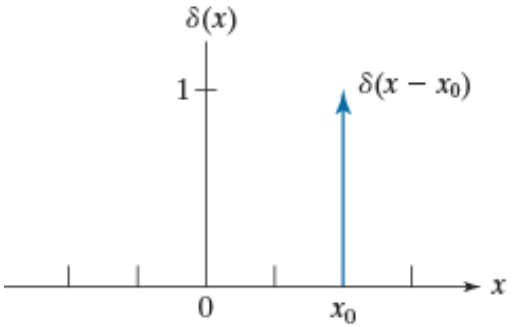
Impulse train

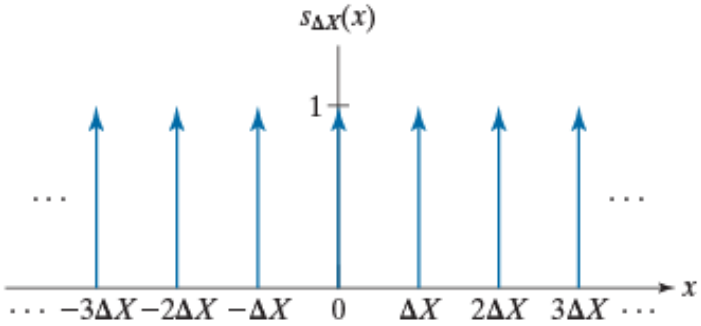
Continuous



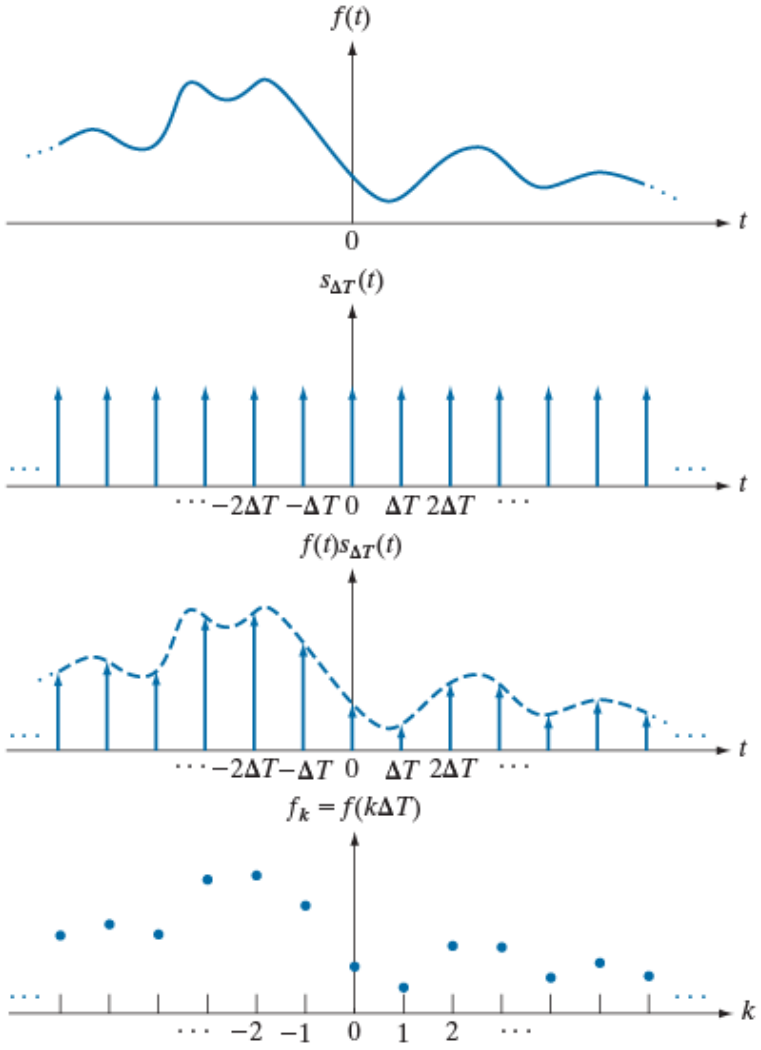


Discrete





Sampling

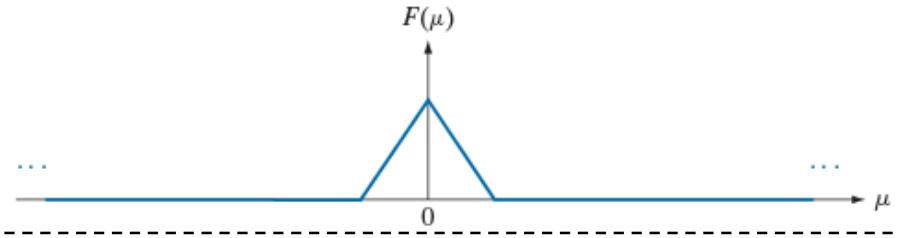


a
b
c
d

FIGURE 4.5
(a) A continuous function. (b) Train of impulses used to model sampling. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of impulses. (The dashed line in (c) is shown for reference. It is not part of the data.)

