

CSE140 HW1, Due Mon. 4/15/2019 by 11:59PM

1 Introduction

The purpose of this assignment is three-fold. First, it aims to help you practice the application of Boolean algebra theorems to transform and reduce Boolean expressions. The second goal is to help you learn how to go from the world of Boolean expressions to the world of digital circuits. The final goal is to help you translate a problem described in words to a Boolean algebraic expression. We hope you can think of why each of these exercises is useful when designing digital circuits.

2 Application of Boolean Algebra Theorems

2.1 Prove the theorems using Boolean algebra:

A. Prove that $A + 1 = 1$.

$$\begin{aligned} A + 1 &= (A + 1) \cdot 1 && \text{Identity Law} \\ &= (A + 1) \cdot (A + A') && \text{Complement Law} \\ &= A + 1 \cdot A' && \text{Distributive Law} \\ &= A + A' && \text{Identity Law} \\ &= 1 && \text{Complement Law} \end{aligned}$$

B. Prove that $A \cdot 0 = 0$.

$$\begin{aligned} A \cdot 0 &= A \cdot 0 + 0 && \text{Identity Law} \\ &= A \cdot 0 + (A \cdot A') && \text{Complement Law} \\ &= A \cdot (0 + A') && \text{Distributive Law} \\ &= A \cdot A' && \text{Identity Law} \\ &= 0 && \text{Complement Law} \end{aligned}$$

2.2 Prove the following equations using Boolean algebra:

A. $a'b + a'cd' + b'cd' = a'b + b'cd'$.

$$\begin{aligned} a'b + a'cd' + b'cd' &= a'b + a'cd'(b + b') + b'cd'(a + a') && \text{Complement, Identity Laws} \\ &= a'b(1 + cd') + a'b'cd' + ab'cd' + a'b'cd' && \text{Distributive Law} \\ &= a'b + (a + a')b'cd' + a'b'cd' && \text{Distributive Law, } A + 1 = 1 \\ &= a'b + (1 + a')b'cd' && \text{Complement, Distributive Laws} \\ &= a'b + b'cd' && \text{Identity Law, } 1 + A = 1 \end{aligned}$$

B. $(a' + b)(a' + c + d' + e)(b' + c + d') = (a' + b)(b' + c + d')$.

$$\begin{aligned}
& (a' + b)(a' + c + d' + e)(b' + c + d') \\
&= (a' + b(c + d' + e))(b' + c + d') && \text{Distributive Law} \\
&= a'b' + a'(c + d') + b(c + d' + e)b' + b(c + d' + e)(c + d') && \text{Distributive Law} \\
&= a'b' + a'(c + d') + b(c + d' + e)(c + d') && \text{Commutative, Complement Laws} \\
&= a'b' + a'(c + d') + b(c + d') && X \cdot (X + Y) = X \\
&= a'(b' + c + d') + b(b' + c + d') && \text{Distributive, Complement Laws} \\
&= (a' + b)(b' + c + d') && \text{Distributive Law}
\end{aligned}$$

We want to show that $X \cdot (X + Y) = X$:

$$\begin{aligned}
X \cdot (X + Y) &= X \cdot X + X \cdot Y && \text{Distributive Law} \\
&= X + X \cdot Y && \text{Distributive Law} \\
&= X \cdot (1 + Y) && \text{Identity, Distributive Laws} \\
&= X && \text{Identity Law, what we have proved in Problem 1}
\end{aligned}$$

Are the above equations related to the consensus theorem? Yes.

2.3 Prove the above two equations using Shannon's expansion.

2.2 A

$$\begin{aligned}
f(a, b, c, d) &= b \cdot f(a, 1, c, d) + b' \cdot f(a, 0, c, d) && \text{by Shannon's Expansion} \\
&= b \cdot (a' + a'cd') + b' \cdot (a'cd' + cd') \\
&= b \cdot (a' \cdot (1 + cd')) + b' \cdot ((a + 1) \cdot cd') && \text{Identity, Distributive Laws} \\
&= b \cdot a' + b' \cdot cd' && \text{Commutative Law} \\
&= a'b + b'cd'
\end{aligned}$$

2.2 B

$$\begin{aligned}
& f(a, b, c, d) \\
&= (b' + f(a, 1, c, d)) \cdot (b + f(a, 0, c, d)) && \text{Shannon's Expansion} \\
&= (b' + 1 \cdot (a' + c + d' + e)(c + d')) \cdot (b + (a' + 0)(a' + c + d' + e)(1 + c + d')) \\
&= (b' + (a' + c + d' + e)(c + d')) \cdot (b + a'(a' + c + d' + e)) && \text{Identity Law} \\
&= (b' + c + d') \cdot (b + a') && X \cdot (X + Y) = X \\
&= (a' + b) \cdot (b' + c + d') && \text{Commutative Law}
\end{aligned}$$

3 From Problem to Boolean Expression

A circuit has four inputs and one output. The inputs (a_3, a_2, a_1, a_0) represent a number from 0 to 15. Output Y is true if the number is prime (0 and 1 are not prime, but 2, 3, 5 and so on are prime).

