Image Compression and Watermarking

Image Processing

CSE 166

Lecture 14

Announcements

- Assignment 4 is due May 22, 11:59 PM
- Reading
  - Chapter 8: Image Compression and Watermarking
    - Sections 8.1, 8.9, 8.10, and 8.12

Data compression

- Data redundancy

$$R = 1 - \frac{1}{C}$$

where compression ratio

$$C = \frac{b}{b'}$$

where

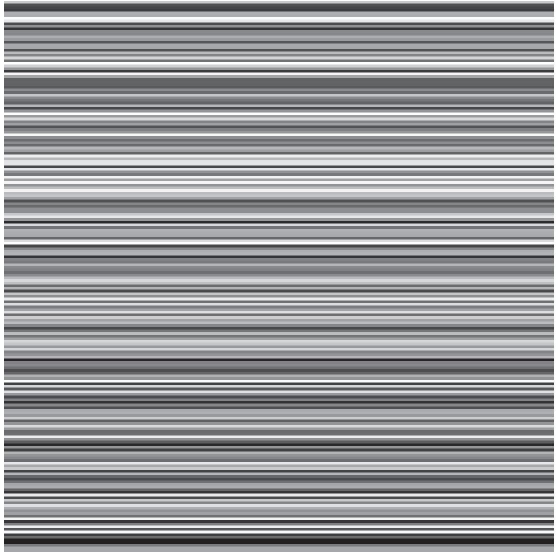
b and b' are the number of bits in two different representations of the same information

Data redundancy in images

Coding redundancy

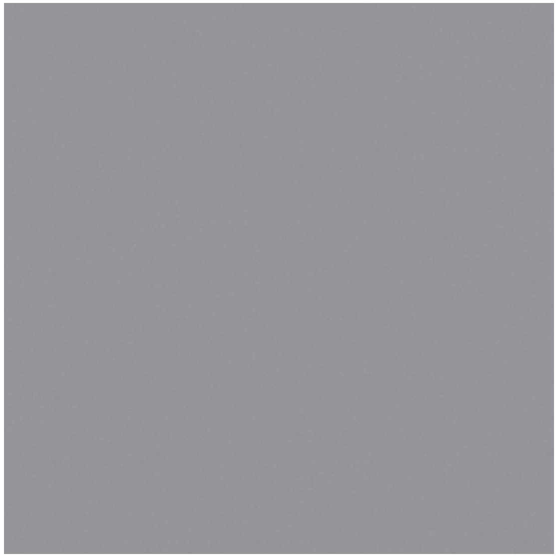
Does not need all 8 bits

Spatial redundancy



Information is unnecessarily replicated

Irrelevant information



Information is not useful

Image information

- Entropy

$$\tilde{H} = -\sum_{k=0}^{L-1} p_r(r_k) \log_2(p_r(r_k))$$

where

L is the number of intensity or gray levels
r_k is input image intensity or gray level value k
$p_r(r_k)$ is normalized histogram of input image

  - It is not possible to encode input image with fewer than $\tilde{H}$ bits/pixel

Fidelity criteria, objective (quantitative)

- Total error

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left(\hat{f}(x,y)-f(x,y)\right)$$

- Root-mean-square error

$$e_{\text{rms}}=\left(\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left(\hat{f}(x,y)-f(x,y)\right)^2\right)^{1/2}$$

- Mean-square signal to noise ratio (SNR)

$$\text{SNR}_{\text{ms}}=\frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left(\hat{f}(x,y)-f(x,y)\right)^2}$$

Fidelity criteria, subjective (qualitative)

TABLE 8.2
Rating scale of the Television Allocations Study Organization. (Frendendall and Behrend.)

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

Approximations

Objective (quantitative) quality
rms error (in intensity levels)