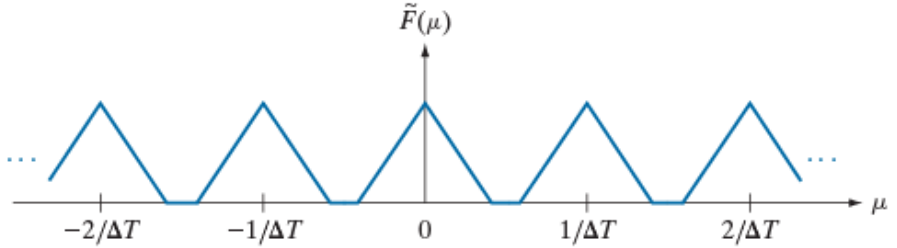
Sampling

Fourier transform of function

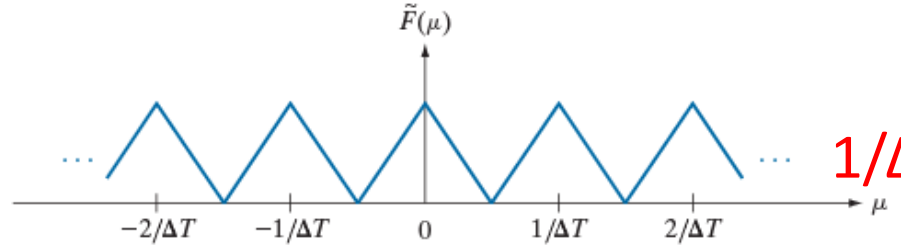


Fourier transforms of **sampled** function

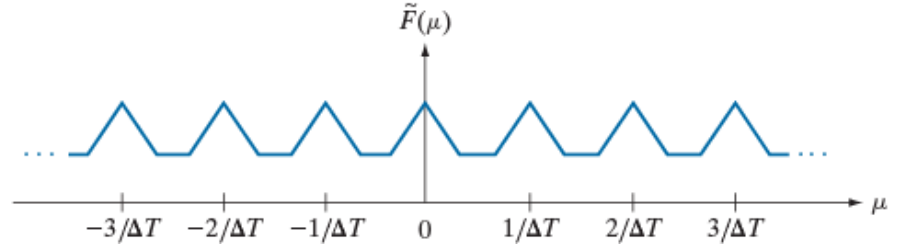
Over-sampled



Critically-sampled



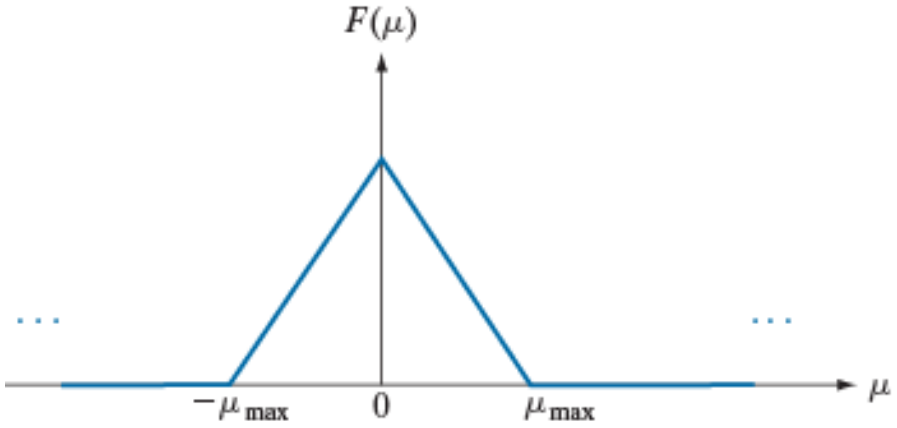
Under-sampled



$1/\Delta T$

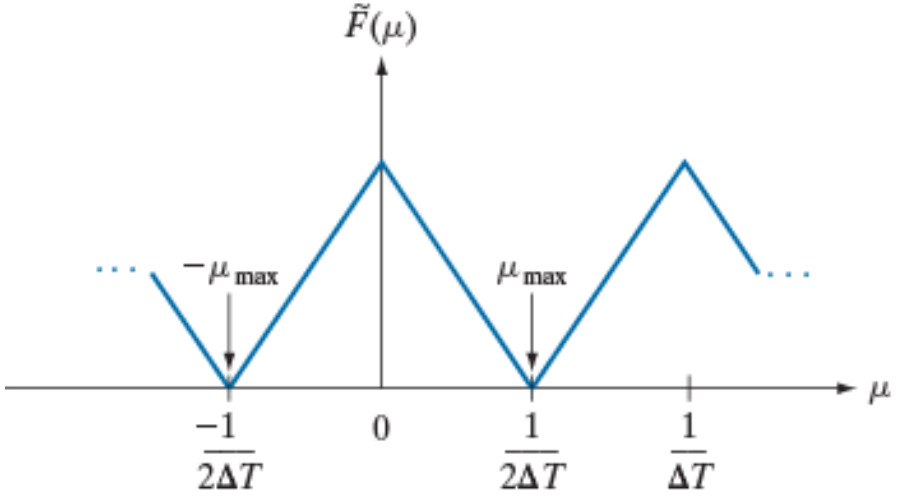
The sampling theorem

Fourier transform of function



Fourier transform of **sampled** function

Critically-sampled



Recovering F(μ) from $\tilde{F}$(μ)

