

CSE140 Homework 2, Due 11:59PM, Mon 4/22/19

Homework 2 covers the CMOS design, combinational logic specification, and implementation. For the first problem, we construct a gate using CMOS designs. For the next three problems, we practice more on the specification, in particular, when the number of input bits is not small, and the implementation may take multiple levels of logic gates. For the last two problems, we practice design minimization using Karnaugh maps in sum of products and product of sums formats.

1 CMOS Designs

Design a CMOS gate with function $y(a, b, c, d) = [a(b + cd)]'$. Draw the schematic diagram using NMOS and PMOS transistors as the basic elements.

2 Voting Machine

A voting machine reads six binary bits $(a_5, a_4, a_3, a_2, a_1, a_0)$ and outputs a binary number (y_2, y_1, y_0) so that the output represents the number of input bits which are 1 instead of 0. For example, when input $(a_5, a_4, a_3, a_2, a_1, a_0) = (1, 1, 1, 1, 1, 0)$, output $(y_2, y_1, y_0) = (1, 0, 1)$. When input $(a_5, a_4, a_3, a_2, a_1, a_0) = (0, 1, 1, 0, 0, 1)$, output $(y_2, y_1, y_0) = (0, 1, 1)$.

- A. Write the truth table of the machine.
- B. Express the function in a minimal Boolean expression.
- C. Use a minimal number of full adders (no other gates) to implement the machine. Depict your design with a schematic diagram.

3 Majority Function

A majority function f reads five binary bits $(a_4, a_3, a_2, a_1, a_0)$ and produces a binary output $f = 1$ when the majority of the input bits are true, otherwise the output $f = 0$. For example, $f(1, 1, 1, 0, 0) = 1$, and $f(1, 1, 0, 0, 0) = 0$.

- A. Express the function in a minimal Boolean expression.
- B. Implement the function using a minimal number of full adders (no other gates). Depict your design with a schematic diagram.

4 Priority Encoder

A priority encoder reads a vector of eight binary bits $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0)$ and produces a binary number (d_2, d_1, d_0) that presents the lowest index of the input bit which is true. For example, when $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) = (0, 0, 1, 0, 1, 0, 0, 0)$, the output $(d_2, d_1, d_0) = (0, 1, 1)$; $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) = (0, 1, 0, 0, 0, 0, 0, 0)$, the output $(d_2, d_1, d_0) = (1, 1, 0)$; and when $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) = (0, 0, 0, 0, 0, 0, 0, 1)$, the output $(d_2, d_1, d_0) = (0, 0, 0)$. However, when none of the input bits asserts a true value, i.e. $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) = (0, 0, 0, 0, 0, 0, 0, 0)$, the output remains to be $(d_2, d_1, d_0) = (0, 0, 0)$.

- A. Derive a Boolean expression of the three output bits (no need to be minimal but try to be concise).
- B. Implement the encoder with AND, OR, NOT gates (no need to be minimal but try to be organized).

5 Minimal Sum of Products Expression

Implementation from truth table to sum of products expressions.

1. Use Karnaugh map to simplify function

$$f(a, b, c) = \sum m(0, 4, 5, 7) + \sum d(1, 2).$$

List **all** possible minimal two-level sum of products expressions. Show the switching functions. No need for the schematic diagram.

2. Use Karnaugh map to simplify function

$$f(a, b, c, d) = \sum m(2, 3, 5, 6, 8, 11, 15) + \sum d(0, 9, 12).$$

List all possible minimal two-level sum of products expressions. Show the switching functions. No need for the schematic diagram.

3. Use Karnaugh map to simplify function

$$f(a, b, c, d, e) = \sum m(6, 7, 8, 10, 12, 13, 15, 16, 21, 22, 24, 26, 31) + \sum d(0, 2, 9, 18, 28, 29)$$

Write **one** minimal two-level sum of products expression. Show the switching functions. No need for the schematic diagram.

6 Minimal Product of Sums Expression

Implementation from truth table to product of sums expressions.

1. Use Karnaugh map to simplify function

$$f(a, b, c) = \sum m(0, 7) + \sum d(1, 2, 5).$$

List all possible minimal two-level product of sums expressions. Show the switching functions. No need for the schematic diagram.

2. Use Karnaugh map to simplify function

$$f(a, b, c, d) = \sum m(1, 2, 4, 9, 12, 15) + \sum d(3, 11, 14).$$

List all possible minimal two-level product of sums expressions. Show the switching functions. No need for the schematic diagram.

3. Use Karnaugh map to simplify function

$$f(a, b, c, d, e) = \sum m(1, 3, 4, 11, 12, 13, 14, 19, 20, 22, 27, 28, 30) + \sum d(7, 9, 23, 24, 26).$$

Write **one** minimal two-level product of sums expression. Show the switching functions. No need for the schematic diagram.