

Lecture 4: Four Input K-Maps

CSE 140: Components and Design Techniques for Digital Systems

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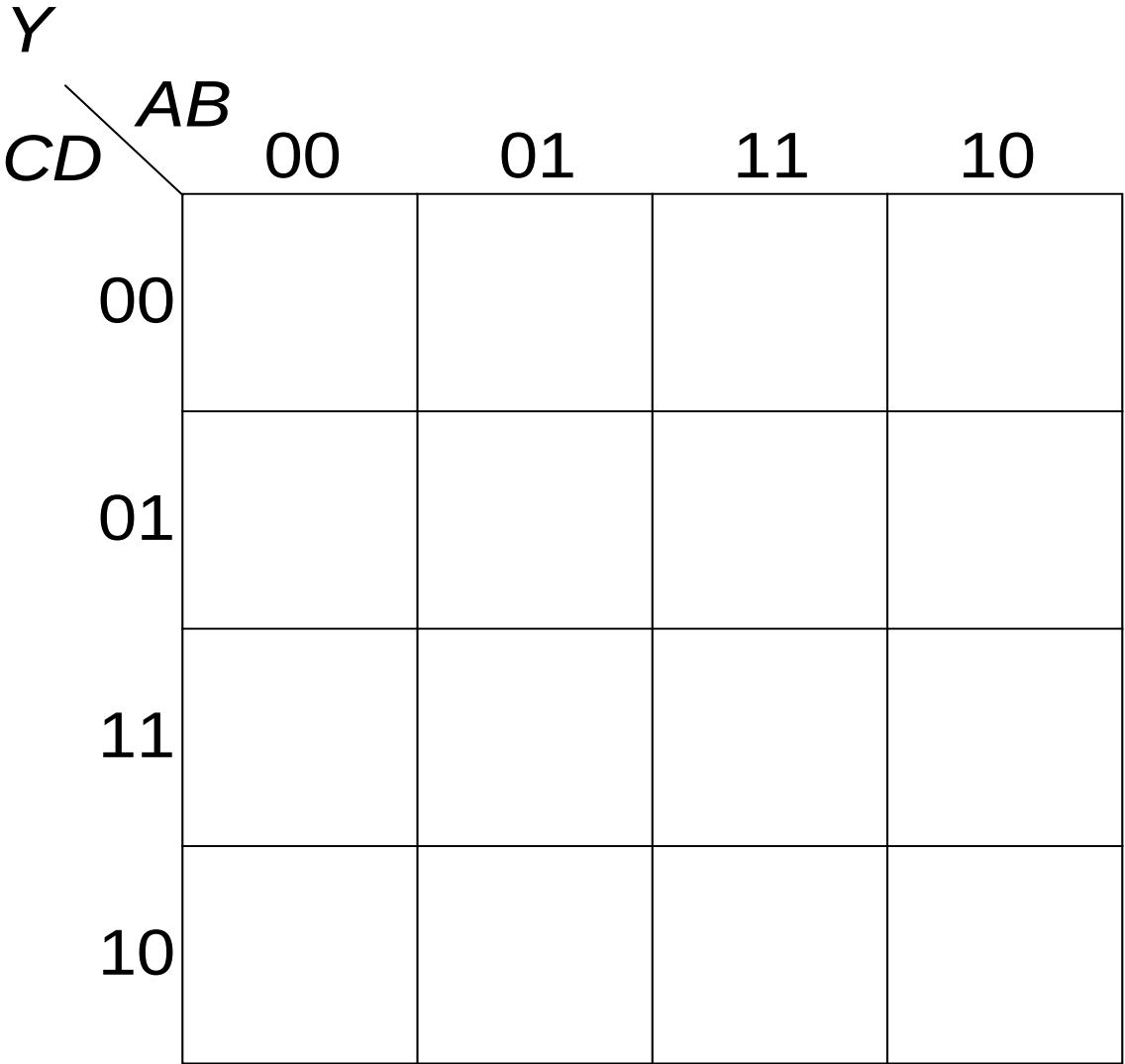
Outlines