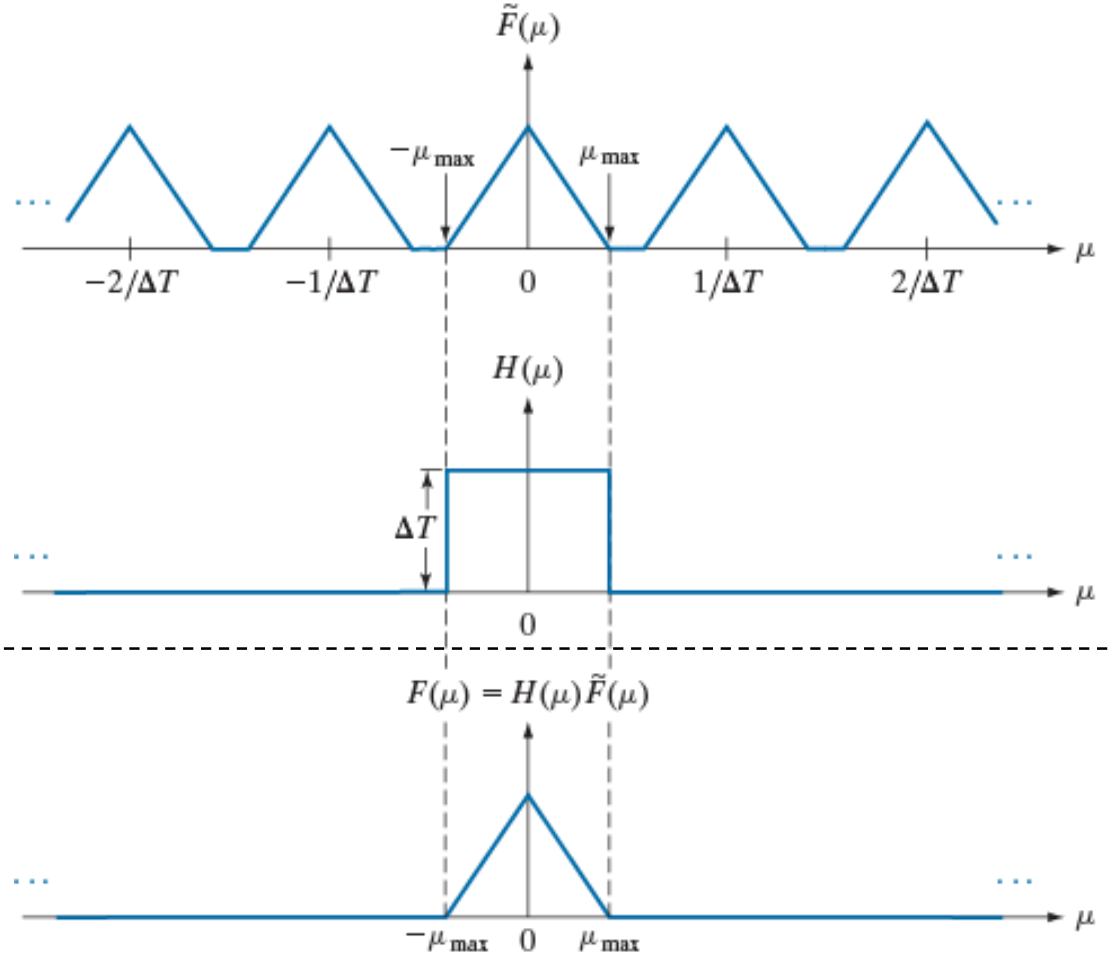
Fourier transform of **sampled** function