Lower is better

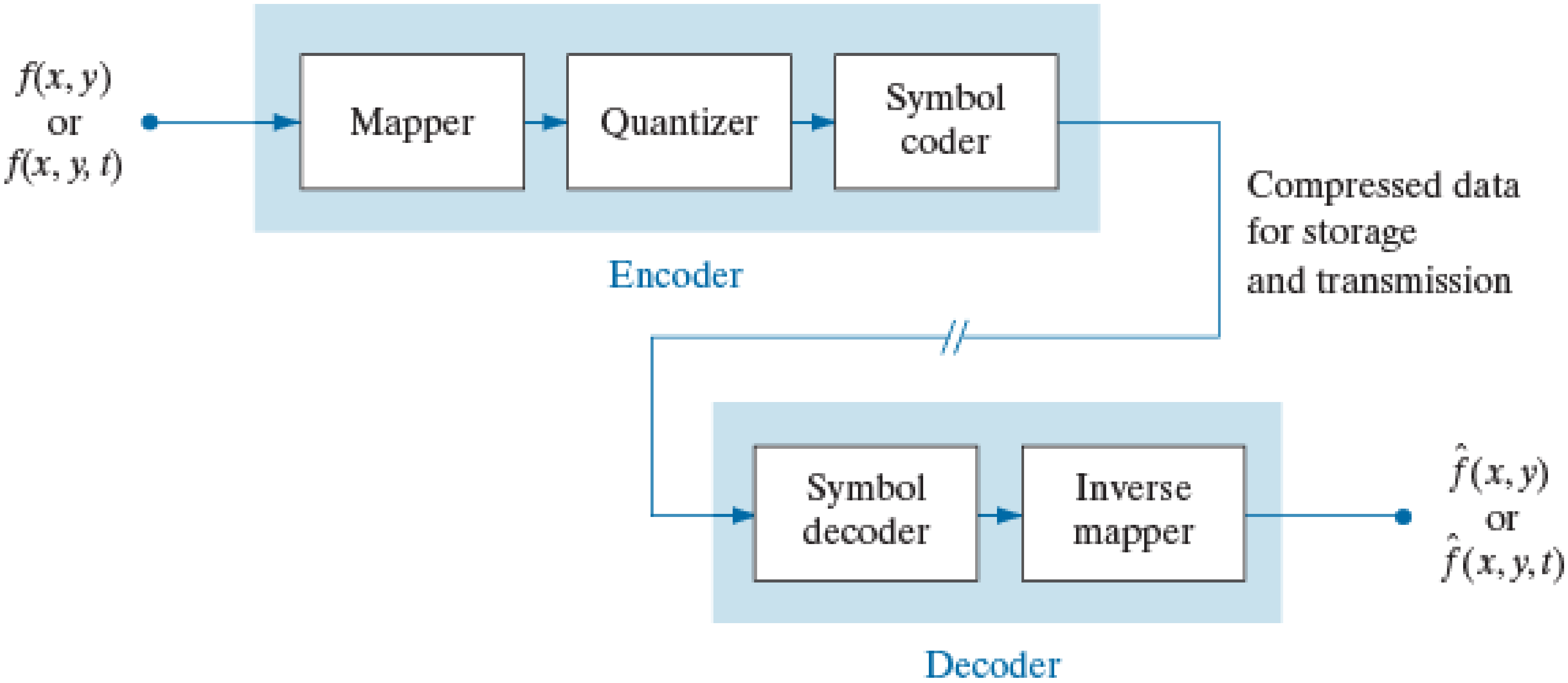
5.17 15.67 14.17

(a) (b) (c)

(a) is better than (b).
(b) is better than (c)

Subjective (qualitative) quality, relative

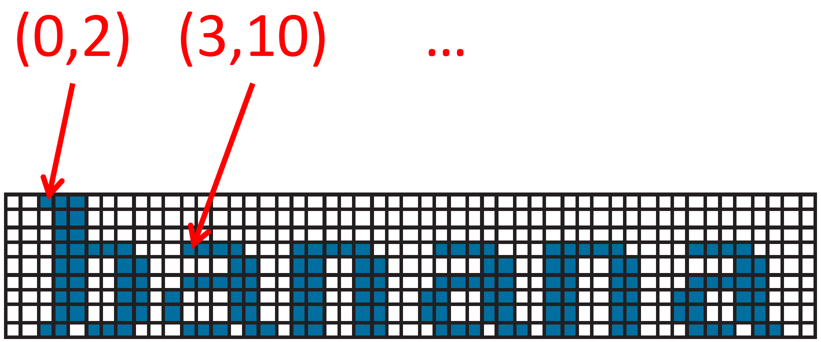
Compression system

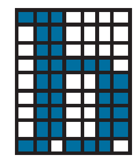
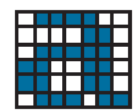
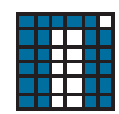


Compression methods

- Huffman coding
- Golomb coding
- Arithmetic coding
- Lempel-Ziv-Welch (LZW) coding
- Run-length coding
- Symbol-based coding
- Bit-plane coding
- Block transform coding
- Predictive coding
- Wavelet coding

