

CSE 140 4/19/19

Discussion Session

Po-Ya Hsu

Outline

- K-map
- CMOS
- XOR
- Full Adder

K-map: all possible minimal two-level sum of products expressions?

		ab			
		00	01	11	10
cd	00	1	0	1	1
	01	1	1	0	0
	11	1	1	1	X
	10	0	X	1	X

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level sum of products expressions?

ab

cd

	00	01	11	10
00	1	0	1	1
01	1	1	0	0
11	1	1	1	X
10	0	X	1	X

Essential Prime Implicants:

$$\sum m(8,10,12,14)$$

$$\sum m(1,3,5,7)$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level sum of products expressions?

ab

cd

	00	01	11	10
00	1 [●]	0	1 [●]	1
01	1	1 [●]	0	0
11	1	1	1 [●]	X
10	0	X	1	X

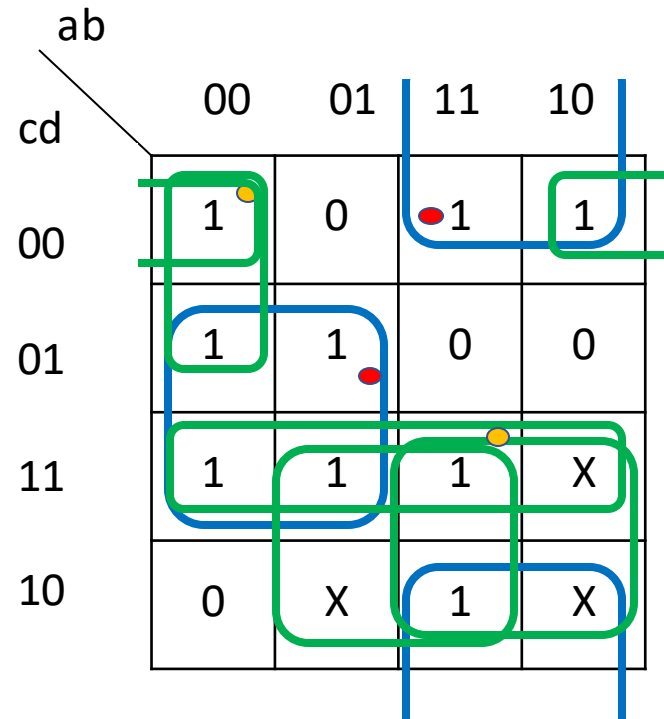
Essential Prime Implicants:

$$\sum m(8,10,12,14)$$

$$\sum m(1,3,5,7)$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level sum of products expressions?



Essential Prime Implicants:

$$\sum m(8,10,12,14)$$

$$\sum m(1,3,5,7)$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

Prime Implicants that cover the yellow circled cells:

$$\sum m(3,7,11,15) \text{ or } \sum m(6,7,14,15) \text{ or } \sum m(10,11,14,15)$$

$$\sum m(0,1) \text{ or } \sum m(0,8)$$

K-map: all possible minimal two-level product of sums expressions?

		ab			
		00	01	11	10
cd	00	1	0	1	1
	01	1	1	0	0
	11	1	1	1	X
	10	0	X	1	X

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level product of sums expressions?

		ab			
		00	01	11	10
cd	00	1	0	1	1
	01	1	1	0	0
	11	1	1	1	X
	10	0	X	1	X

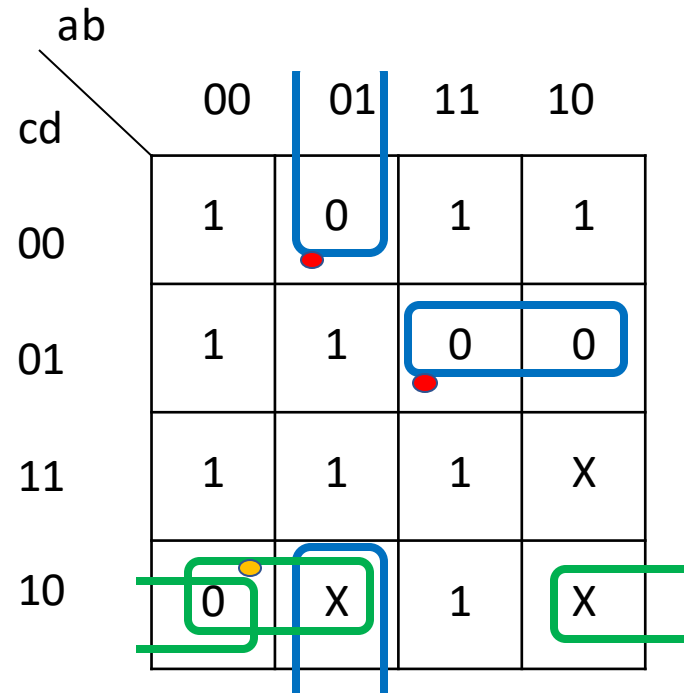
Essential Prime Implicates:

$$\prod_{M(9,13)}$$

$$\prod_{M(4,6)}$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

K-map: all possible minimal two-level product of sums expressions?