- Boolean Algebra vs. Karnaugh Maps
 - Algebra: variables, product terms, minterms, consensus theorem
 - Map: planes, rectangles, cells, adjacency
- Definitions: implicants, prime implicants, essential prime implicants
- Implementation Procedures

An example

id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

4-input K-map



4-input K-map

id	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	0

Y $CD \backslash AB$					
		00	01	11	10
00	1	0	0	1	
01	0	1	0	1	
11	1	1	0	0	
10	1	1	0	1	



4-input K-map

- Arrangement of variables
- Adjacency and partition

Y CD \ AB		AB			
		00	01	11	10
00	00	0 1	4 0	12 0	8 1
	01	1 0	5 1	13 0	9 1
	11	3 1	7 1	15 0	11 0
	10	2 1	6 1	14 0	10 1

K-Map vs. Boolean Expression

Boolean Expression	K-Map
Variable x_i and complement x_i'	Half planes Rx_i and Rx_i'
Product term $P = \Pi_i x_i^*$	Rectangle $R_p = \cup_{i \in P} x_i^*$
Each minterm	One element cell
Two adjacent minterms	Two neighboring cells
Each minterm has n adjacent minterms	Each cell has n neighbors
Boolean algebra	Two dimensional tool
?	D Don't Care set handling

Procedure for finding the minimal function via K-maps (layman terms)

1. Convert truth table to K-map
2. Group adjacent ones: In doing so include the largest number of adjacent ones (Prime Implicants)
3. Create new groups to cover all ones in the map: create a new group only to include at least one cell (of value 1) that is not covered by any other group (Essential Prime Implicants)
4. Select the groups that result in the minimal sum of products (we will formalize this because its not straightforward)

		Y			
		AB			
CD		00	01	11	10
	00	1	0	0	1
	01	0	1	0	1
	11	1	1	0	0
	10	1	1	0	1

Reading the reduced K-map

Y CD \ AB		AB			
		00	01	11	10
CD	00	0 1	4 0	12 0	8 1
	01	1 0	5 1	13 0	9 1
	11	3 1	7 1	15 0	11 0
	10	2 1	6 1	14 0	10 1

$$\sum m(2,3,6,7)$$

$$\sum m(5,7)$$

$$\sum m(8,9)$$

$$\sum m(0,2,8,10)$$

$$Y = \overline{A}\overline{C} + \overline{A}BD + A\overline{B}\overline{C} + \overline{B}\overline{D}$$



Definitions: implicant, prime implicant, essential prime implicant

- **Implicant**: A product term that has non-empty intersection with **on-set F** and does not intersect with off-set R .
- **Prime Implicant**: An implicant that is **not covered** by any other implicant.
- **Essential Prime Implicant**: A prime implicant that has **an element in on-set F** but this element is not covered by any other prime implicants.

Examples of Primes and Essential Primes

Implicant, Prime, Essential Prime

Y		AB			
CD		00	01	11	10
		00	01	11	10
00	0	1	4	12	8
01	1	0	5	13	9
11	3	1	7	15	11
10	2	1	6	14	10

$$Y = \overline{A}\overline{C} + \overline{A}BD + A\overline{B}\overline{C} + \overline{B}\overline{D}$$

Prime

$$\sum m(2,3,6,7)$$

$$\sum m(5,7)$$

$$\sum m(8,9)$$

$$\sum m(0,2,8,10)$$

Definition: Prime Implicant

1. Implicant: A product term that has non-empty intersection with on-set F and does not intersect with off-set R .
2. Prime Implicant: An implicant that is not covered by any other implicant.

Y		AB			
CD		00	01	11	10
	00	1	0	0	1
	01	0	1	0	1
	11	1	1	0	0
	10	1	1	0	1

Q: How about this one? Is it a prime implicant?