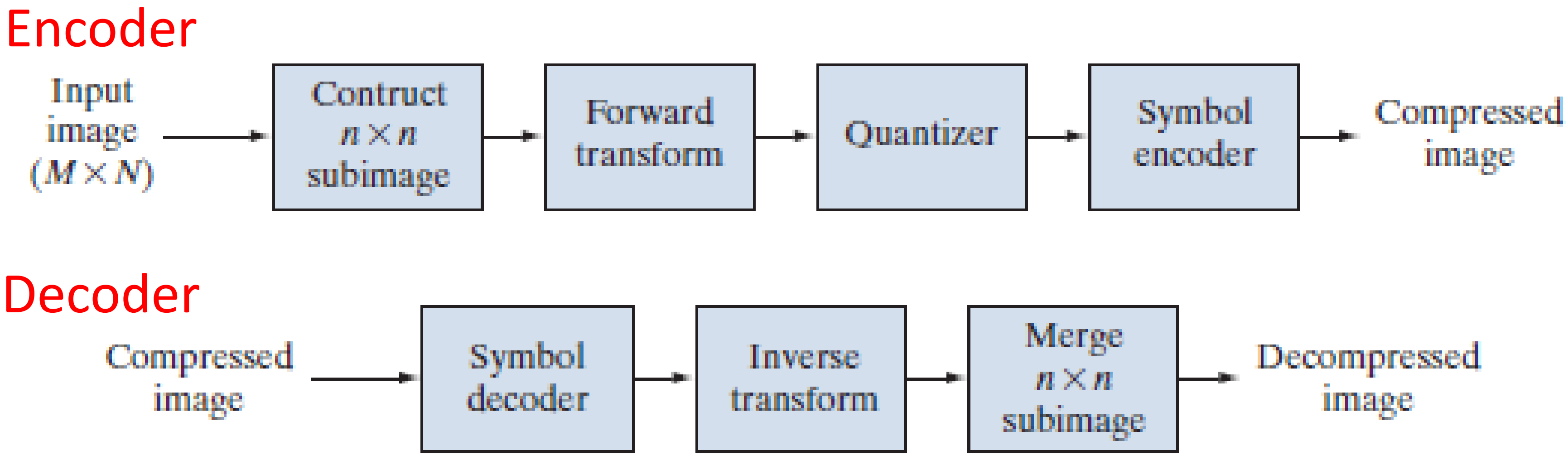
Symbol-based coding



Token	Symbol
0	
1	
2	

Triplet
(0, 2, 0)
(3,10, 1)
(3, 18, 2)
(3, 26, 1)
(3, 34, 2)
(3, 42, 1)

Block-transform coding



Block-transform coding

- Example: discrete cosine transform

$n \times n$ subimage $g(x, y)$

$$T(u,v) = \sum_{x=0}^{n-1}\sum_{y=0}^{n-1} g(x,y)\alpha(u)\alpha(v)\cos\left(\frac{(2x+1)u\pi}{2n}\right)\cos\left(\frac{(2y+1)v\pi}{2n}\right)$$

where

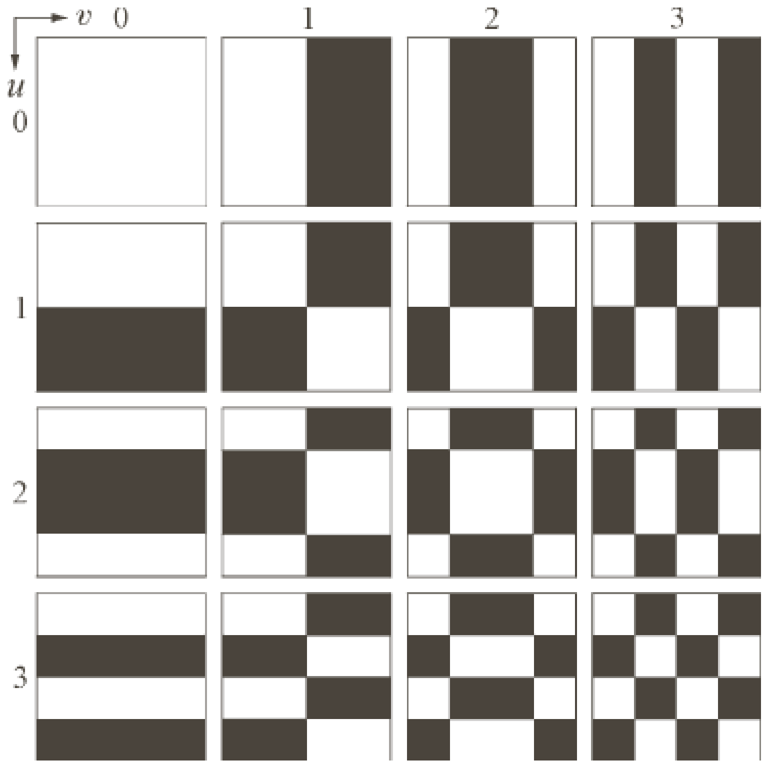
$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{n}} & \text{if } u = 0 \\ \sqrt{\frac{2}{n}} & \text{otherwise} \end{cases} \text{ and } \alpha(v) = \begin{cases} \sqrt{\frac{1}{n}} & \text{if } v = 0 \\ \sqrt{\frac{2}{n}} & \text{otherwise} \end{cases}$$

- Inverse

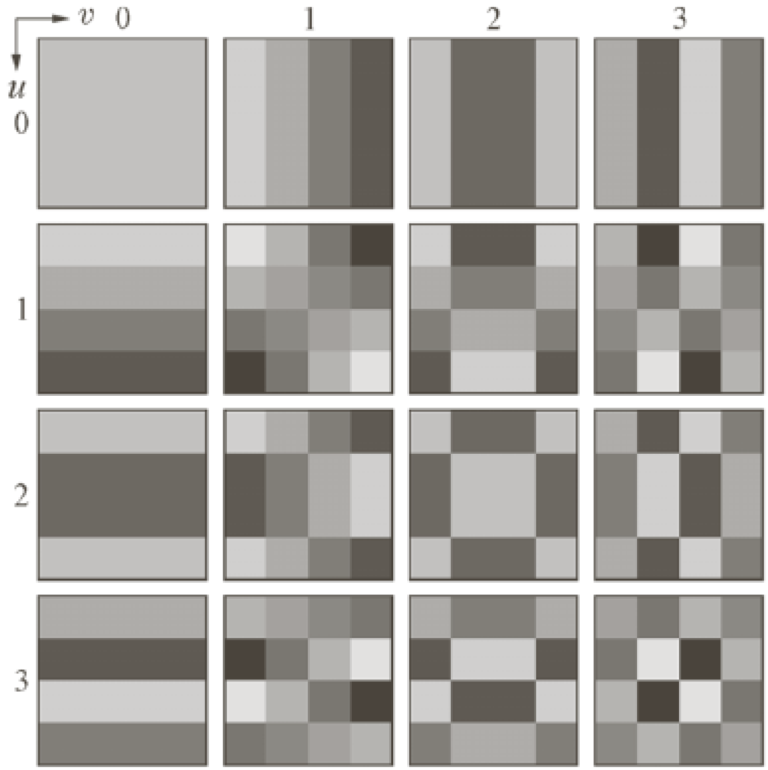
$$g(x,y) = \sum_{u=0}^{n-1}\sum_{v=0}^{n-1} T(u,v)\alpha(u)\alpha(v)\cos\left(\frac{(2x+1)u\pi}{2n}\right)\cos\left(\frac{(2y+1)v\pi}{2n}\right)$$