Essential Prime Implicates:

$$\prod M(9,13)$$

$$\prod M(4,6)$$

Prime Implicates that cover the yellow circled cells:

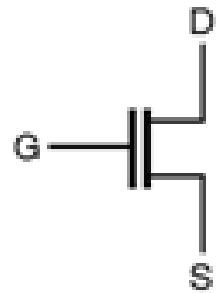
$$\prod M(2,6) \text{ or } \prod M(6,10)$$

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

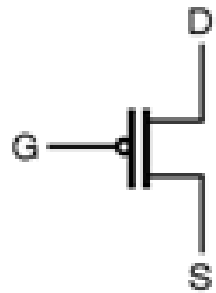
Problem: $f(a, b, c) = \sum m(0, 2, 5) + \sum d(3, 6)$

- all possible minimal two-level product of sums expressions?

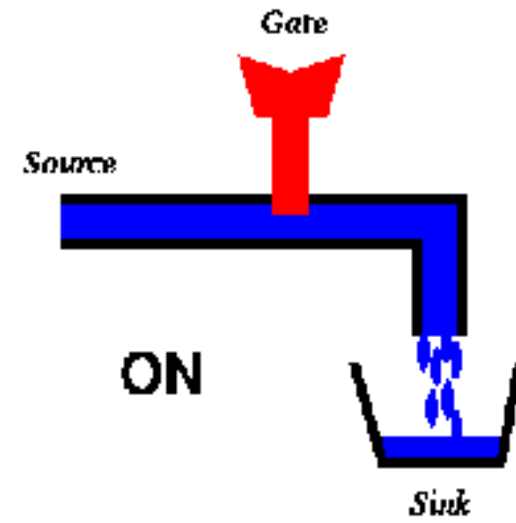
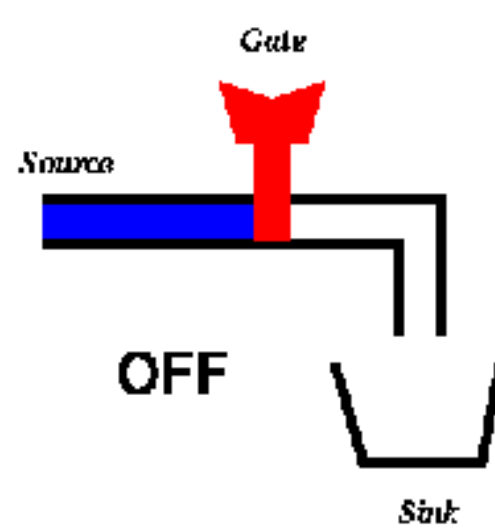
CMOS



