CSE 105, Spring 2019 - Homework 2

Due: Monday 4/15 midnight

Instructions

Upload a single file to Gradescope for each group. All group members’ names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Sipser Sections 1.1 and 1.2

Key Concepts Deterministic finite automata (DFA), state diagram, computation trace, accept / reject, language of an automaton, regular language, union of languages, concatenation of languages, star of a language, closure of the class of regular languages under certain operations, nondeterministic finite automata (NFA), nondeterministic computation, ε arrows, equivalence of NFA and DFA.
Problem 1 (10 points)

Draw the state diagram of the DFA that recognizes the language over $\Sigma = \{0, 1\}$:

$A = \{w \in \{0, 1\}^* : w \text{ does not contain the string 1010 as a substring}\}$

For full credit your DFA should have no more than five states.

![State Diagram](image)
Problem 2 (10 points)

Draw the state diagram of the DFA that recognizes the following language over $\Sigma = \{0, 1\}$:

$$L = \{0^m1^n : m \text{ is at least 2, and } n \text{ is even}\}$$

Problem 3 (10 points)

Recall, for a language $L \subseteq \Sigma^*$ its complement is the set of strings over $\Sigma^*$ not in $L$, denoted as $\overline{L} = \{w \in L^* : w \not\in L\}$. Also, let $-$ denote the set difference, ie: $L_1 - L_2 = \{w \in L_1 : w \not\in L_2\}$.

Let $A$ be the language above (from Problem 1) and let:

$$B = \{w \in \{0, 1\}^* : w \text{ has an even number of zeros}\}.$$

Draw the state diagram of the DFA of the following language. For full credit, your DFA should have no more than 10 states. Hint: use the construction from the book proving the union of two regular languages is regular.

(a) $\overline{A} \cup B$
(b) $A - B$
Problem 4 (10 points)

We first review some definitions.
• The concatenation of two languages \( L_1, L_2 \) over \( \Sigma \) is \( L_1 \circ L_2 = \{ x_1 x_2 | x_1 \in L_1, x_2 \in L_2 \} \).
• Lastly, the language \( L^* = \{ x_1 x_2 x_3 ... x_k | x_i \in L, k \geq 0 \} \)

Let \( A \) and \( B \) be the languages above. Draw the NFA state diagrams of the following languages:
(a) \( A \circ B \)

(b) \( (\overline{A})^* \circ B \)
Problem 5 (10 points)

Let \( w_2 \in \Sigma^* \) be a fixed but arbitrary string, and let \( S(k) \) denote the following statement:

For all \( w_1 \in \Sigma^* \) such that \( |w_1| = k \), \( \delta^*(q, w_1 \circ w_2) = \delta^*(\delta^*(q, w_1), w_2) \).

We will complete our proof by using induction to show that \( S(k) \) is true for all \( k \geq 0 \).

For the base case \((k = 0)\), note that the empty string is the only string of length 0. We know that \( \varepsilon \circ w_2 = w_2 \), and therefore \( \delta^*(q, w_1 \circ w_2) = \delta^*(q, \varepsilon \circ w_2) = \delta^*(q, w_2) \). Furthermore, using the recursive definition of \( \delta^* \), we know that \( \delta^*(\delta^*(q, w_1), w_2) = \delta^*(\delta^*(q, \varepsilon), w_2) = \delta^*(q, w_2) \), which proves that \( S(0) \) is true.

Next, we assume that \( S(k) \) is true for some \( k \geq 0 \) (this is our inductive hypothesis). We wish to show that \( S(k + 1) \) must be true. Let \( w_1 \) be any string of length \( k + 1 \). If the first symbol in \( w_1 \) is denoted \( f \), and the remaining \( k \) symbols in \( w_1 \) are denoted \( r \) (that is, \( w_1 = f \circ r \), with \( |f| = 1 \) and \( |r| = k \)), then we have

\[
\delta^*(q, w_1 \circ w_2) = \delta^*(q, (f \circ r) \circ w_2) = \delta^*(q, f \circ (r \circ w_2))
\]

(by associativity of concatenation)

\[
= \delta^*(\delta(q, f), r \circ w_2)
\]

(by the definition of \( \delta^* \))

\[
= \delta^*(\delta^*(q, f), w_2)
\]

(In the above line, \( |r| = k \), and so this is merely an application of the inductive hypothesis; i.e., the assumption that \( S(k) \) is true)

\[
= \delta^*(\delta^*(q, f \circ r), w_2)
\]

(by the definition of \( \delta^* \))

\[
= \delta^*(\delta^*(q, w_1), w_2)
\]

Then, by induction, we know that \( S(k) \) is true for all \( k \geq 0 \). In particular, the statement we are trying to prove is true regardless of the value of \( w_1 \). Lastly, because \( w_2 \) was assumed to be arbitrary, we know that the statement is also true regardless of the value of \( w_2 \).