Over-sampled

Ideal lowpass filter

Product of above

Recovered

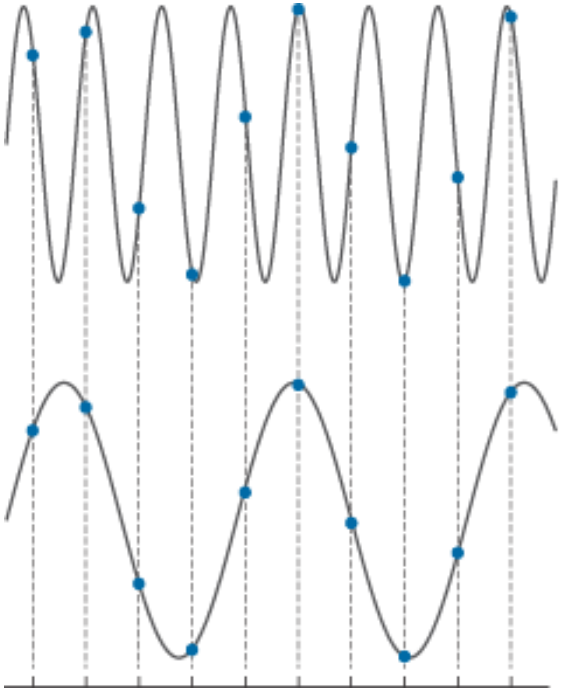


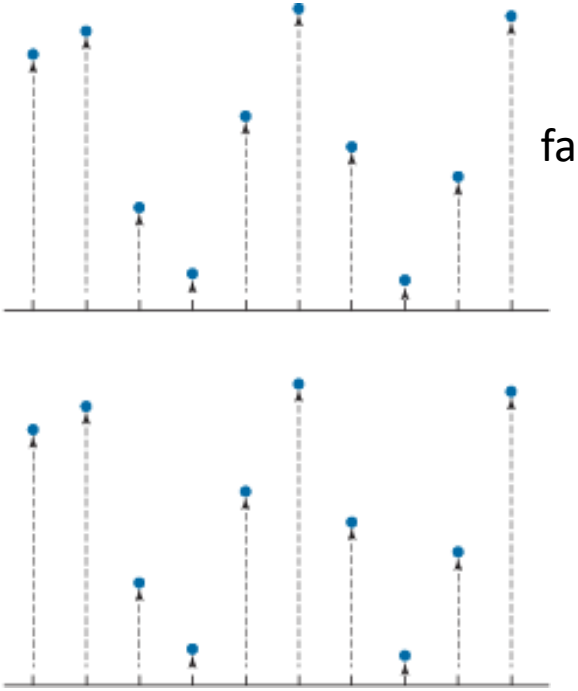
Aliasing

Continuous

Discrete