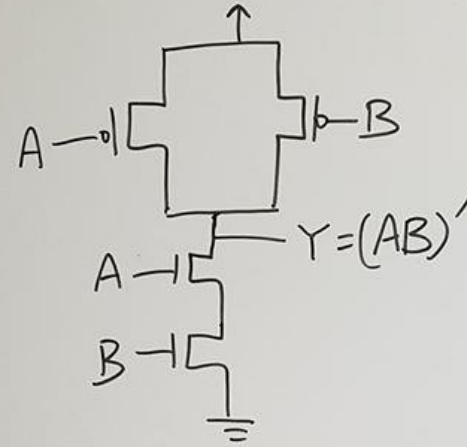
(e)
NMOS used in
Digital circuits



(f)
PMOS used in
Digital circuits

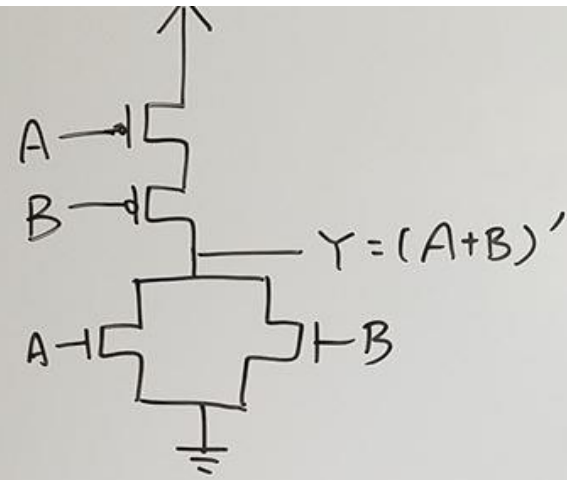


CMOS: 2-input NAND



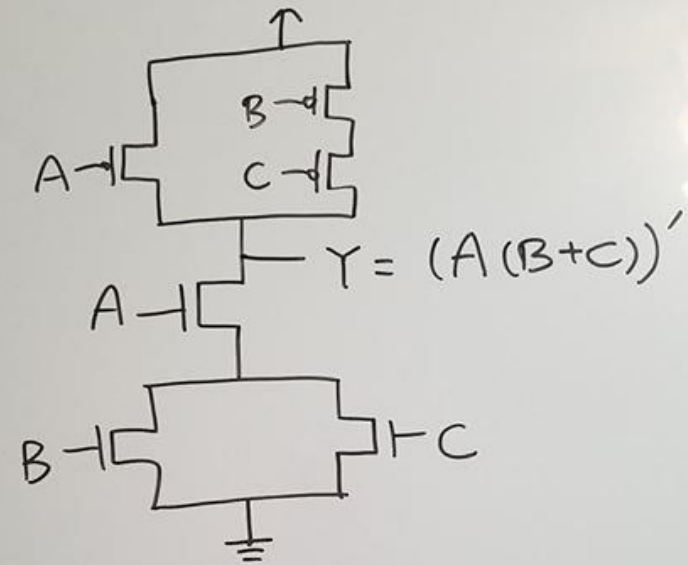
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

CMOS: 2-input NOR



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

$$\text{CMOS: } Y = [A(B+C)]'$$



A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

CMOS Design: $y(a,b,c,d)=[a(bc+d)]'$

XOR

a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

XOR - Properties

1. Commutative: $x \oplus y = y \oplus x$
2. Associative: $(x \oplus y) \oplus z = x \oplus (y \oplus z)$
3. $1 \oplus x = x'$; $0 \oplus x = x$
4. $x \oplus x = 0$; $x \oplus x' = 1$

XOR - Problem

- Given a binary string with the numbers of bit 1 and bit 0 being n and m , what is the output of XOR every bit in this binary string?

Full Adder

a	b	c _i	c _o	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Full Adder

a	b	c_i	c_o	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Trick:

Add the three bits a, b, and, c_i and then express the result with base=2

e.g. $a=1, b=1, c_i=1 \Rightarrow$

$a+b+c_i = 3$

3 (base 2) = 11

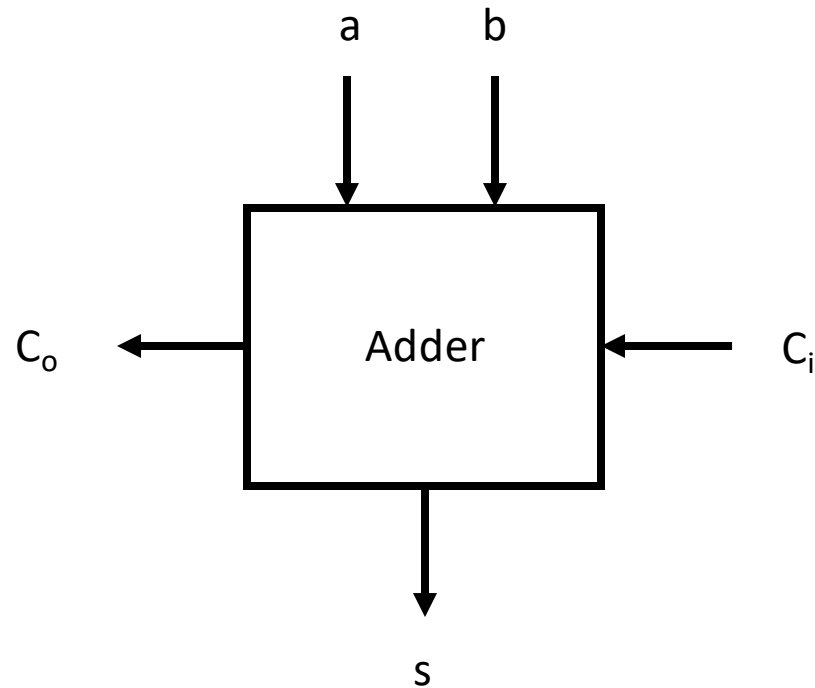
Full Adder – Boolean Expression

a	b	c _i	c _o	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$S = a \oplus b \oplus c_i$$