Block-transform coding

4x4 subimages (4x4 basis images)



Walsh-Hadamard transform



Discrete cosine transform

Block-transform coding

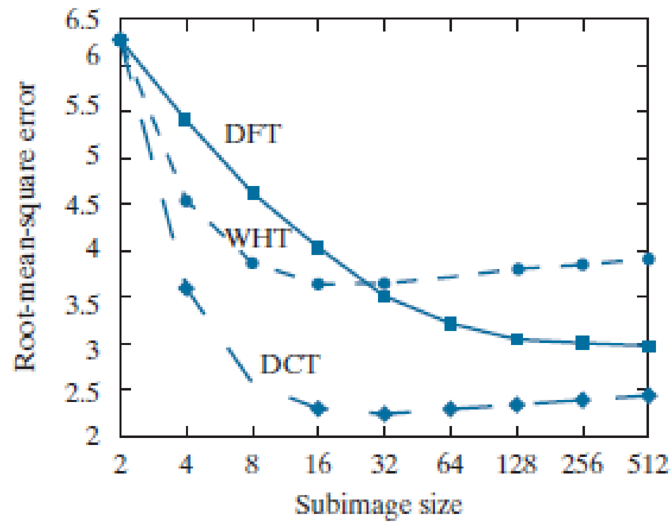
8x8 subimages

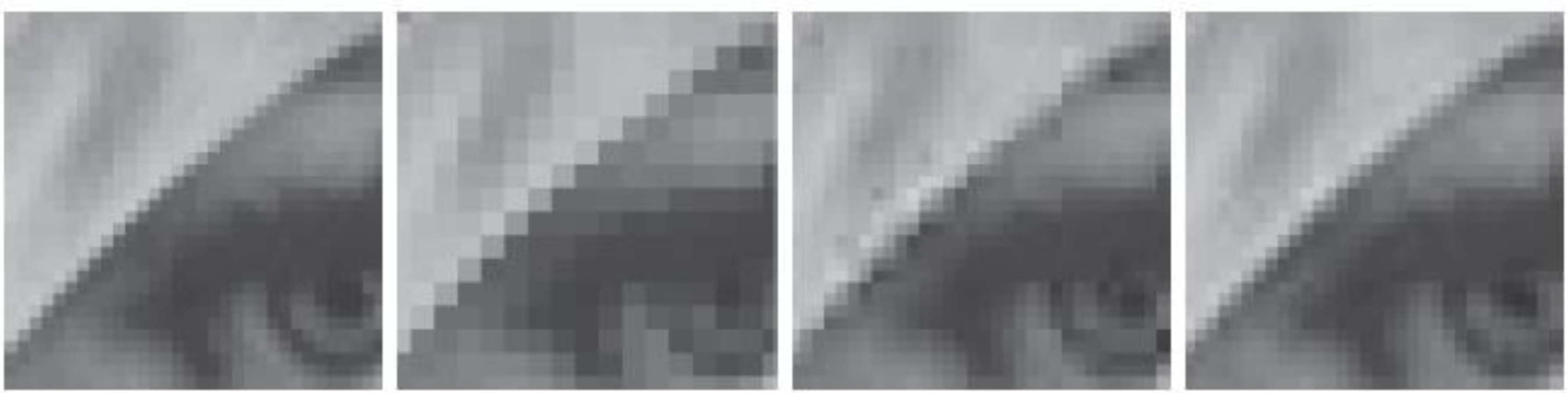
	Fourier transform	Walsh-Hadamard transform	cosine transform
Retain 32 largest coefficients			
Error image			
rms error	2.32	1.78	1.13

