

Name of student to your left _____

Name of student to your right _____

Please read the following instructions carefully: If you are unclear about any of the questions on the exam, make the most plausible assumption to answer the question. Instructors and proctors will not answer questions on the exam material. Please write your name on the top of each page. Write the names of the students to your left and right in the space provided. **All the Best!**

1 CMOS Circuits

Design a CMOS gate with function $y(a, b, c, d) = [(a + b)cd]'$. Draw the schematic diagram using NMOS and PMOS transistors as the basic elements.

Rubrics

- 10 points for correct design
- 5 points deducted for wrong PMOS/NMOS logic
- 1 or 2 points deducted for minor mistakes
- no points for illogical design

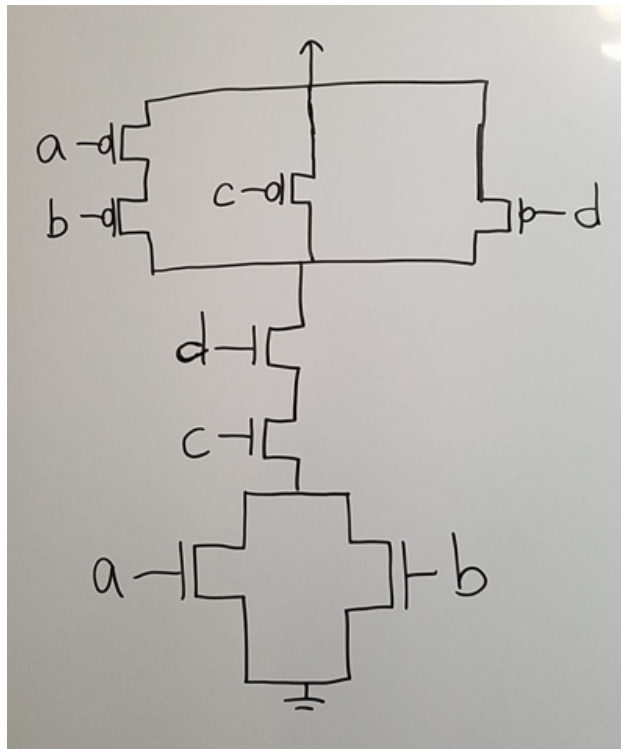


Figure 1: CMOS

2 Boolean Algebra

1. State the definition of Boolean algebra including the five laws.

Rubrics

- 10 points for correctly stating the ten equations (two for each law)
- 1 point deducted for each missing equation; the deduction is up to 5 points
- If only the names of the five laws are given, 5 points will be given.
- 0.5 point deducted if XOR operations are mentioned
- 1 point deducted if $\{ +, \cdot \}$ operations not mentioned or fail to deliver the correct set definition of Boolean algebra

A branch of algebra in which the values of the variables belong to a set B (e.g. 0, 1), has two operations $\{+, \cdot\}$ that satisfy the following five sets of laws.

- Associative laws: $(a + b) + c = a + (b + c), (a \cdot b) \cdot c = a \cdot (b \cdot c)$
- Commutative laws: $a + b = b + a, a \cdot b = b \cdot a$
- Distributive laws: $a + (b \cdot c) = (a + b) \cdot (a + c), a \cdot (b + c) = a \cdot b + a \cdot c$
- Identity laws: $a + 0 = a, a \cdot 1 = a$
- Complement laws: $a + a' = 1, a \cdot a' = 0$

2. Minimize the following Boolean expression using Boolean algebra. State the laws and/or theorems (e.g. consensus theorem) used in the derivation.

