Binary Image Processing

Introduction to Computer Vision

CSE 152

Lecture 5

Announcements

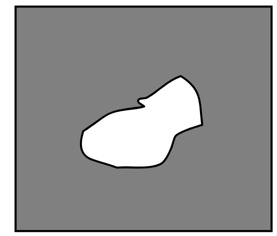
- Homework 2 is due Apr 25, 11:59 PM
- Reading:
  - Szeliski, Chapter 3 Image processing, Section 3.3 More neighborhood operators

Binary System Summary

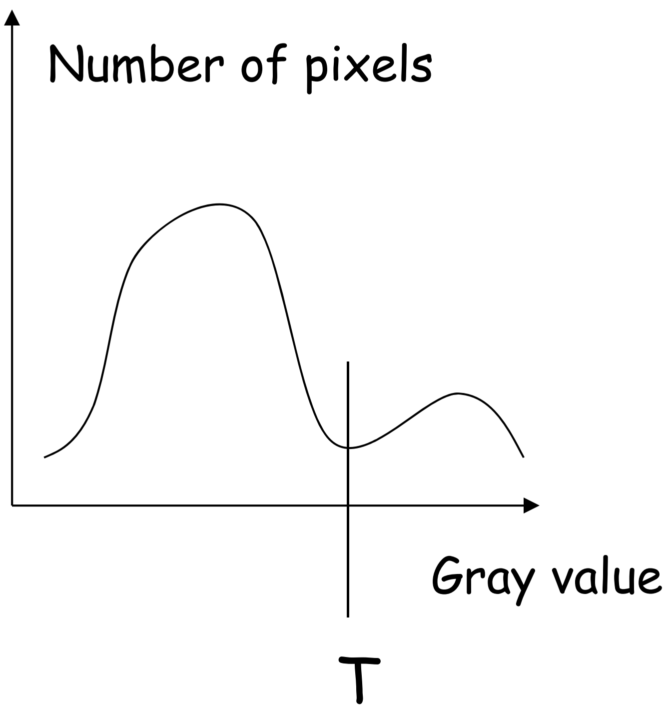
1. Acquire images and binarize (tresholding, color labels, etc.)
2. Possibly clean up image using morphological operators
3. Determine regions (blobs) using connected component exploration
4. Compute position, area, and orientation of each blob using moments
5. Compute features that are rotation, scale, and translation invariant using Moments (e.g., Eigenvalues of normalized moments)

Histogram-based Segmentation

Ex: bright object on dark background:



Histogram



- Select threshold
- Create binary image:

$$I(x,y) < T \;\rightarrow O(x,y) = 0$$

$$I(x,y) \geq T \;\rightarrow O(x,y) = 1$$

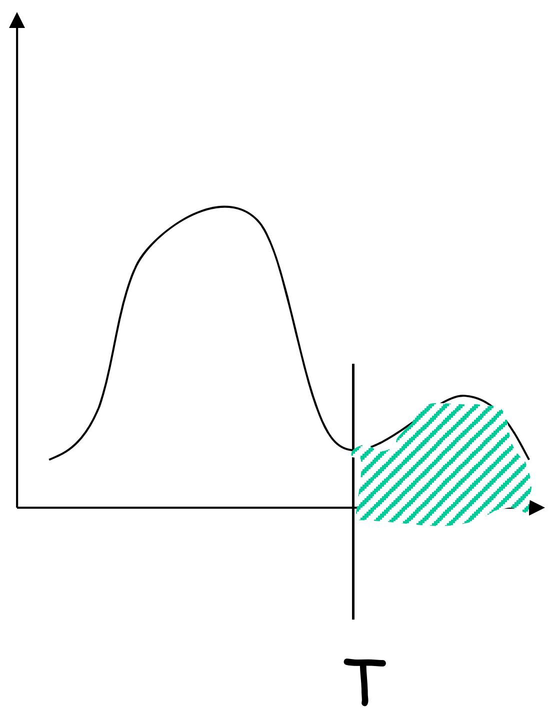
[From Octavia Camps]

How do we select a Threshold?

- Manually determine threshold experimentally
  - Good when lighting is stable and high contrast
- Automatic thresholding
  - P-tile method
  - Mode method
  - Otsu’s method

P-Tile Method

- If the *size* of the object is approximately known, pick T such that the area under the histogram corresponds to the size of the object:



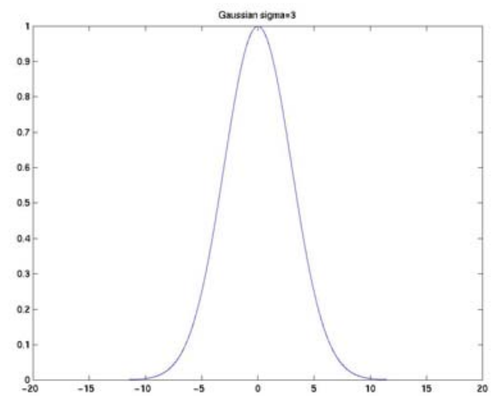
[From Octavia Camps]

Mode Method

- Model intensity in each region R_i as
  "constant" + $N(0,\sigma_i)$:

$$\text{If } (x,y) \in R_i \text{ then, } I(x,y) = \mu_i + n_i(x,y)$$

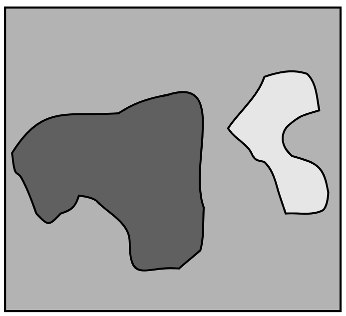
$$p(n_i) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\frac{n_i^2}{\sigma_i^2}}$$



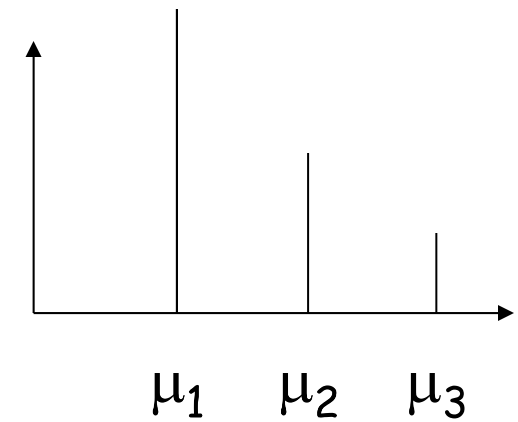
$$E(n_i) = 0 \qquad E(n_i^2) = \sigma_i^2$$

[From Octavia Camps]

Example: Image with 3 regions

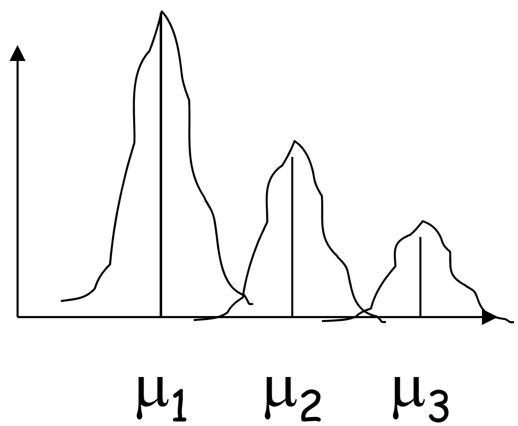


Ideal histogram:



- Approximate histogram as being comprised of multiple Gaussian modes.
  - How many modes?
  - Where are they centered, width

If above image is noisy, histogram looks like

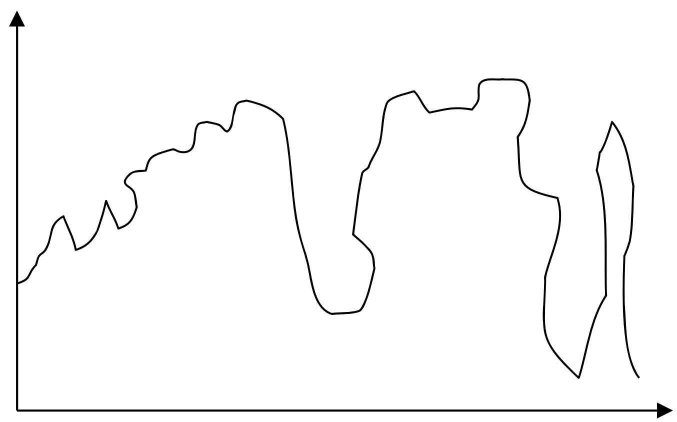


- Altenatively,the valleys are good places for thresholding to separate regions.