Lower is better

Block-transform coding

Reconstruction error versus subimage size



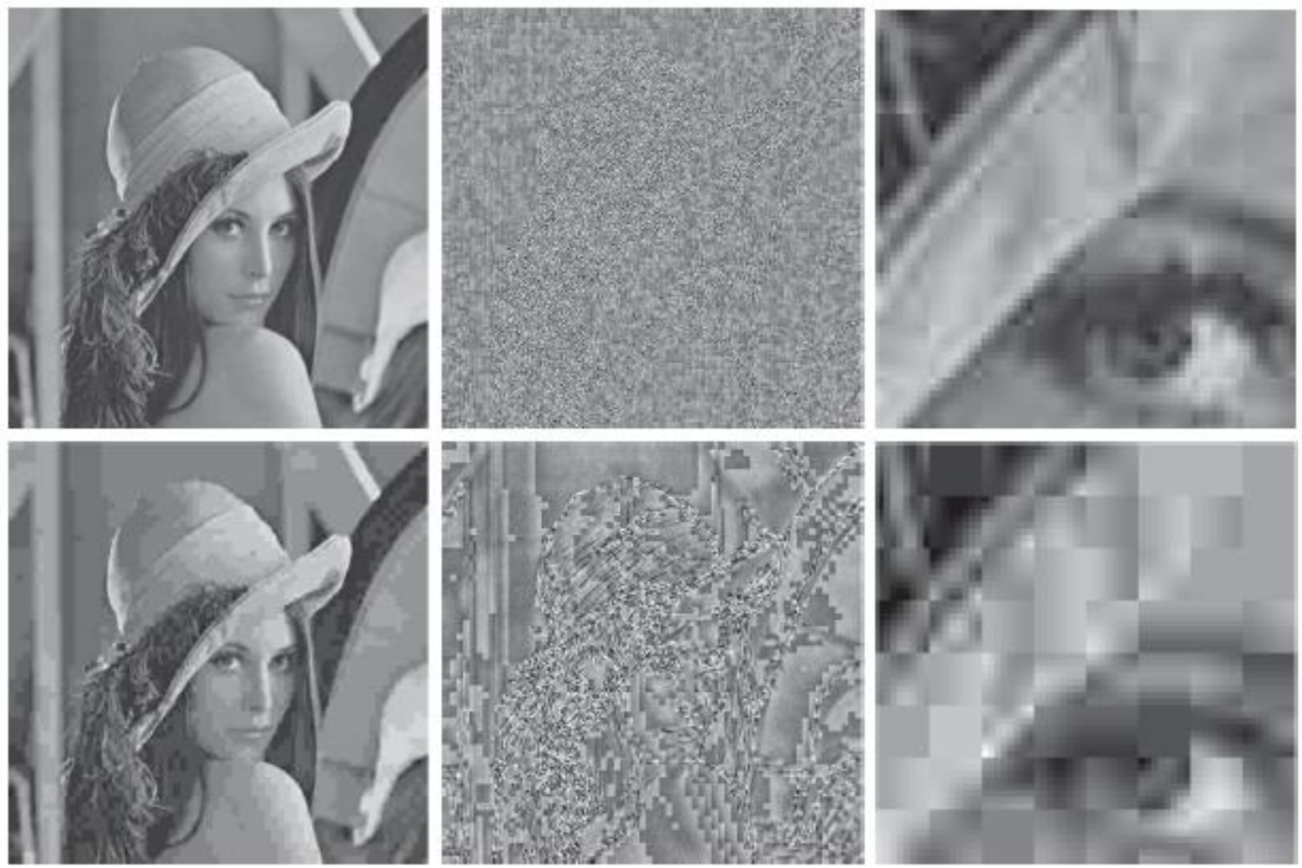


DCT subimage size: 2x2 4x4 8x8

JPEG uses block DCT-based coding

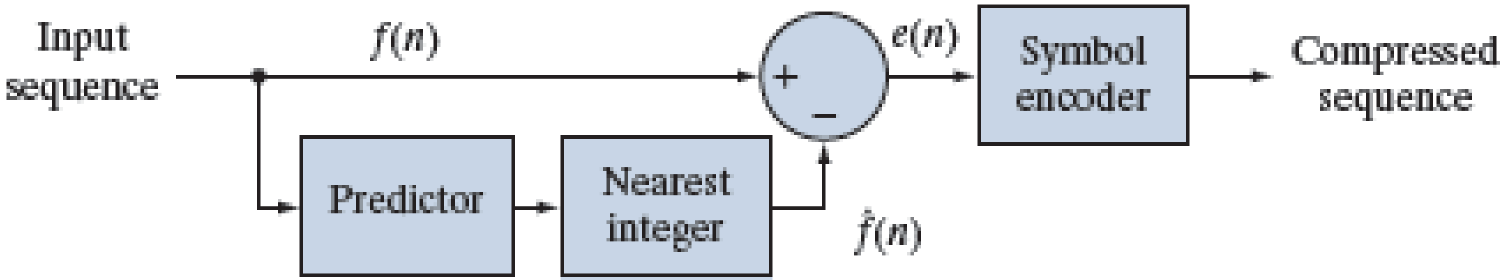
Compression reconstruction | Scaled error image | Zoomed compression reconstruction