Rubrics

- 1 point deducted for each incorrect or missing step if less than 3 incorrect or missing steps
- 1 point rewarded for each correct (and useful) step for 3 or more incorrect or missing steps

$$f(a, b, c, d) = a'c'd + abd + a'bcd.$$

$$\begin{aligned} & a'c'd + abd + a'bcd \\ &= a'c'd + abd + a'bcd + a'bd \\ &= a'c'd + bd(a + a'c + a') \\ &= a'c'd + bd(1 + a'c) \\ &= a'c'd + bd \end{aligned}$$

Consensus Theorem terms 1&3
Distributive, Associative Laws
Complement Law
Annulment Law

3 Circuit Specification: Voting Machine

A voting machine reads four binary bits (a_3, a_2, a_1, a_0) and outputs a binary number (y_2, y_1, y_0) so that the output represents the number of input bits which are 1 instead of 0. For example, when input $(a_3, a_2, a_1, a_0) = (1, 1, 1, 1)$, output $(y_2, y_1, y_0) = (1, 0, 0)$. When input $(a_3, a_2, a_1, a_0) = (0, 1, 1, 0)$, output $(y_2, y_1, y_0) = (0, 1, 0)$.

1. Write the truth tables and Karnaugh maps of the output functions.

Rubrics

- 10 points for 3.1
- 10 points for 3.2
- 4 points for the truth table; 1 point deducted for each wrong entry
- 2 points for each K-map; 1 point deducted for each wrong cell
- 1 point deducted for mislabeled variables or unexplained blanks in K-maps
- For 3.2, 1 point deducted for each wrong, missing, or extra term
- 1 point deducted for wrongly formatted essential prime implicants (but we can tell you know the correct answer from your wrongly formatted answer)

a_3	a_2	a_1	a_0	y_2	y_1	y_0
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	0	0	1
1	0	0	1	0	1	0
1	0	1	0	0	1	0
1	0	1	1	0	1	1
1	1	0	0	0	1	0
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	0	0

2. List all **essential prime implicants** for each of the above maps.

- $y_2 : a_3 a_2 a_1 a_0$
- $y_1 : No$
- $y_0 : a'_3 a'_2 a'_1 a_0, a'_3 a'_2 a_1 a'_0, a'_3 a_2 a'_1 a'_0, a'_3 a_2 a_1 a_0, a_3 a'_2 a'_1 a'_0, a_3 a'_2 a_1 a_0, a_3 a_2 a'_1 a'_0, a_3 a_2 a_1 a'_0$

y_2

$a_3 a_2$					
$a_1 a_0$		00	01	11	10
00	0	0	0	0	0
01	0	0	0	0	0
11	0	0	1	0	0
10	0	0	0	0	0

Figure 2: Voting Machine y_2

y_1

$a_3 a_2$					
$a_1 a_0$		00	01	11	10
00	0	0	1	0	0
01	0	1	1	1	1
11	1	1	0	1	1
10	0	1	1	1	1

Figure 3: Voting Machine y_1

y_0

		a_3a_2			
		00	01	11	10
a_1a_0	00	0	1	0	1
	01	1	0	1	0
	11	0	1	0	1
	10	1	0	1	0

Figure 4: Voting Machine y_0

4 Karnaugh Map

Rubrics

- 5 points for each K-map; 1 point deducted for each wrong cell
- 6 points for minimal expressions in 4.1
- 4 points for minimal expressions in 4.2
- 1 point deducted for each wrong, missing, or extra expression

4.1 Sum of Products Expressions

Use Karnaugh map to simplify function

$$f(a,b,c) = \sum m(1,3,5,6) + \sum d(2,4).$$

Write the K-map and list all possible minimal sum of products expressions. No need for the logic diagram.

- $f(a,b,c) = a'c + b'c + bc'$
- $f(a,b,c) = a'c + b'c + ac'$
- $f(a,b,c) = a'c + ab' + bc'$
- $f(a,b,c) = a'c + ab' + ac'$
- $f(a,b,c) = b'c + a'b + bc'$
- $f(a,b,c) = b'c + a'b + ac'$