[From Octavia Camps]

Finding the peaks and valleys

- It is a not trivial problem:



[From Octavia Camps]

Otsu’s Method

- Each region (called a class) is modeled by a Gaussian distribution
- Exhaustively search for threshold *t* such that the **between** class variance σ_b^2 is maximized
  – Which also minimizes the **within** class variance σ_w^2
- Linear Discriminant Analysis (LDA)

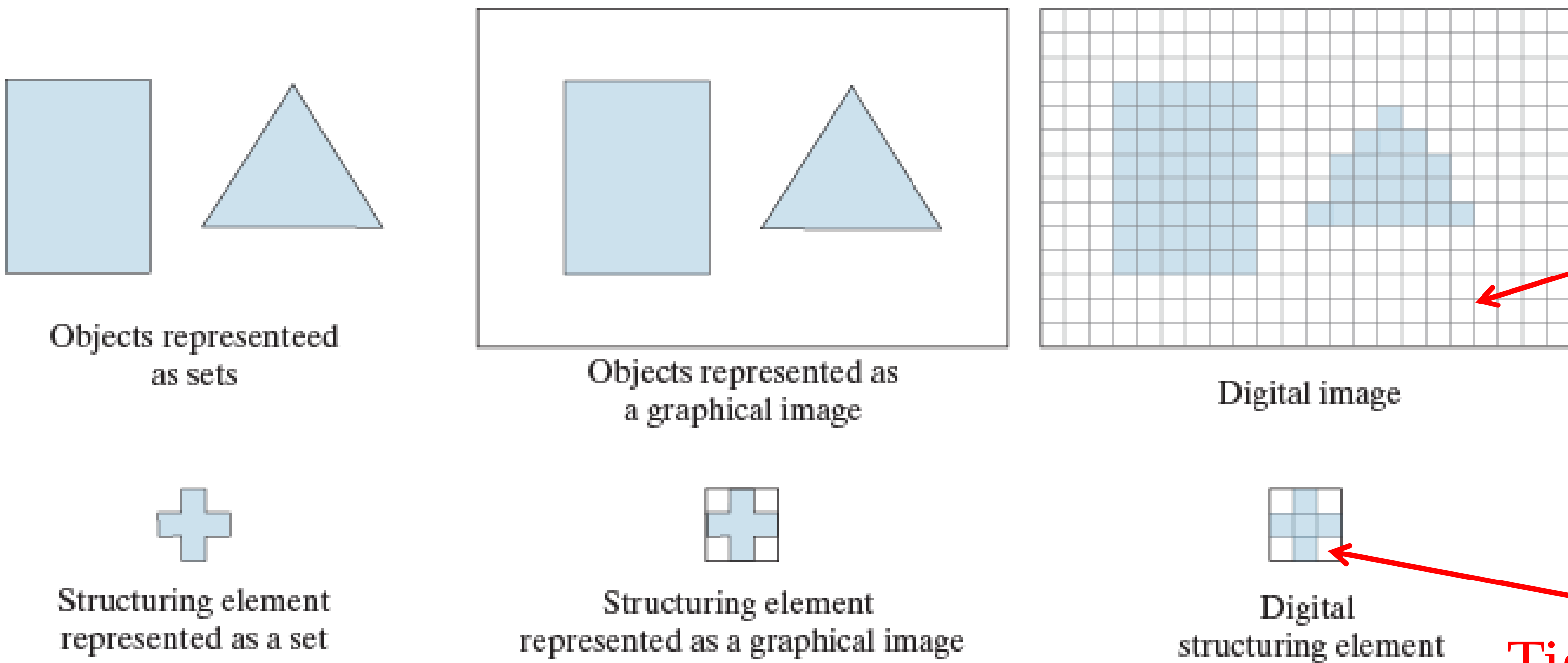
Otsu's Method, 2 classes

1. Compute histogram, then probability p_k of each intensity level k
2. Compute vector of cumulative sum of class 1 probabilities $P_1(k)$
3. Compute vector of cumulative mean $m(k)$
   – Use probabilities
   – Global mean μ_G is last element of vector
4. Compute vector of between class variance $\sigma_b^2(k)$
5. Threshold $k*$ is the value of k for which between class variance $\sigma_b^2(k)$ is maximum

Morphological Operations

Sets of pixels:
objects and structuring elements (SEs)

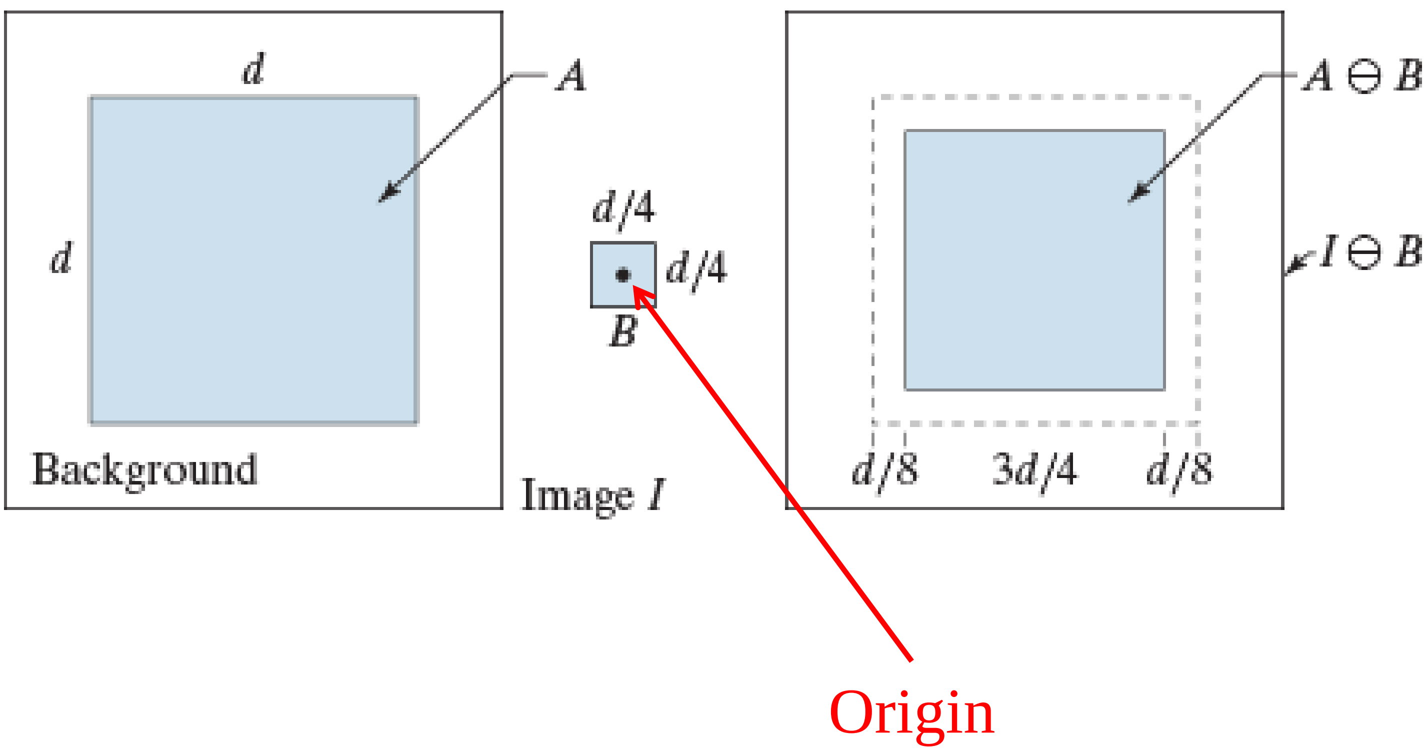
Border of background pixels around objects