A. Describe the truth table.

a_3	a_2	a_1	a_0	output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

B. Write the function in sum-of-products canonical form.

$$\begin{aligned} F(a_3, a_2, a_1, a_0) &= \Sigma m(2, 3, 5, 7, 11, 13) \\ &= a'_3 a'_2 a'_1 a'_0 + a'_3 a'_2 a_1 a_0 + a'_3 a_2 a'_1 a_0 \\ &\quad + a'_3 a_2 a_1 a_0 + a_3 a'_2 a_1 a_0 + a_3 a_2 a'_1 a_0 \end{aligned}$$

C. Write the function in product-of-sums canonical form.

$$\begin{aligned} F(a_3, a_2, a_1, a_0) &= \Pi M(0, 1, 4, 6, 8, 9, 10, 12, 14, 15) \\ &= (a_3 + a_2 + a_1 + a_0) \cdot (a_3 + a_2 + a_1 + a'_0) \cdot (a_3 + a'_2 + a_1 + a_0) \\ &\quad \cdot (a_3 + a'_2 + a'_1 + a_0) \cdot (a'_3 + a_2 + a_1 + a_0) \cdot (a'_3 + a_2 + a_1 + a'_0) \\ &\quad \cdot (a'_3 + a_2 + a'_1 + a_0) \cdot (a'_3 + a'_2 + a_1 + a_0) \cdot (a'_3 + a'_2 + a'_1 + a_0) \\ &\quad \cdot (a'_3 + a'_2 + a'_1 + a'_0) \end{aligned}$$

4 Boolean Algebra and Implementation

A. Simplify each of the following two Boolean equations.

B. Sketch a reasonably simple combinational circuit implementing the simplified equation.

C. List the numbers of literals and operators versus the numbers of gates, nets, and pins in the schematic diagrams

i. $ac'd' + ab'cd' + a'bcd' + bd + a'bc'd + abc$

$$\begin{aligned}
 & ac'd' + ab'cd' + a'bcd' + bd + a'bc'd + abc \\
 &= ad'(c' + b'c) + a'bcd' + bd(1 + a'c') + abc && \text{Distributive Law} \\
 &= ab'd' + ac'd' + a'bcd' + bd + abc && \text{Distributive Law, } A + 1 = 1 \\
 &= ab'd' + ac'd' + a'bcd' + bd + abc + a'bc && \text{Consensus Theorem, terms 3,4} \\
 &= ab'd' + ac'd' + a'bcd' + bd + bc && \text{Distributive, Complement Laws} \\
 &= ab'd' + ac'd' + a'bcd' + bd + bc + acd' && \text{Consensus Theorem, terms 1,5} \\
 &= ab'd' + ad' + a'bcd' + bd + bc && \text{Distributive, Complement Laws} \\
 &= ad' + a'bcd' + bd + bc && \text{Distributive, Law } A + 1 = 1 \\
 &= ad' + bd + bc && \text{Distributive Law, } A + 1 = 1
 \end{aligned}$$

The combinational circuit is illustrated in Figure 1.

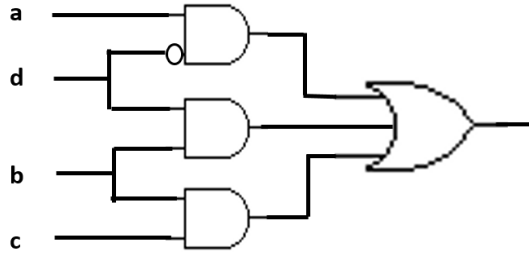


Figure 1: (i) Circuit Diagram

#Variable = 4

#Literal = 6

#Operator = 4

#Net = #Variable + #Operator = 4 + 4 = 8

#Pin = #Literal + 2 * #Operator - 1 = 6 + 2 * 4 - 1 = 13

ii. $(b' + ac)(a'c' + b)(ab + c)'$

$$\begin{aligned}
 & (b' + ac)(a'c' + b)(ab + c)' \\
 &= (b' + ac)(a'c' + b)(a' + b')c' && \text{DeMorgan's Theorem} \\
 &= b'c'(a'c' + b)(a' + b') && \text{Complement, Commutative Law} \\
 &= (a'b'c')(a' + b') && \text{Complement, Distributive Laws} \\
 &= a'b'c' && X \cdot (X + Y) = X, Y = 0, \text{ Distributive Law}
 \end{aligned}$$

The combinational circuit is illustrated in Figure 2.

#Variable = 3

#Literal = 3

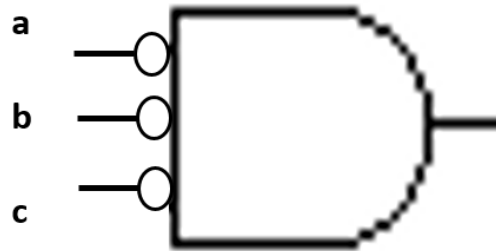


Figure 2: (ii) Circuit Diagram

$\#Operator = 1$

$\#Net = \#Variable + \#Operator = 3 + 1 = 4$

$\#Pin = \#Literal + 2 * \#Operator - 1 = 3 + 2 * 1 - 1 = 4$

5 Assignment Turn-in

Once you're finished with the assignment, turn it in as a pdf on *Gradescope*.