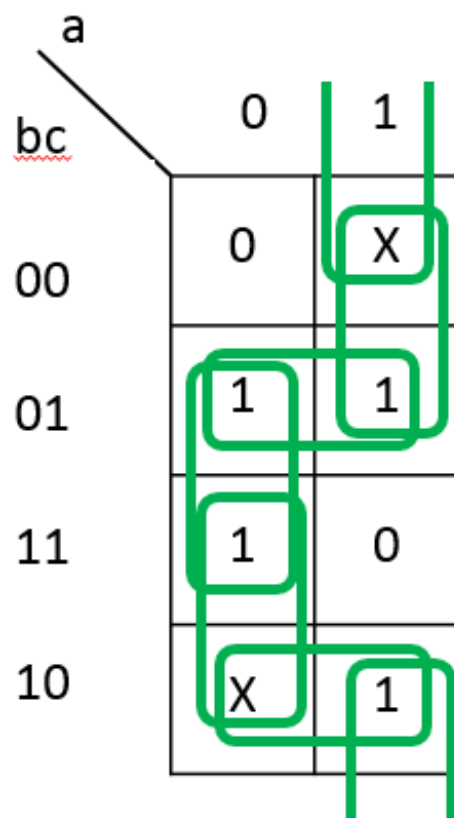


Figure 5: K-map of 4.1

4.2 Product of Sums Expressions

Use Karnaugh map to simplify function

$$f(a,b,c) = \sum m(1,6) + \sum d(5).$$

Write the K-map and list all possible minimal product of sums expressions. No need for the logic diagram.

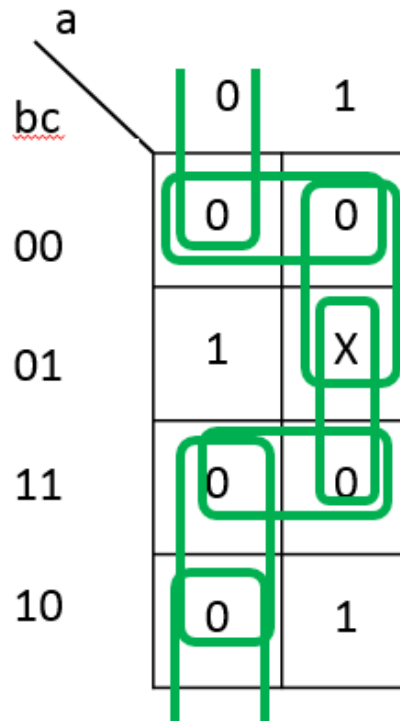


Figure 6: K-map of 4.2

- $f(a,b,c) = (b+c)(b'+c')(a+b')$
- $f(a,b,c) = (b+c)(b'+c')(a+c)$
- $f(a,b,c) = (b+c)(a+b')(a'+c')$
- $f(a,b,c) = (a+c)(b'+c')(a'+b)$

5 Other Types of Gates

5.1 Universal Set of Gates

Circle the following items which are universal sets of gates

A.{AND, OR}; B.{NOR}; C. $\{f(x,y) = xy + x'y\}$; D. $\{f(x,y) = x' + y\}$; E. $\{f(x,y,z) = x'y' + x'z' + y'z'\}$.

Rubrics

- 3 points for each choice

BDE

5.2 Other Types of Gates

Consider the function $f(x,y)$ where $\oplus$ is an Exclusive OR operator and the priority of the operators is AND (first), Exclusive OR and OR (last):

$$f(x,y) = x'y \oplus (x + y') \oplus (x' + y') \oplus xy' \oplus y \oplus xy.$$

Simplify the function into a minimal sum of products expression using switching function techniques e.g. Shannon's expansion. Show your derivation.

Rubrics

- 15 points for correct answer
- 3 points deducted for using Shannon's expansion but arriving at a wrong answer
- 7 points for making derivation but arriving at a wrong answer
- no points for any wrong answer without the derivation

$$\begin{aligned} f(x,y) &= xf(1,y) + x'f(0,y) \quad \text{Shannon's Expansion} \\ &= x[0 \oplus 1 \oplus y' \oplus y' \oplus y \oplus y] + x'[y \oplus y' \oplus 1 \oplus 0 \oplus y \oplus 0] \\ &= x[1] + x'[y] \\ &= x + y \quad (\text{Distributive Law}) \end{aligned}$$

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