

Lecture 5: K-Map minimization in larger input dimensions and K-map minimization using max terms

CSE 140: Components and Design Techniques for Digital Systems

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Part I. Combinational Logic

1. Specification
2. Implementation

K-map:

Sum of products

Product of sums

Definitions: Sum of Products and Product of Sums Expressions

Sum of Products Expression

Implicant: A **product term** that has non-empty intersection with on-set $\mathcal{F}$ and does not intersect with off-set $\mathcal{R}$.

Prime Implicant: An **implicant** that is not a proper subset of any other **implicant**.

Essential Prime Implicant: A prime **implicant** that has an element in on-set $\mathcal{F}$ but this element is not covered by any other prime **implicants**.

Product of Sums Expression

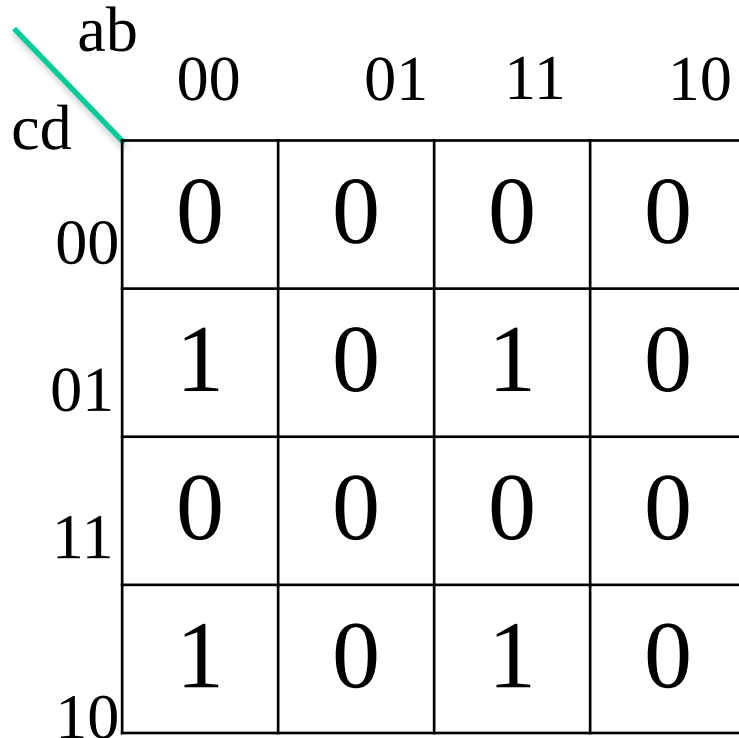
Implicate: A **sum term** that has non-empty intersection with off-set $\mathcal{R}$ and does not intersect with on-set $\mathcal{F}$.

Prime Implicate: An **implicate** that is not a proper subset of any other implicate.

Essential Prime Implicate: A prime **implicate** that has an element in off-set $\mathcal{R}$ but this element is not covered by any other prime **implicates**.

K-Map to Minimized Product of Sums

- Sometimes it is easier to reduce the K-map by considering the offset



ab \ cd	00	01	11	10
00	0	0	0	0
01	1	0	1	0
11	0	0	0	0
10	1	0	1	0

$$F_1 = a'b'c'd + abc'd + a'b'cd' + abcd'$$

$$F_2 = (a+b')(a'+b)(c'+d')(c+d)$$

iClicker:

Which function is simpler for a two level logic implementation?

- A. F_1
- B. F_2
- C. Two are the same

K-Map to Minimized Product of Sums

- Sometimes it is easier to reduce the K-map by considering the offset

$$F_1 = a'b'c'd + abc'd + a'b'cd' + abcd'$$

$$F_2 = (a+b')(a'+b)(c'+d')(c+d)$$

ab	00	01	11	10
cd				
00	0	0	0	0
01	1	0	1	0
11	0	0	0	0
10	1	0	1	0

Another min product of sums example

Given $\mathcal{R}(a,b,c,d) = \Sigma m(3, 11, 12, 13, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m(4, 8, 10)$

K-map



ab		00	01	11	10
cd	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Another min product of sums example

Given $\mathcal{R}(a,b,c,d) = \Sigma m(3, 11, 12, 13, 14)$
 $\mathcal{D}(a,b,c,d) = \Sigma m(4, 8, 10)$

Is $\pi M(12,14)$ a prime implicate?

A: Yes

B: No

		ab			
		00	01	11	10
cd	00	⁰ 1	⁴ X	¹² 0	⁸ X
	01	¹ 1	⁵ 1	¹³ 0	⁹ 1
	11	³ 0	⁷ 1	¹⁵ 1	¹¹ 0
	10	² 1	⁶ 1	¹⁴ 0	¹⁰ X

a

d

Prime Implicates:

$\Pi M(0,8)$, $\Pi M(11,15)$, $\Pi M(12,13,14,15)$, $\Pi M(6,14)$

PI Q: Which of the following is a not an essential prime implicate?