Under-sampled





Alias: a false identity

Different

Identical

Over-sampled

Sampled at same rate

Aliasing

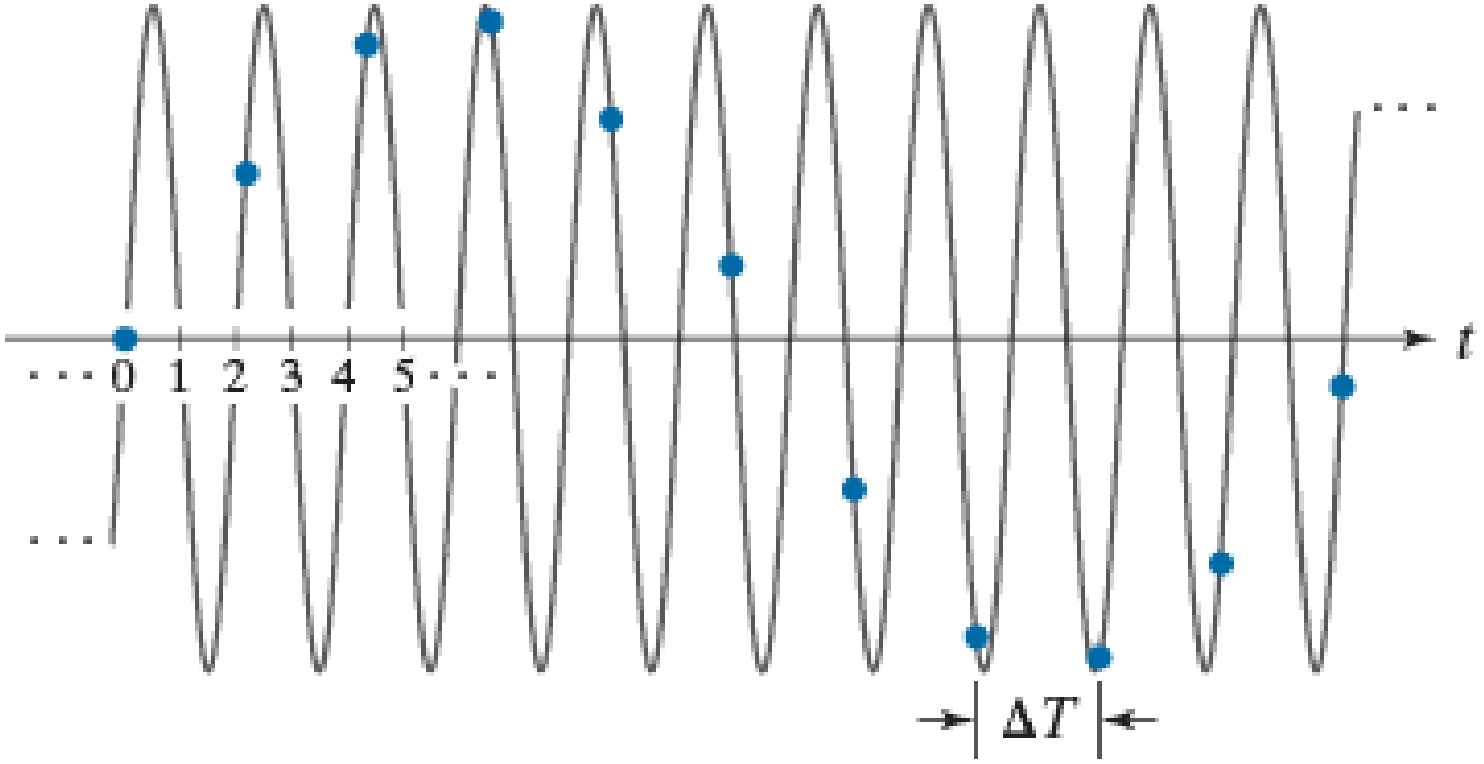
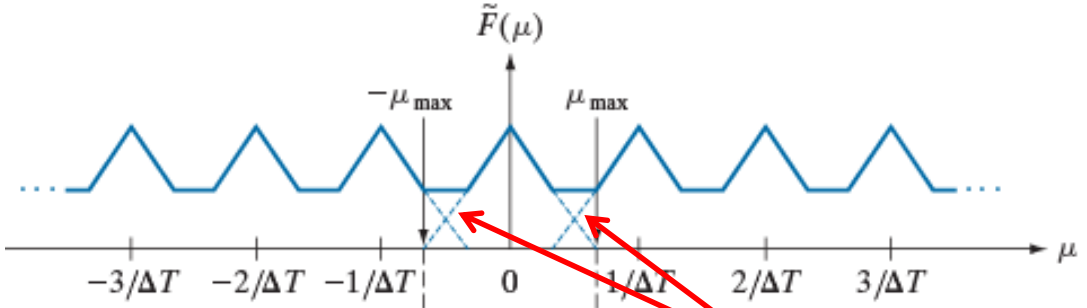