Tight border around SE

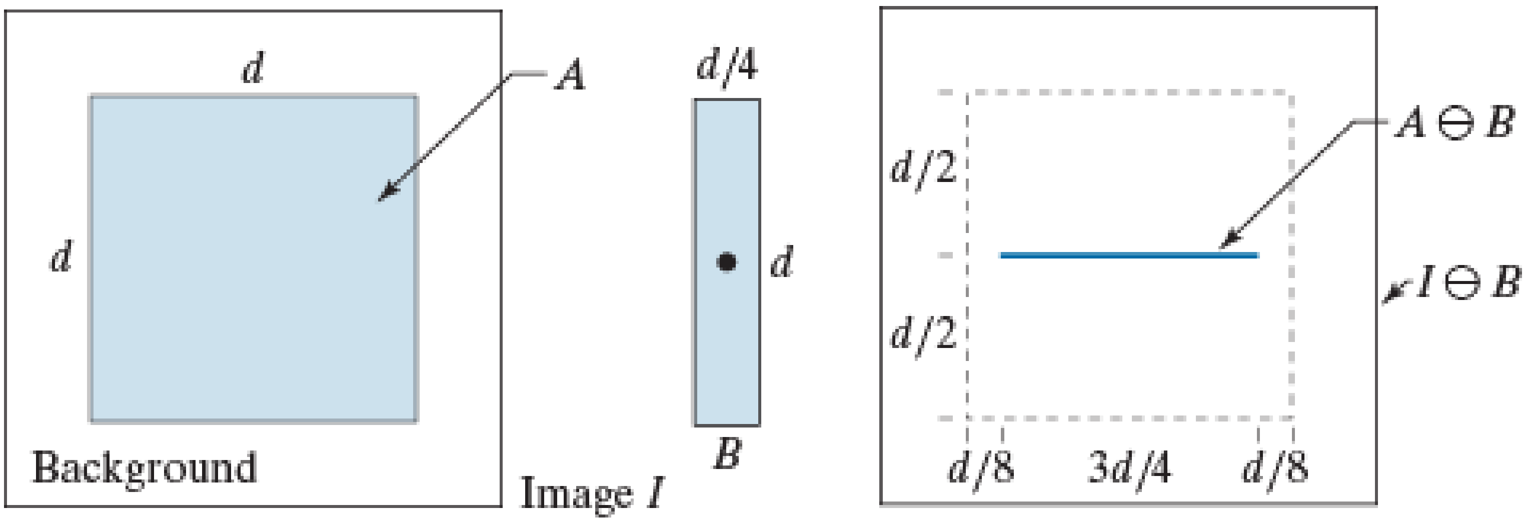
Erosion

Example: square SE



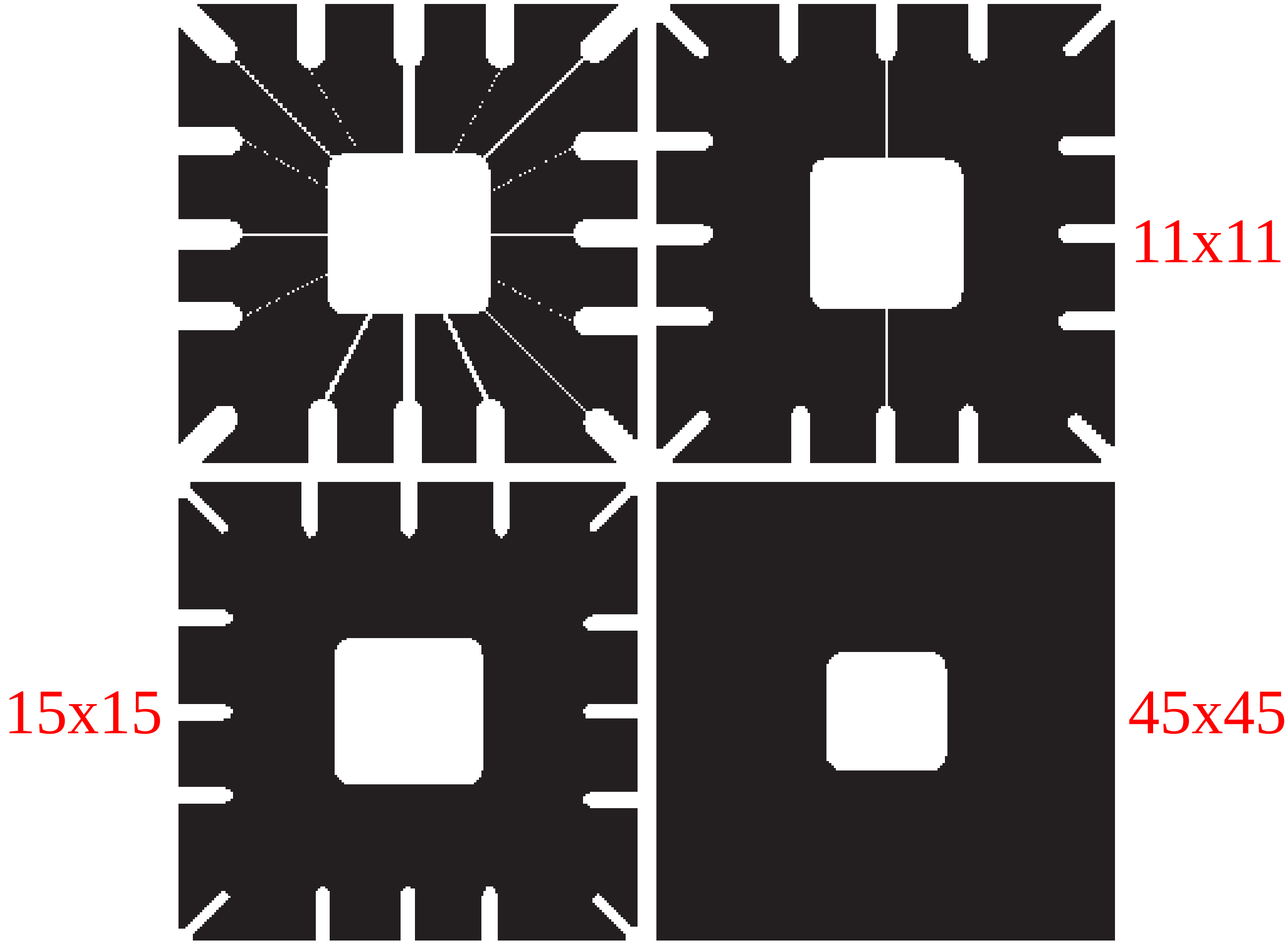
Erosion

Example: elongated SE



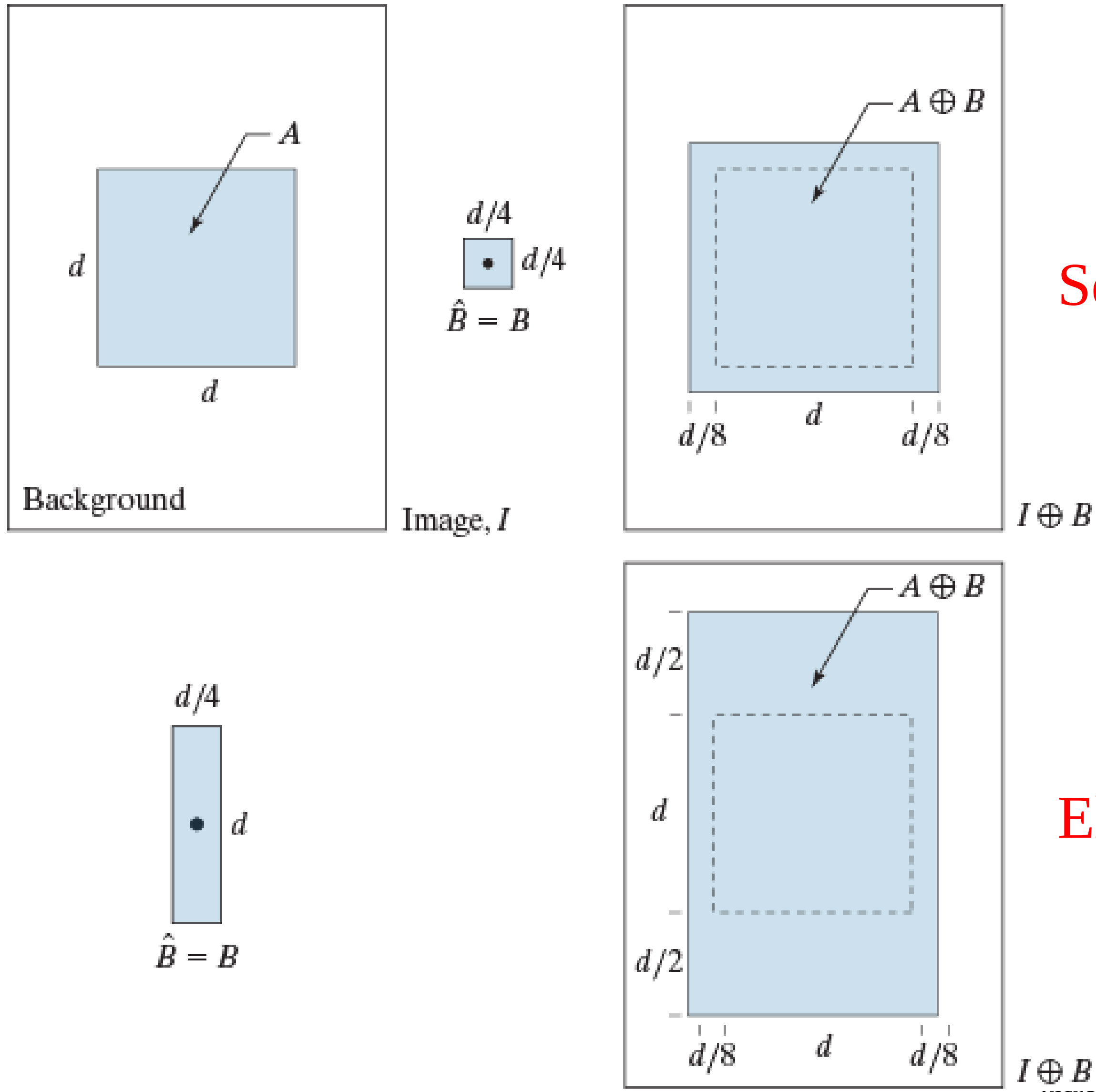
Erosion

Shrinks



Dilation

Examples