Compression ratio: 25:1, 52:1

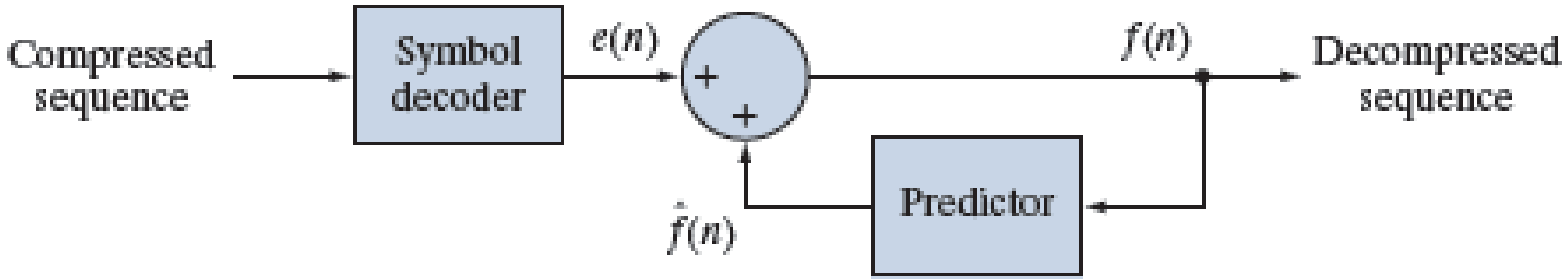


Predictive coding model

Encoder



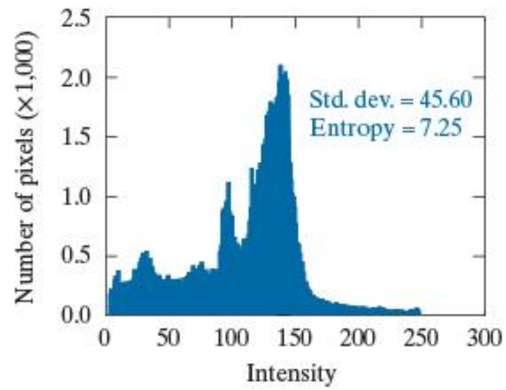
Decoder



Predictive coding

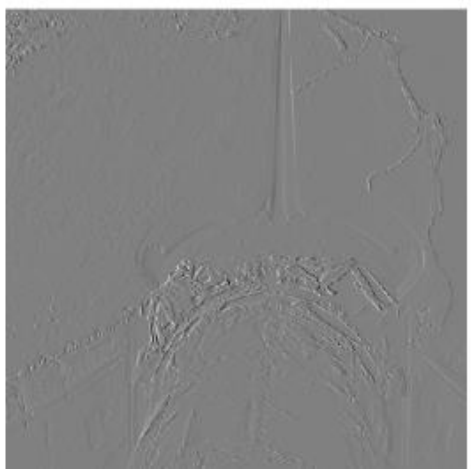
Example: previous pixel coding

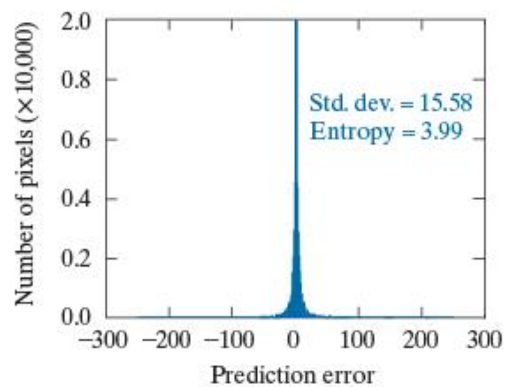
Input image



Histograms

Prediction error image

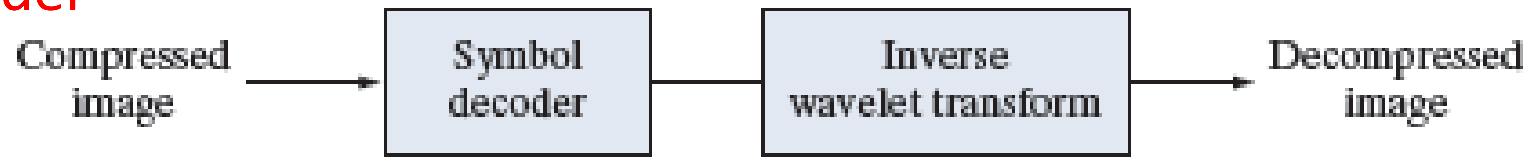




Wavelet coding

Encoder

Decoder



Wavelet coding

Detail coefficients below 25 are truncated to zero

TABLE 8.15 Decomposition level impact on wavelet coding the 512×512 image of Fig. 8.9(a).

Decomposition Level (Scales or Filter Bank Iterations)	Approximation Coefficient Image	Truncated Coefficients (%)	Reconstruction Error (rms)
1	256×256	74.7%	3.27
2	128×128	91.7%	4.23
3	64×64	95.1%	4.54
4	32×32	95.6%	4.61
5	16×16	95.5%	4.63