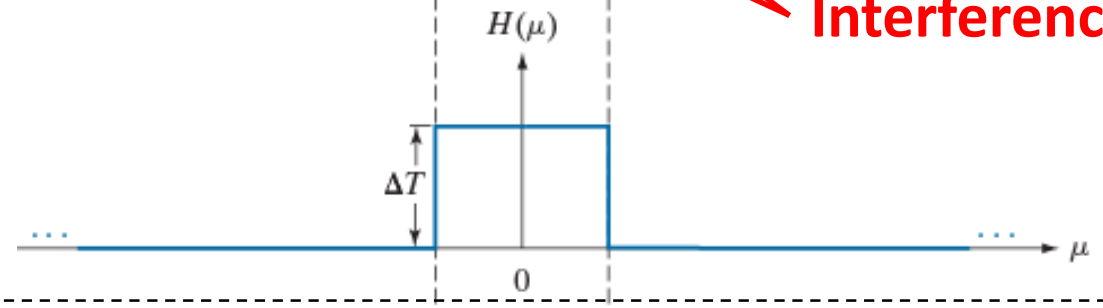
FIGURE 4.11 Illustration of aliasing. The under-sampled function (dots) looks like a sine wave having a frequency much lower than the frequency of the continuous signal. The period of the sine wave is 2 s, so the zero crossings of the horizontal axis occur every second. ΔT is the separation between samples.

Aliasing

Fourier transform of **under-sampled** function

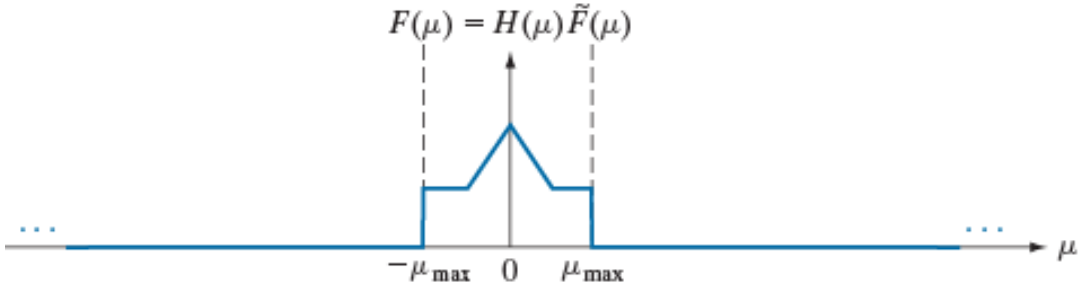


Ideal lowpass filter



Product of above

Imperfect recovery



Next Lecture

- Filtering in the frequency domain
- Reading
  - Chapter 4: Filtering in the Frequency Domain