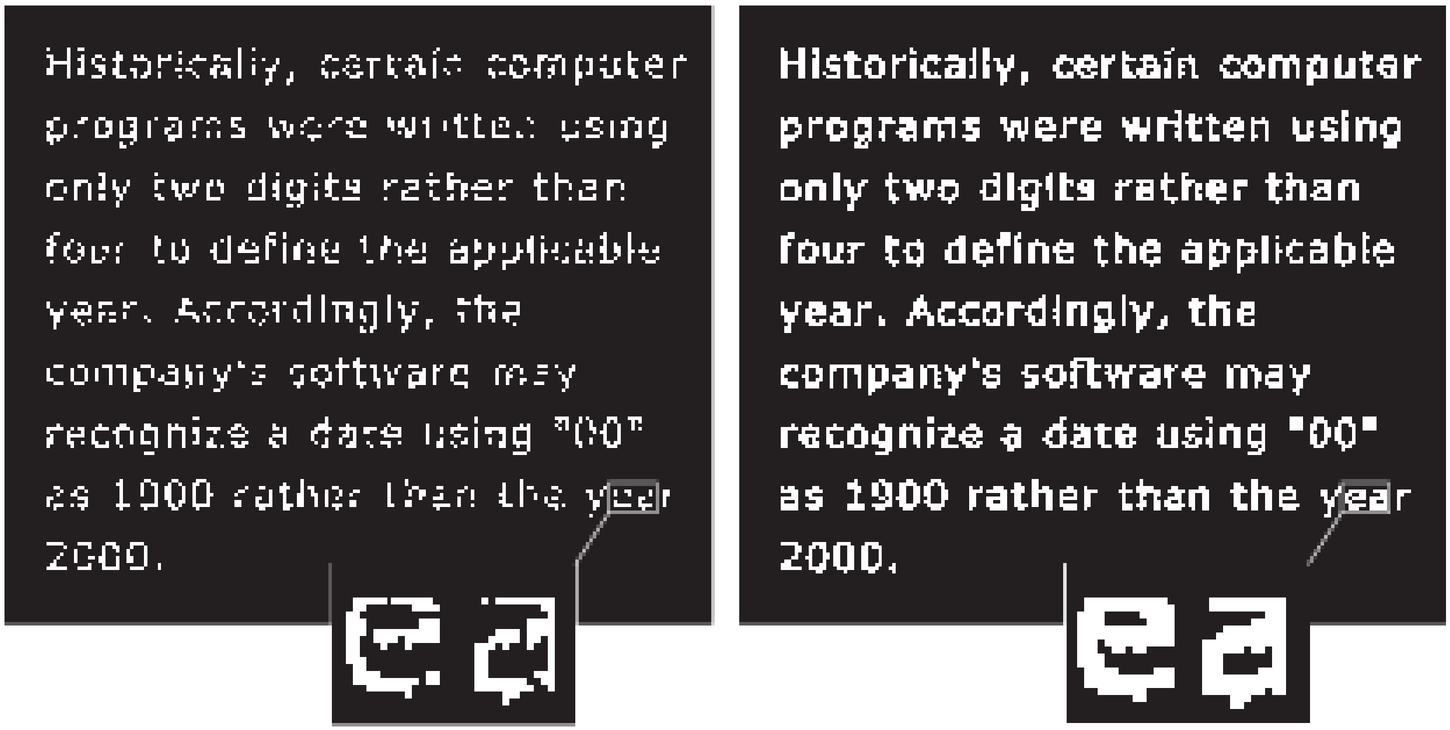


Square SE

Elongated SE

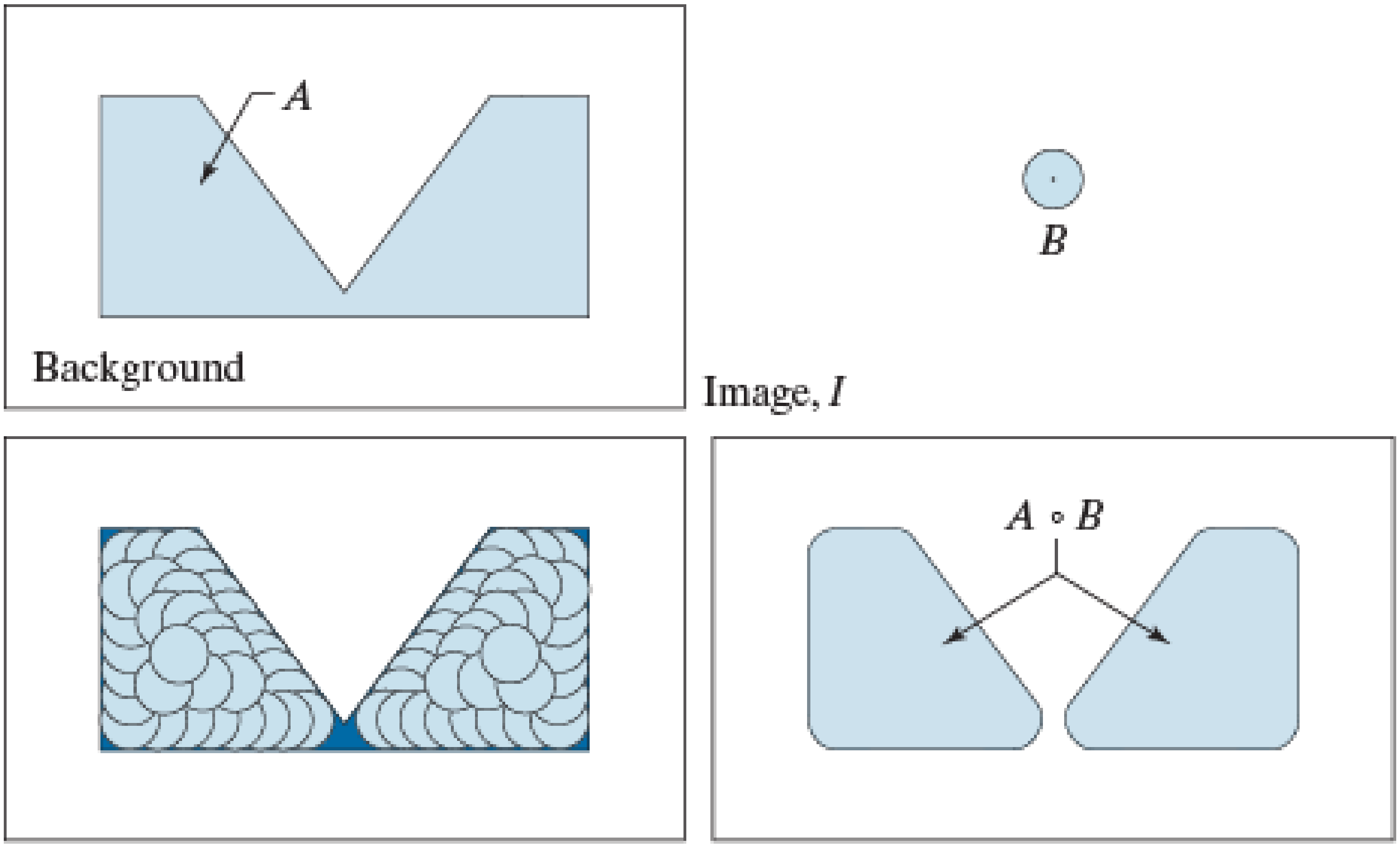
Dilation

Expands



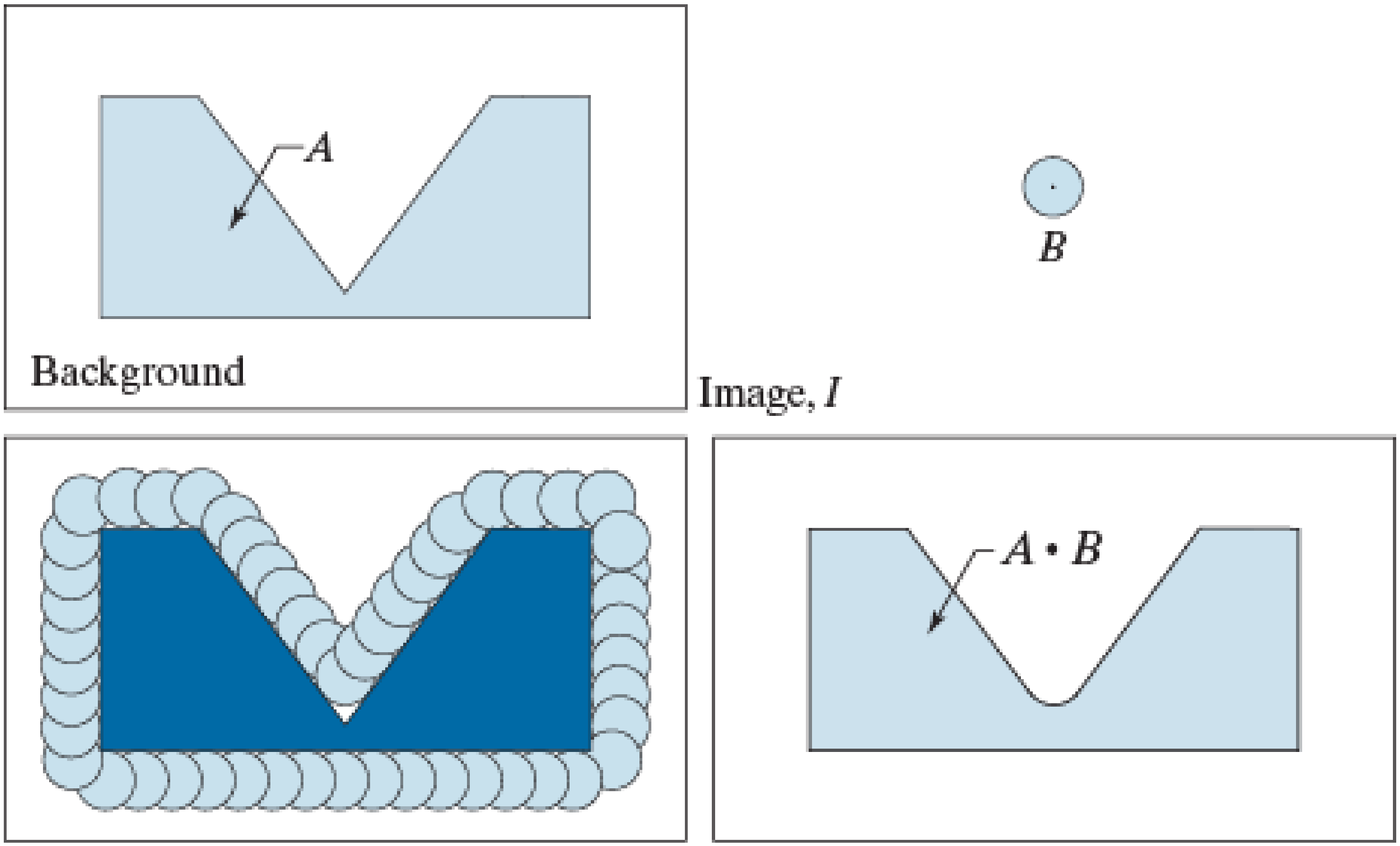
1	1	1
1	1	1
1	1	1

Opening



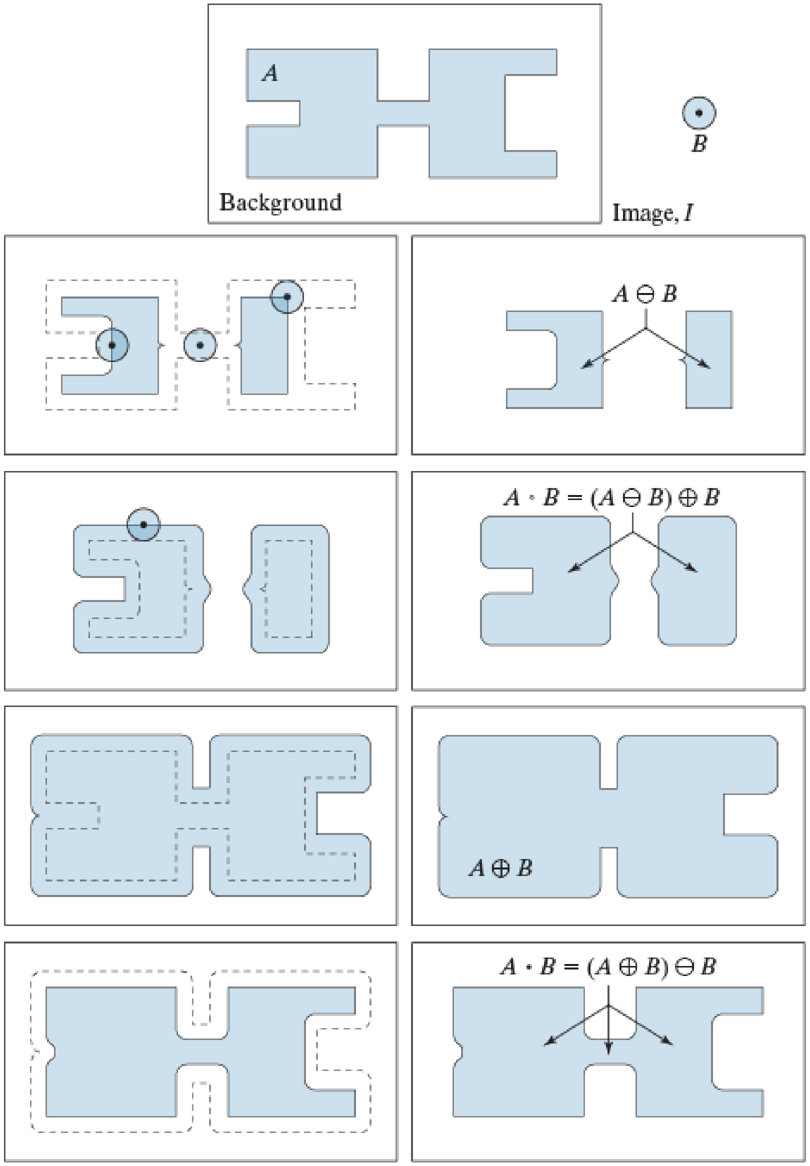