$$c_o = c_i(a \oplus b) + ab$$

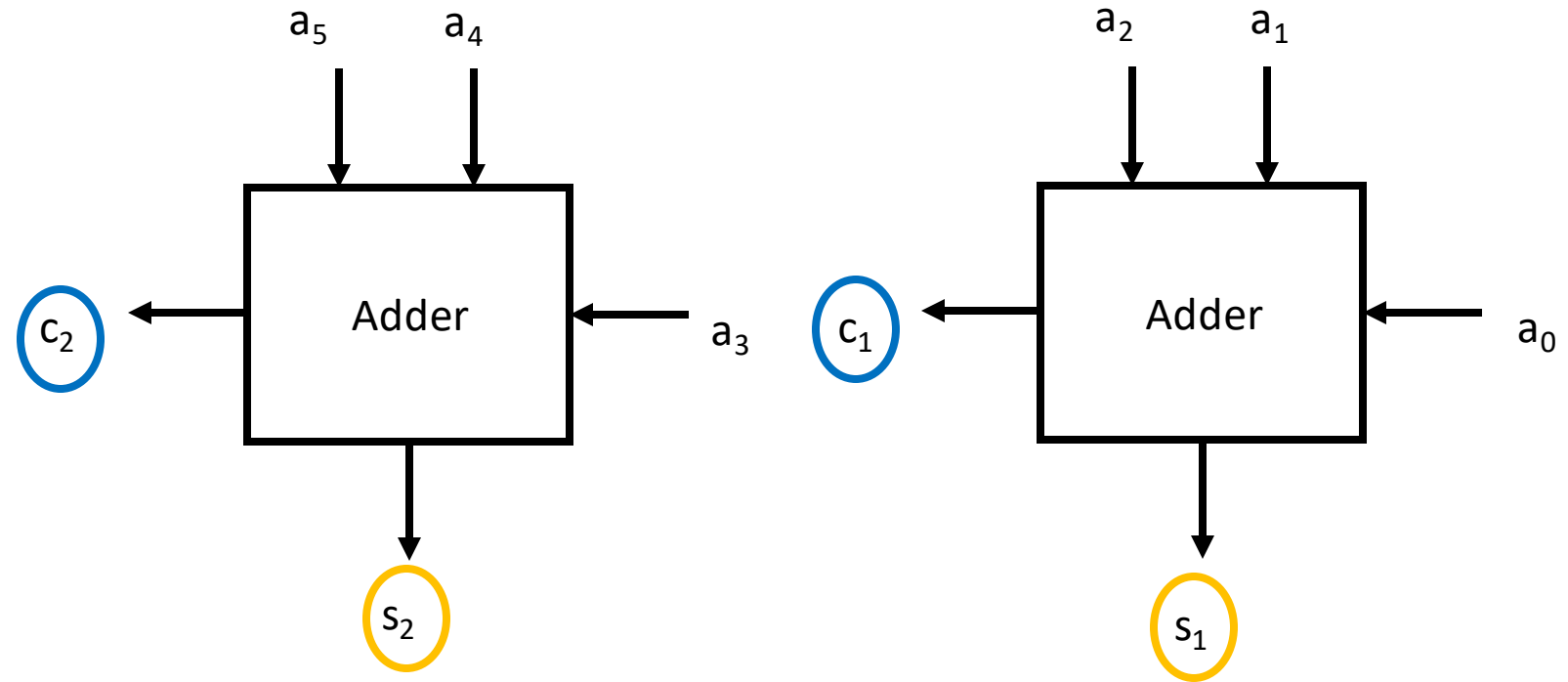
Hints on Homework 2



Weight(ing) Representation:

When carry-out bit is equal to 1, it means at least two of the input bits is 1.

Hints on Homework 2



1. Are there patterns of parity?
2. What are the “**weight(ing)s**” of the input and output bits of a full adder?
3. If one full adder is not sufficient, how about two, or more, or stacking FA together?
4. You are allowed to assign 0/1 to FA according to your design.