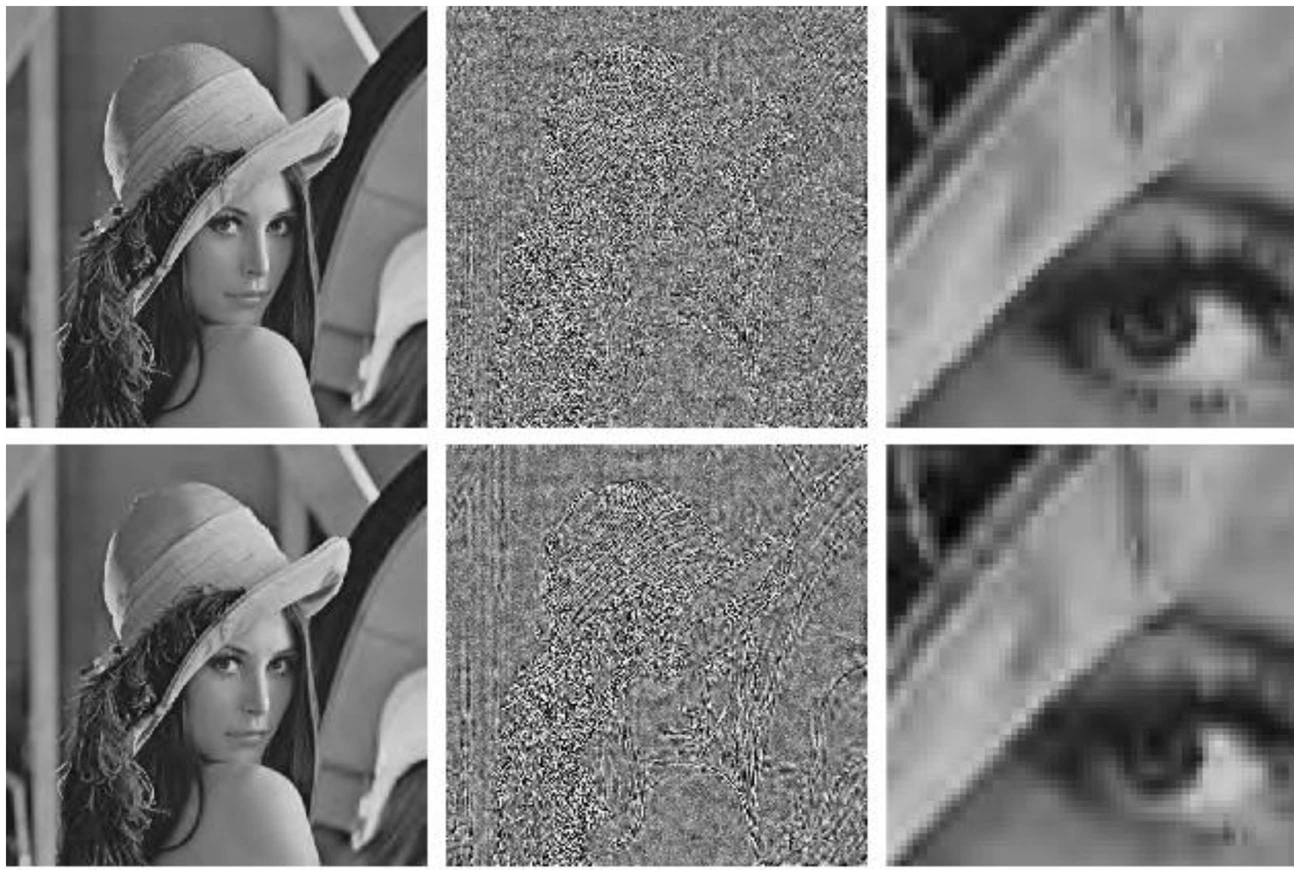
JPEG-2000 uses wavelet-based coding

Compression reconstruction | Scaled error image | Zoomed compression reconstruction

Compression ratio

25:1

52:1



JPEG-2000 uses wavelet-based coding

Compression reconstruction | Scaled error image | Zoomed compression reconstruction