Structuring element rolls along **inner** boundary

Closing



Structuring element rolls along **outer** boundary

Opening and closing



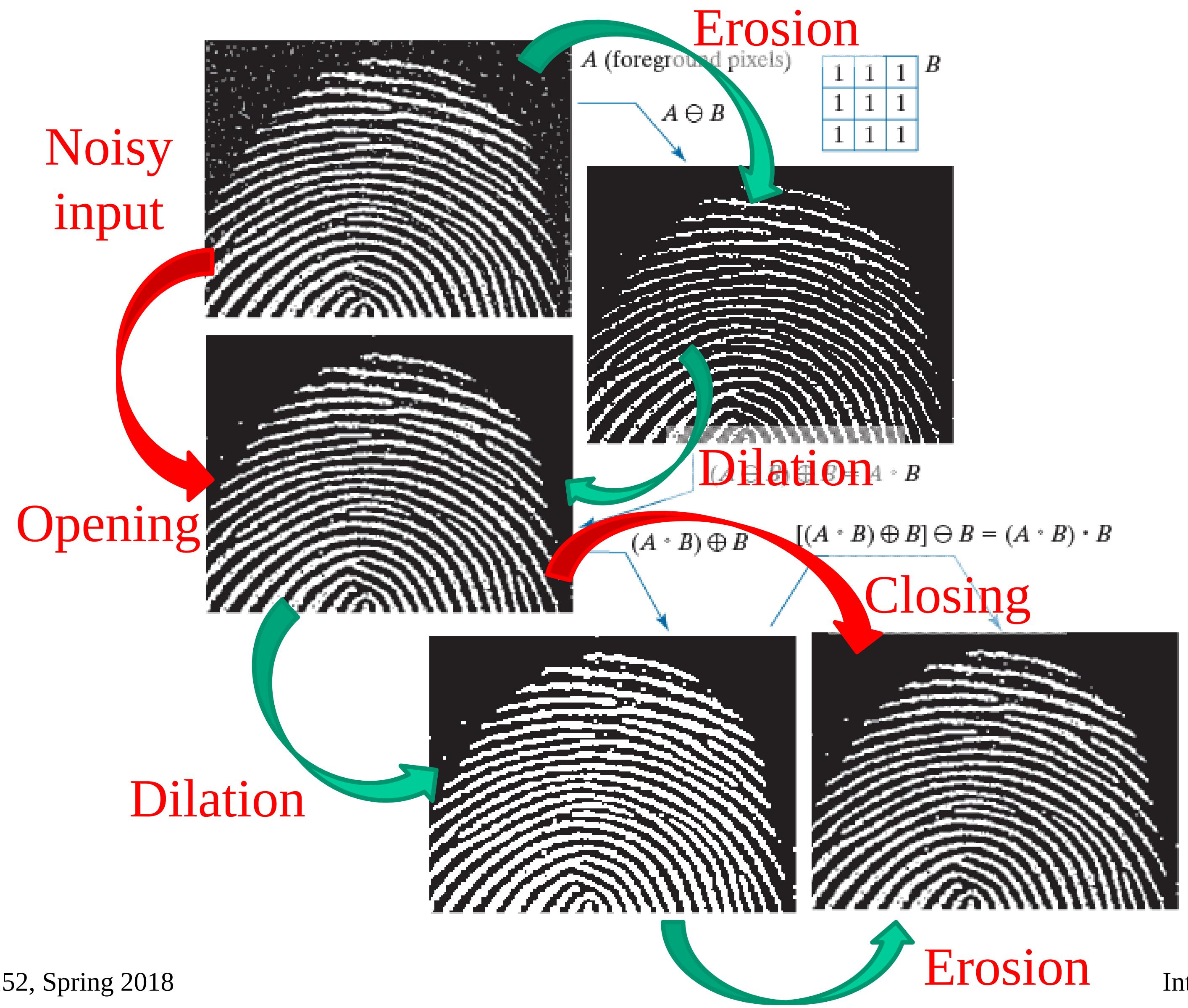
Erosion

Opening

Dilation

Closing

Morphological image processing



Regions

What is a region?