A. $\Pi M(0,8)$

B. $\Pi M(11,15)$

C. $\Pi M(12,13,14,15)$

D. $\Pi M(6,14)$

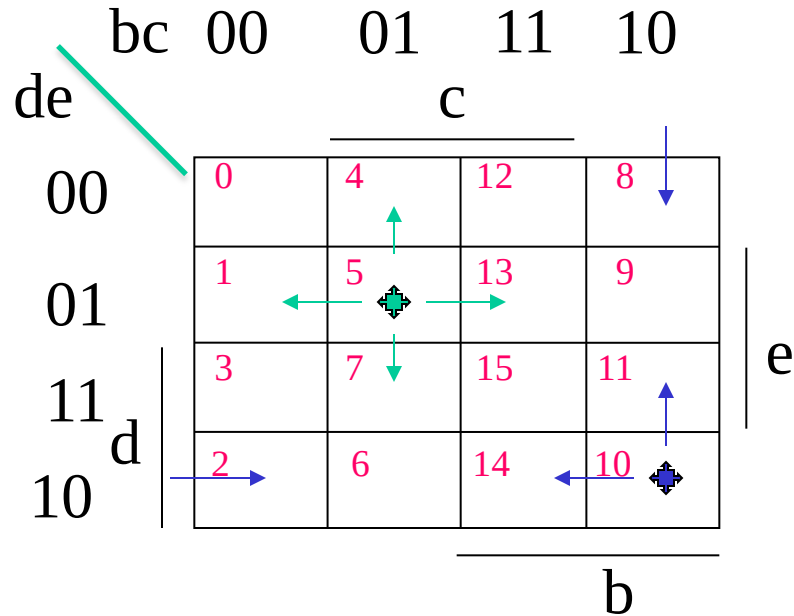
		ab			
		00	01	11	10
cd	00	⁰ 0	⁴ 1	¹² 0	⁸ X
	01	¹ 1	⁵ 1	¹³ 0	⁹ 1
	11	³ 1	⁷ 1	¹⁵ 0	¹¹ 0
	10	² 1	⁶ X	¹⁴ 0	¹⁰ 1

d

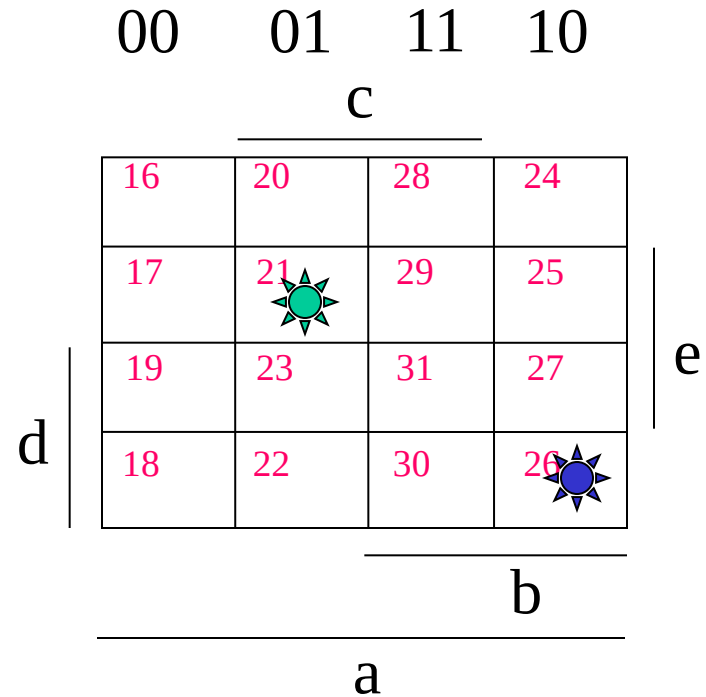
a

Five variable K-map

a=0



a=1



Neighbors of m_5 are: minterms 1, 4, 7, 13, and 21

Neighbors of m_{10} are: minterms 2, 8, 11, 14, and 26



Reading a Five variable K-map: An example

a=0

		c			
bc		00	01	11	10
de	00	0 1	4	12 1	8 1
	01	1	5 1	13 1	9
	11	3	7	15 1	11 1
	10	2 1	6	14 1	10 1
		b			

d

e

a=1

		c			
		00	01	11	10
d	00	16 1	20	28	24 1
	01	17	21	29	25
	11	19	23 1	31 1	27 1
	10	18 1	22	30 1	26 1
		b			
		a			

e

Six variable K-map