Compression ratio: 75:1, 105:1

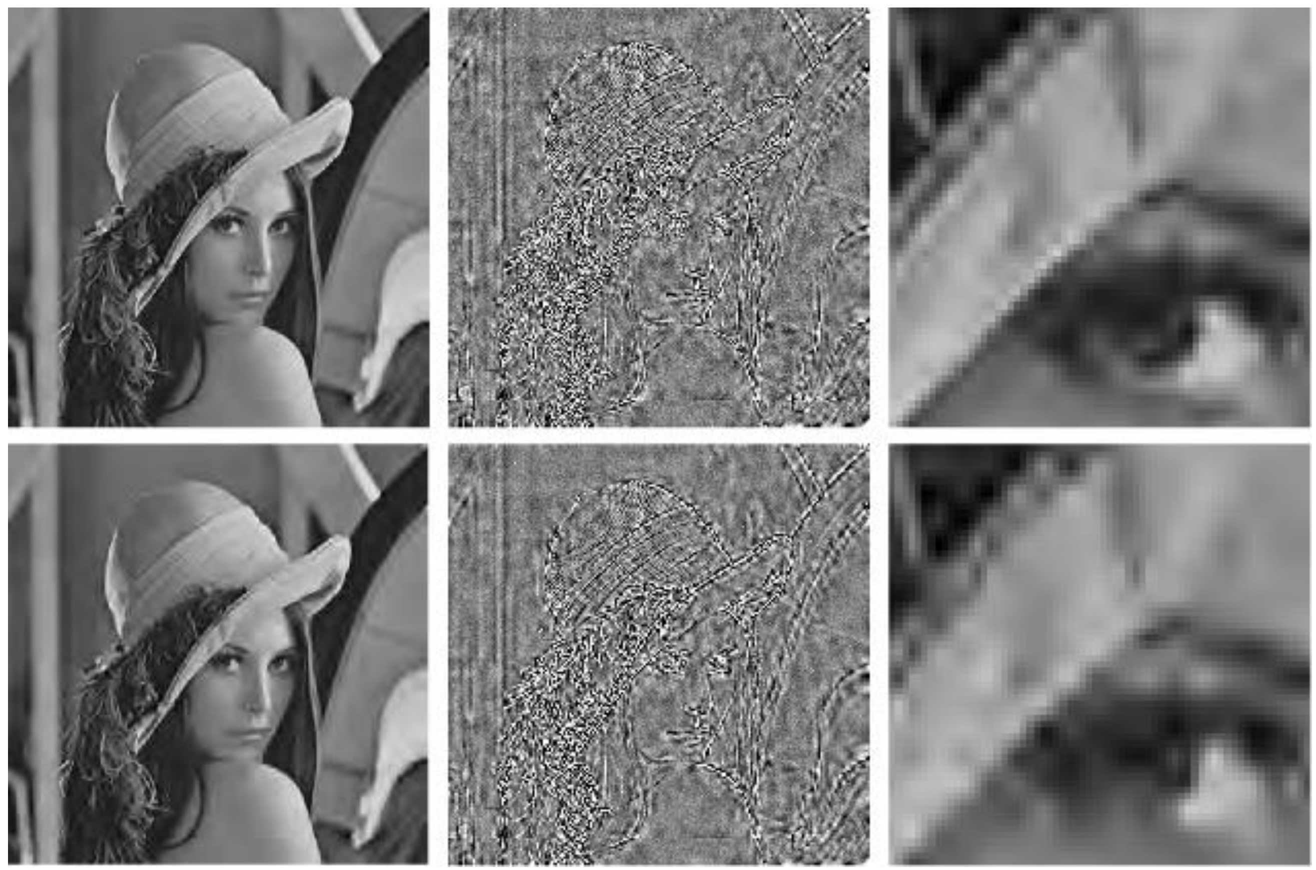


Image watermarking

- Visible watermarks
- Invisible watermarks

Visible watermark

Watermark

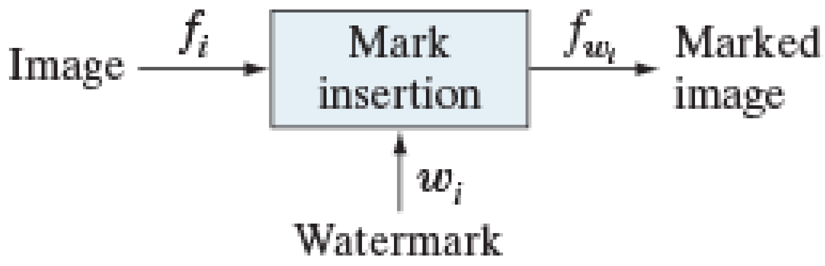
Watermarked image

Original image minus watermark

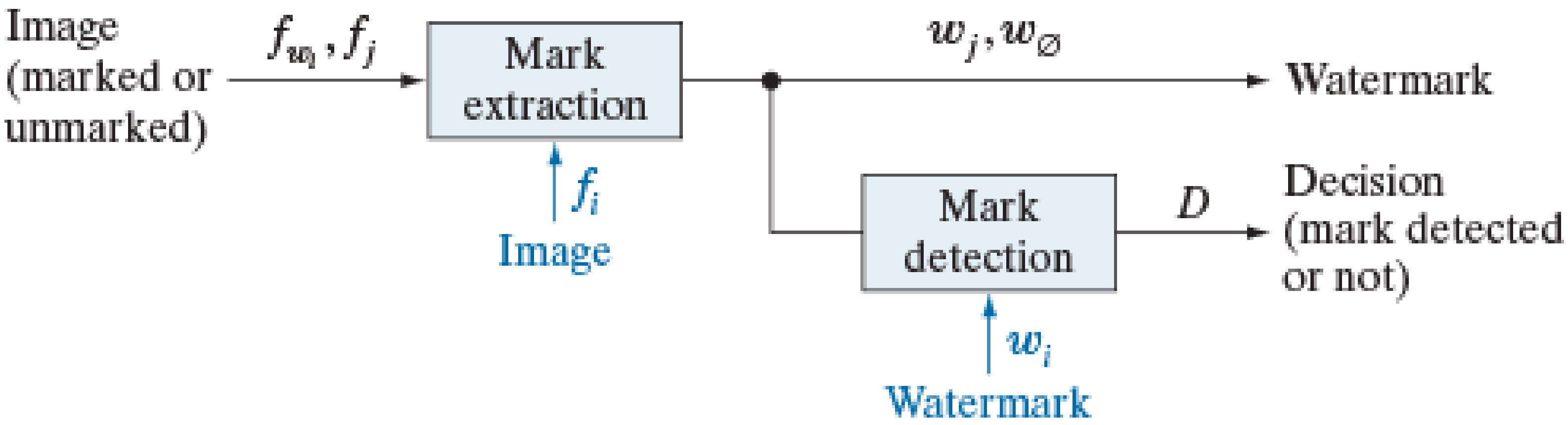
$$f_w = (1 - \alpha) f + \alpha w$$

Invisible image watermarking system

Encoder



Decoder



Invisible watermark

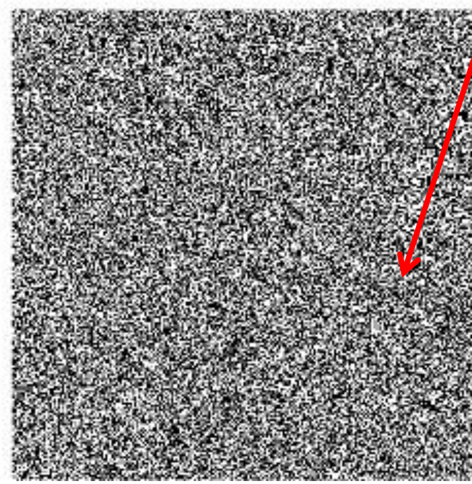
Example: watermarking using two least significant bits

Original image

Extracted watermark

Two least significant bits

JPEG compressed



Fragile invisible watermark

Invisible watermark

Example: DCT-based watermarking

Watermarked images

Extracted robust invisible watermark

Next Lecture

- Morphological image processing
- Reading
  - Chapter 9: Morphological image processing
    - Sections 9.1, 9.2, 9.3, and 9.5 (through subsection connected components)