- “Maximal connected set of points in the image with same brightness value” (e.g., 1)
- Two points are *connected* if there exists a continuous path joining them
- Regions can be
  – *simply connected* (for every pair of points in the region, all smooth paths can be smoothly and continuously deformed into each other)
  – *multiply connected* (holes), otherwise

Connected Regions

		1	1	1											
		1	1	1						1	1	1	1		
		1	1	1					1	1			1		
					1	1			1	1			1	1	
				1	1	1				1	1	1	1		
					1										

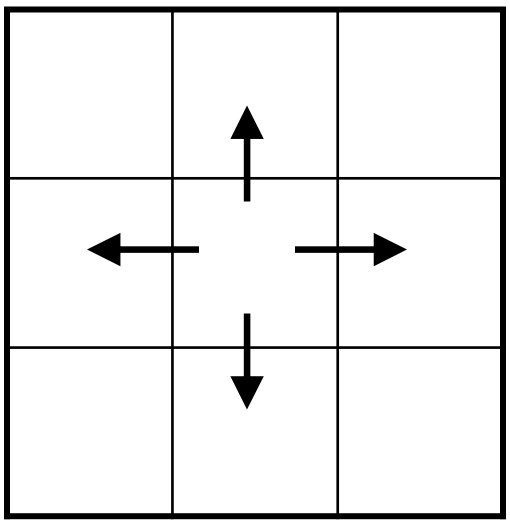
- What are the connected regions in this binary image?
- Which regions are contained within which region?

Connected Regions

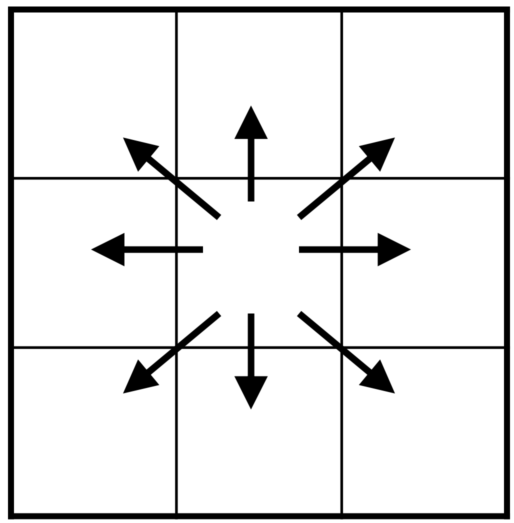
		1							1	1	1	1	1		
	1	1						1					1		
	1	1					1	1					1	1	
	1	1							1	1	1	1	1		

- What are the connected regions in this binary image?
- Which regions are contained within which region?

Four & Eight Connectedness



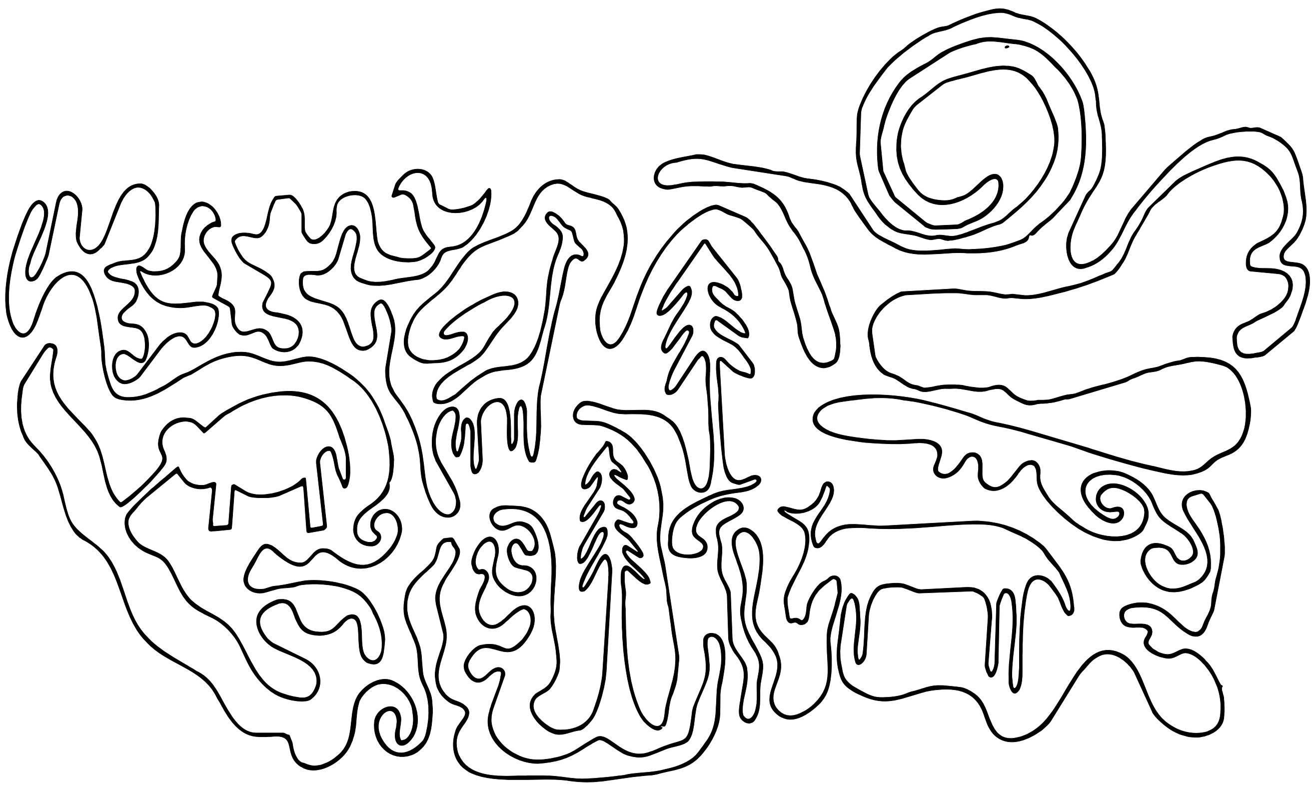
Four Connected



Eight Connected

Jordan Curve Theorem

- "Every closed curve in R^2 divides the plane into two region, the 'outside' and 'inside' of the curve."



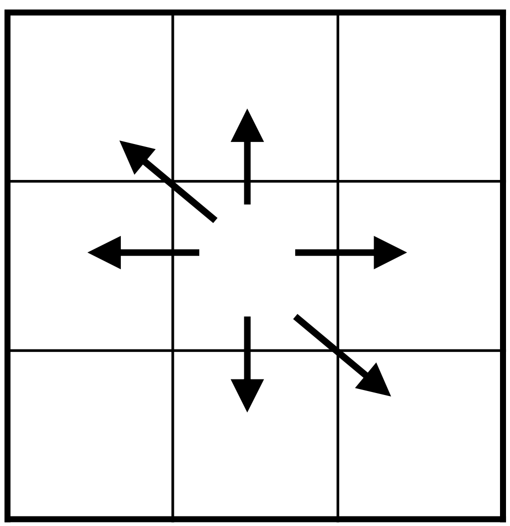
Problem of 4/8 Connectedness

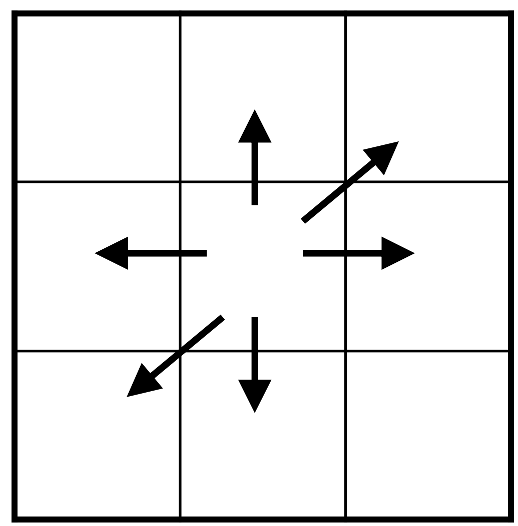
		1	1	1		
	1				1	
	1				1	
		1	1	1		

- 8 Connected:
  - Ones form a closed curve, but background only forms one region
- 4 Connected
  - Background has two regions, but ones form four “open” curves (no closed curve)

