Matrix-Based Transforms

Image Processing

CSE 166

Lecture 12

Announcements

- Assignment 4 will be released May 15
- Reading
  - Chapter 6: Wavelet and Other Image Transforms
    - Section 6.2

Matrix-based transforms

$$T(u) = \sum_{x=0}^{N-1} f(x) r(x,u)$$ Forward transform

$$f(x) = \sum_{u=0}^{N-1} T(u) s(x,u)$$ Inverse transform

where

x is a spatial variable

u is a transform variable

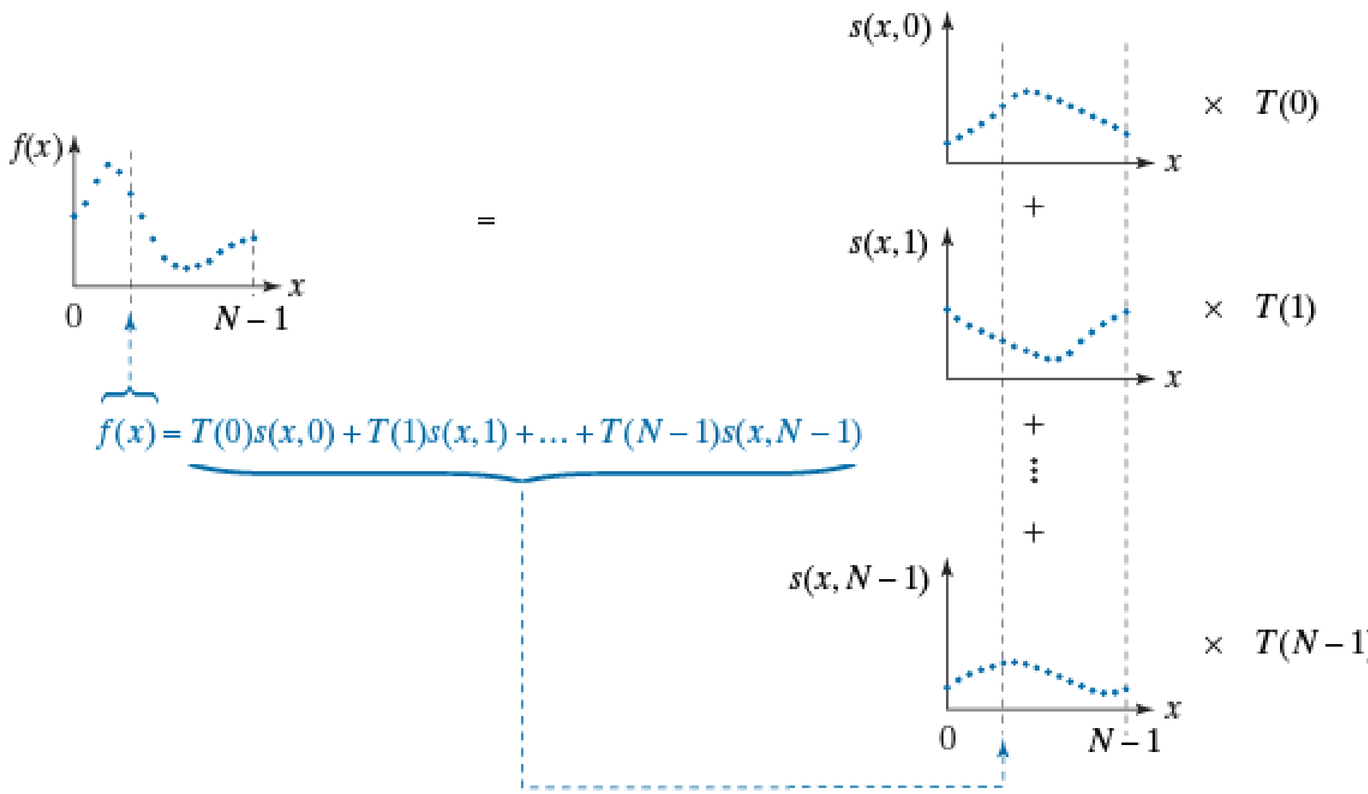
$T(u)$ is the transform of $f(x)$

$f(x)$ is the inverse transform of $T(u)$

$r(x,u)$ is a forward transformation kernel

$s(x,u)$ is an inverse transformation kernel

General inverse transform using basis vectors



Matrix-based transforms using orthonormal basis vectors

- In vector form

$$T(u) = \langle s(x,u), f(x) \rangle$$

$$T(u) = \langle \mathbf{s}_u, \mathbf{f} \rangle$$

$$\text{where } \mathbf{s}_u = \begin{bmatrix} s(0,u) \\ s(1,u) \\ \vdots \\ s(N-1,u) \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{bmatrix}$$

