CSE 105, Spring 2019 - Homework 1

Due: Monday 4/8 midnight

Instructions

Upload a single file to Gradescope for each group. All group members' names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Sipser Chapter 0 and Section 1.1

Key Concepts Sets, integers, sequences, functions, relations, predicates, graphs, trees, strings, languages, lexicographic ordering, boolean logic, proof by construction, proof by contradiction, proof by induction, finite automata (DFA), computation trace, accept / reject, language of an automaton, regular language, union of languages, concatenation of languages, star of a language.
Problem 1 (10 points)

1. In this problem, we’ll use the following definitions (from page 44) of operations on languages (sets of strings) $A$, $B$:

- **Union** $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- **Concatenation** $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$
- **Star** $A^* = \{x_1x_2\ldots x_k \mid k \in \mathbb{Z} \text{ and } k \geq 0 \text{ and each } x_i \in A\}$

For each of the following sets of strings over the alphabet $\{a, b\}$, answer the following questions:

1. Is $\epsilon$ (the empty string) in the set?
2. What is an example of a string over $\{a, b\}$ of length at least 2 that is in the set (or why isn’t there such an example)?
3. What is an example of a string over this alphabet of length at least 2 that is not in the set (or why isn’t there such an example)?

(a) $\{ w \mid w \in \{a,b\}^*\}$
(b) $\{ w \mid w \in (ab)^*\}$
(c) $\{ a, aa \} \circ \{ b, bb \}$
(d) $\{ a \} \cup \{ aa \} \cup \{ ab \} \cup \{ ba \}$
Problem 2 (10 points)

(a) What is the language recognized by M? Give an informal description in English and briefly justify your answer.

(b) Are there strings \( x \in L(M) \), such that if the bits of \( x \) are flipped (changing 0 to 1 and 1 to 0), the resulting string will also be in \( L(M) \)? Why or why not? Is this true for every string \( x \in L(M) \)?

(c) Write the formal definition of \( M = (Q, \Sigma, \delta, q_0, F) \). Use a table to define \( \delta \).

Problem 3 (10 points)

CSE 105 is a fairly proof heavy class, so the goal of this problem is to help refresh you on different proof techniques. In particular, proof by induction and closure proofs will be important in this class.

(a) Prove by induction that \( 1 + 2 + 4 + 8 + \ldots + 2^n = 2^{n+1} - 1 \)

(b) Prove that the set of even integers is closed under subtraction, but the set of odd integers is not closed under subtraction.
Problem 4 (10 points)

(a) Write the formal definition of $M = (Q, \Sigma, \delta, q_0, F)$. Use a table to define $\delta$.
(b) Give an example of a string $x$ such that both $x$ and $x^R$ ($x^R$ is the reverse of $x$) are in $L(M)$.
(c) Give an example of a string $y$ such that neither $y$ nor $y^R$ are in $L(M)$.
(d) What is the language recognized by $M$? Give an informal description in English.
(e) Can you find an example of a string $w$ such that $w$ is in $L(M)$ but $w^R$ is not in $L(M)$? If so, write the example. If not, explain why.

Problem 5 (10 points)

You have an incoming bitstream (sequence of 0s and 1s) that might be truncated at any time. Create a DFA (State diagram) to determine if the number represented (in binary, reading bits from left-to-right) is a multiple of two (2) but not a multiple of four (4). (If you need to you can use http://madebyevan.com/fsm/ to draw your diagram.)