To achieve consistency with respect to Jordan Curve Theorem

1. Treat background as 4-connected and foreground as 8-connected
2. Use 6-connectedness





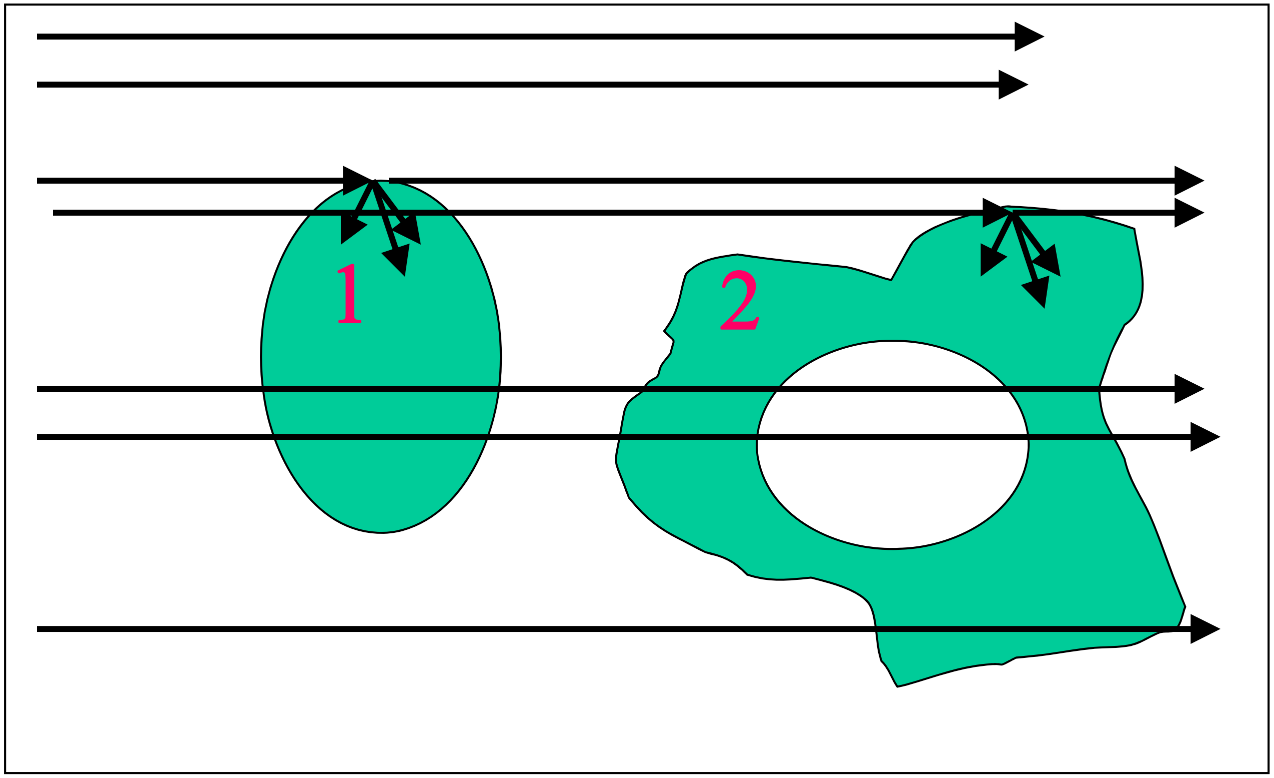
Recursive Labeling
Connected Component Exploration

```
Procedure Label (Pixel)
BEGIN
  Mark(Pixel) <- Marker;
  FOR neighbor in Neighbors(Pixel) DO
     IF Image (neighbor) = 1 AND Mark(neighbor)=NIL THEN
             Label(neighbor)
END

BEGIN Main
  Marker <- 0;
  FOR Pixel in Image DO
     IF Image(Pixel) = 1 AND Mark(Pixel)=NIL THEN
             BEGIN
                 Marker <- Marker + 1;
                 Label(Pixel);
             END;
END
```

Globals:
Marker: integer
Mark: Matrix same size as Image,
initialized to NIL

Recursive Labeling
Connected Component Exploration



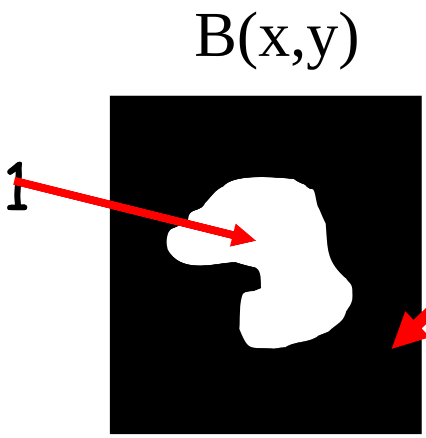
Some notes

- Once labeled, you know how many regions (the value of Marker)
- From Mark matrix, you can identify all pixels that are part of each region (and compute area)
- How deep does stack go?
- Iterative algorithms
- Parallel algorithms

Properties extracted from binary image

- A tree showing containment of regions
- Properties of a region
  1. Genus – number of holes
  2. Centroid
  3. Area
  4. Perimeter
  5. Moments (e.g., measure of elongation)
  6. Number of “extrema” (indentations, bulges)
  7. Skeleton

Moments



The region S is defined as:

$$S = \{(x,y) | \mathrm{B}(x,y) = 1\}$$

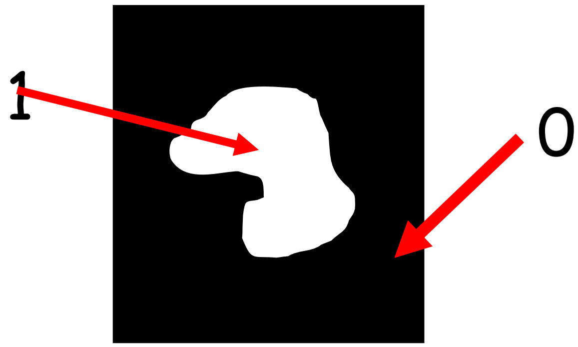
Given a pair of non-negative integers (j,k) the *discrete* (j,k)$^{\text{th}}$ *moment* of S is defined as:

$$M_{jk}(S) = \sum_{(x,y)\in S} x^j y^k$$

$$M_{jk} = \sum_{x=1}^{n}\sum_{y=1}^{m} B(x,y) x^j y^k$$

- Fast way to implement computation over n by m image or window
- One object

Moments: Area



$$S = \{(x, y) | f(x, y) = 1\}$$

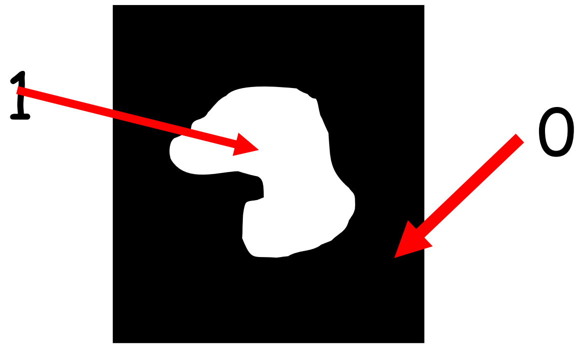
$$M_{jk}(S) = \sum_{(x,y)\in S} x^j y^k$$

Example:

$$M_{oo}(S) = \sum_{(x,y)\in S} x^0 y^0 = \sum_{(x,y)\in S} 1 = \#(S)$$

Area of S

Moments: Centroid



$$S = \{(x,y) | f(x,y) = 1\}$$

$$M_{jk}(S) = \sum_{(x,y)\in S} x^j y^k$$

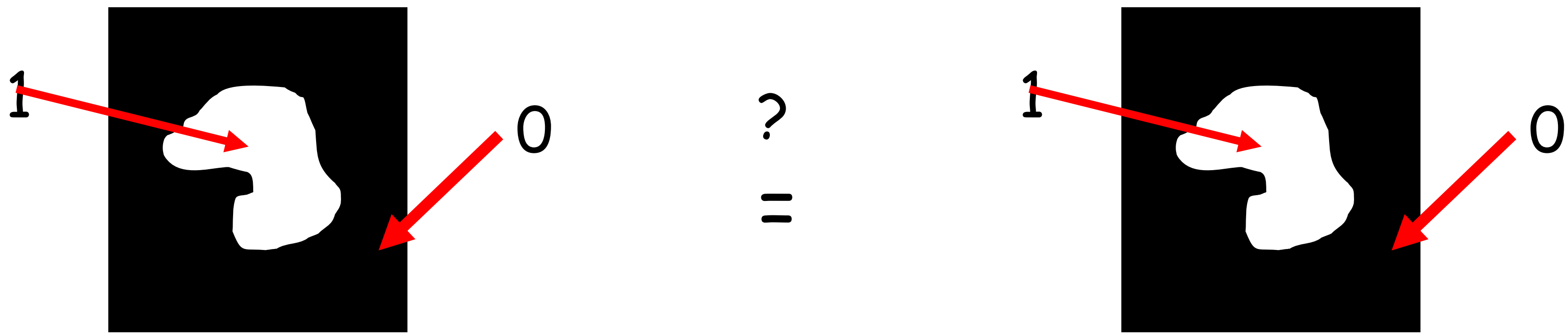
Example:

$$M_{10}(S) = \sum_{(x,y)\in S} x^1 y^0 = \sum_{(x,y)\in S} x \qquad M_{01}(S) = \sum_{(x,y)\in S} x^0 y^1 = \sum_{(x,y)\in S} y$$

$$\frac{M_{10}(S)}{M_{oo}(S)} = \frac{\sum_{(x,y)\in S} x}{\#(S)} = \bar{x} \qquad \frac{M_{01}(S)}{M_{oo}(S)} = \frac{\sum_{(x,y)\in S} y}{\#(S)} = \bar{y}$$