$$T(u) = \mathbf{s}_u^H \mathbf{f} \text{ for complex vectors}$$

$$T(u) = \mathbf{s}_u^T \mathbf{f} \text{ for real vectors}$$

Matrix-based transforms using orthonormal basis vectors

- In matrix form

$$\begin{bmatrix} T(0) \\ T(1) \\ \vdots \\ T(N-1) \end{bmatrix} = \begin{bmatrix} \mathrm{s}_0^H \\ \mathrm{s}_1^H \\ \vdots \\ \mathrm{s}_{N-1}^H \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{bmatrix} \text{ for complex vectors}$$

$$\begin{bmatrix} T(0) \\ T(1) \\ \vdots \\ T(N-1) \end{bmatrix} = \begin{bmatrix} \mathrm{s}_0^T \\ \mathrm{s}_1^T \\ \vdots \\ \mathrm{s}_{N-1}^T \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{bmatrix} \text{ for real vectors}$$

$$\mathbf{t} = \mathtt{A}\mathbf{f}$$

$$\mathbf{f} = \mathtt{A}^H\mathbf{t} \text{ for complex vectors}$$

$$\mathbf{f} = \mathtt{A}^T\mathbf{t} \text{ for real vectors}$$

Matrix-based transforms using biorthonormal basis vectors

- In matrix form

$$\begin{bmatrix} T(0) \\ T(1) \\ \vdots \\ T(N-1) \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{s}}_0^H \\ \tilde{\mathbf{s}}_1^H \\ \vdots \\ \tilde{\mathbf{s}}_{N-1}^H \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{bmatrix} \text{ for complex vectors}$$

$$\begin{bmatrix} T(0) \\ T(1) \\ \vdots \\ T(N-1) \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{s}}_0^T \\ \tilde{\mathbf{s}}_1^T \\ \vdots \\ \tilde{\mathbf{s}}_{N-1}^T \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{bmatrix} \text{ for real vectors}$$