Center of gravity (centroid, mean) of S

Shape recognition by Moments

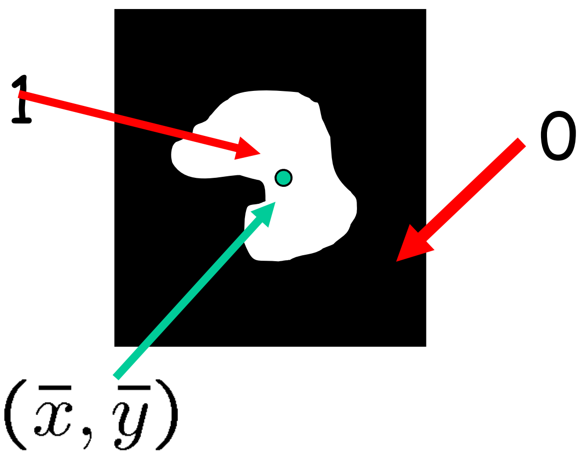


Recognition could be done by comparing moments

However, moments M_{jk} are not invariant under:

- Translation
- Scaling
- Rotation
- Skewing

Central Moments



$$S = \{(x, y) | f(x, y) = 1\}$$

$$\bar{x} = \frac{M_{10}(S)}{M_{oo}(S)} \qquad \bar{y} = \frac{M_{01}(S)}{M_{oo}(S)}$$

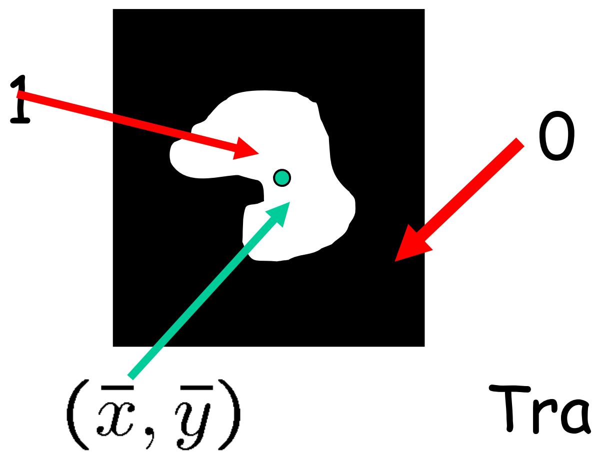
Given a pair of non-negative integers (j,k) the *central* (j,k)th *moment* of S is given by:

$$\mu_{jk}(S) = \sum_{(x,y)\in S} (x - \bar{x})^j (y - \bar{y})^k$$

Or the central moments can be computed from precomputed regular moments

$$\mu_{jk} = \sum_{m=1}^{i} \sum_{n=1}^{j} \binom{i}{m} \binom{j}{n} (-\bar{x})^{(i-m)} (-\bar{y})^{(j-n)} M_{mn}$$

Central Moments

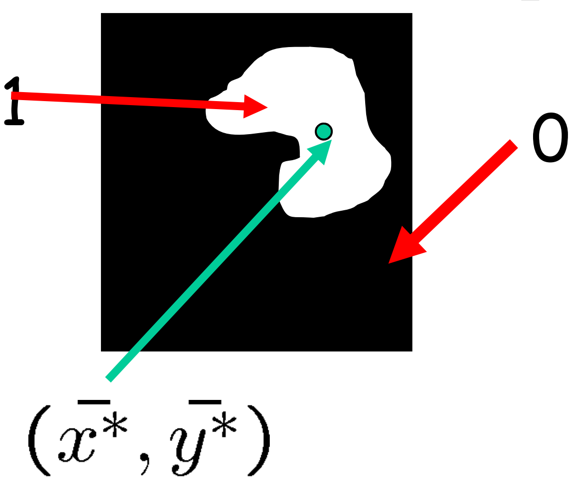


$$S = \{(x,y) | f(x,y) = 1\}$$

$$\mu_{jk}(S) = \sum_{(x,y)\in S} (x-\bar{x})^j (y-\bar{y})^k$$

Translation by T = (a,b) :

$$S_T = \{(x^*, y^*) | x^* = x + a, y^* = y + b, (x,y) \in S\}$$

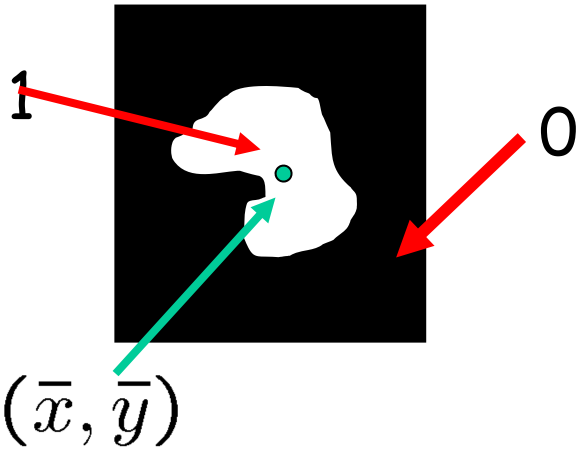


$$\bar{x^*} = \frac{M_{10}(S_T)}{M_{oo}(S_T)} = \bar{x} + a \qquad \bar{y^*} = \frac{M_{01}(S_T)}{M_{oo}(S_T)} = \bar{y} + b$$

$$\mu_{jk}(S_T) = \mu_{jk}(S)$$

Translation INVARIANT

Normalized Moments



$$S = \{(x, y) | f(x, y) = 1\}$$

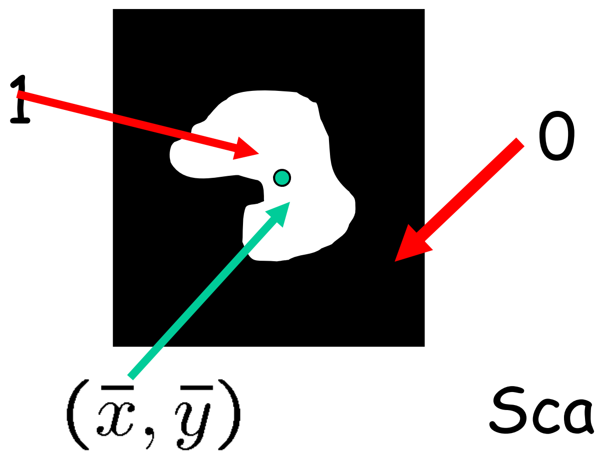
$$\mu_{jk}(S) = \sum_{(x,y) \in S} (x - \bar{x})^j (y - \bar{y})^k$$

$$\sigma_x = \sqrt{\frac{\mu_{20}(S)}{M_{oo}(S)}} \qquad \sigma_y = \sqrt{\frac{\mu_{02}(S)}{M_{oo}(S)}}$$

Given a pair of non-negative integers (j,k) the ***normalized*** (j,k)th ***moment*** of S is given by:

$$m_{jk}(S) = \sum_{(x,y) \in S} \left(\frac{x - \bar{x}}{\sigma_x} \right)^j \left(\frac{y - \bar{y}}{\sigma_y} \right)^k$$

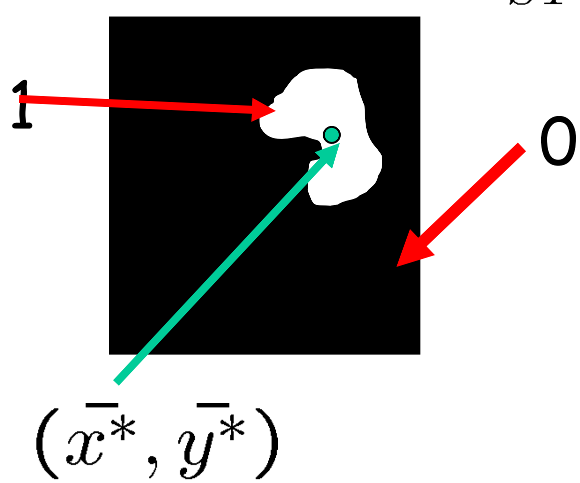
Normalized Moments



$$S = \{(x,y) | f(x,y) = 1\}$$

Scaling by (a,c) and translating by T = (b,d) :

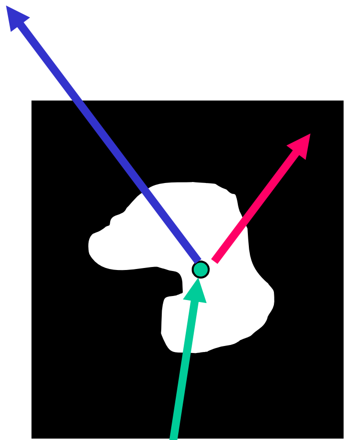
$$S_{ST} = \{(x^*, y^*) | x^* = ax{+}b, y^* = cy{+}d, (x,y) \in S\}$$



$$m_{jk}(S_{ST}) = m_{jk}(S)$$

Scaling and translation INVARIANT

Region orientation from Second Moment Matrix



$$(\bar{x}, \bar{y})$$

1. Compute second centralized moment matrix

$$\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix}$$

- Symmetric, positive definite matrix
- Positive Eigenvalues
- Orthogonal Eigenvectors

1. Compute Eigenvectors of Moment Matrix to obtain orientation
2. Eigenvalues are independent of orientation and translation

Next Lecture

- Early vision
  - Linear filters
- Reading:
  - Chapter 4: Linear filters