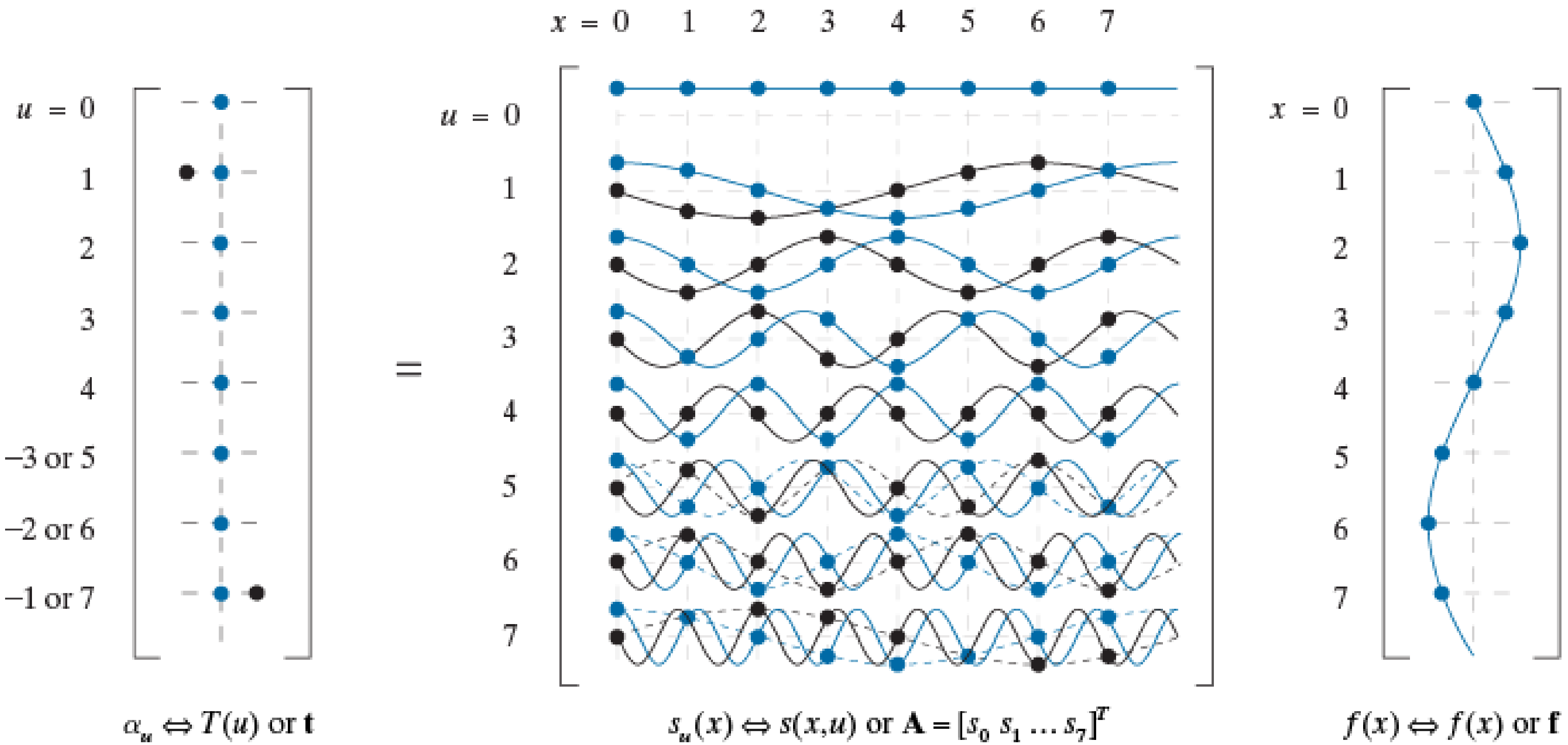
$$\mathbf{t} = \tilde{\mathtt{A}}\mathbf{f}$$

$$\mathbf{f} = \mathtt{A}^H\mathbf{t} \text{ for complex vectors}$$

$$\mathbf{f} = \mathtt{A}^T\mathbf{t} \text{ for real vectors}$$

Matrix-based transform

Example: 8-point DFT of $f(x) = \sin(2\pi x)$



real part + imaginary part

Matrix-based transforms in two dimensions

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) r(x,y,u,v)$$ Forward transform

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u,v) s(x,y,u,v)$$ Inverse transform

where

x, y are spatial variables

u, v are transform variables

$T(u,v)$ is the transform of $f(x,y)$

$f(x,y)$ is the inverse transform of $T(u,v)$

$r(x,y,u,v)$ is a forward transformation kernel

$s(x,y,u,v)$ is an inverse transformation kernel

Matrix-based transforms in two dimensions

- If r and s are separable and symmetric, and M = N, then
  - For orthonormal basis vectors

$$
\begin{aligned}
\mathtt{T} &= \mathtt{AFA}^T \\
\mathtt{F} &= \mathtt{A}^{*T}\mathtt{TA}^* \text{ for complex vectors} \\
\mathtt{F} &= \mathtt{A}^T\mathtt{TA} \text{ for real vectors}
\end{aligned}
$$

  - For biorthonormal basis vectors

$$
\begin{aligned}
\mathtt{T} &= \tilde{\mathtt{A}}\mathtt{F}\tilde{\mathtt{A}}^T \\
\mathtt{F} &= \mathtt{A}^{*T}\mathtt{TA}^* \text{ for complex vectors} \\
\mathtt{F} &= \mathtt{A}^T\mathtt{TA} \text{ for real vectors}
\end{aligned}
$$

Next Lecture

- Basis images
- Wavelet transforms
- Reading
  - Chapter 6: Wavelet and Other Image Transforms
    - Sections 6.5 and 6.10