- A. Yes
B. No

Definition: Essential Prime

- Essential Prime Implicant: A prime implicant that has an element in on-set F but this element is not covered by any other prime implicants.

Y $CD \backslash AB$					
		00	01	11	10
00		1	0	0	1
01		0	1	0	1
11		1	1	0	0
10		1	1	0	1

Q: Is the blue group an essential prime?

- A. Yes
B. No

Definition: Essential Prime

Q: Which of the following product term(s) is (are) not an essential prime for the given K-map ?

		ab			
		00	01	11	10
cd	00	0 1	4	12 1	8 1
	01	1	5 1	13 1	9
	11	3	7	15 1	11 1
	10	2 1	6	14 1	10 1

- A. $bc'd$ $\sum m(5,13)$
- B. $d'b'$ $\sum m(0,2,8,10)$
- C. ac $\sum m(10,11,14,15)$
- D. ab $\sum m(12,13,14,15)$
- E. ad' $\sum m(8,10,12,14)$

Procedure for finding the minimal function via K-maps (formal terms)

1. Convert truth table to K-map
2. Include all essential primes
3. Include non essential primes as needed to completely cover the onset (all cells of value one)

Y $CD \backslash AB$					
		00	01	11	10
CD	00	1	0	0	1
	01	0	1	0	1
	11	1	1	0	0
	10	1	1	0	1

K-maps with Don't Cares

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Y</i>
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

<i>Y</i> <i>CD</i> \ <i>AB</i>					
		00	01	11	10
<i>CD</i>	00				
	01				
	11				
	10				

K-maps with Don't Cares

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>Y</i>
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

<i>Y</i> <i>CD</i> \ <i>AB</i>		00	01	11	10
<i>CD</i>	00	1	0	X	1
	01	0	X	X	1
	11	1	1	X	X
	10	1	1	X	X

K-maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

		AB			
		00	01	11	10
Y CD	00	1	0	X	1
	01	0	X	X	1
	11	1	1	X	X
	10	1	1	X	X

$$Y = A + \bar{B}\bar{D} + C$$

Reducing Canonical expressions

Given $F(a,b,c,d) = \sum m(0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \sum m(9, 10)$

1. Draw K-map

ab \ cd	00	01	11	10
00				
01				
11				
10				

Reducing Canonical Expressions

Given $F(a,b,c,d) = \sum m(0, 1, 2, 8, 14)$

$D(a,b,c,d) = \sum m(9, 10)$

1. Draw K-map

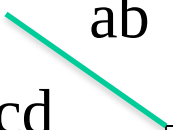
	ab	00	01	11	10
cd	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Reducing Canonical Expressions

Given $F(a,b,c,d) = \sum m(0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \sum m(9, 10)$

1. Draw K-map



ab		00	01	11	10
cd	00	⁰ 1	⁴ 0	¹² 0	⁸ 1
	01	¹ 1	⁵ 0	¹³ 0	⁹ X
	11	³ 0	⁷ 0	¹⁵ 0	¹¹ 0
	10	² 1	⁶ 0	¹⁴ 1	¹⁰ X

Reducing Canonical Expressions

1. Draw K-map
2. Identify Prime implicants
3. Identify Essential Primes

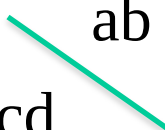
ab					
cd	00	01	11	10	
	0 1	4 0	12 0	8 1	
	1 1	5 0	13 0	9 X	
	3 0	7 0	15 0	11 0	
	2 1	6 0	14 1	10 X	

PI Q: How many primes (P) and essential primes (EP) are there?

- A. Four (P) and three (EP)
- B. Three (P) and two (EP)
- C. Three (P) and three (EP)
- D. Four (P) and Four (EP)

Reducing Canonical Expressions

1. Prime implicants: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
2. Essential Primes: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$



	ab	00	01	11	10
cd	00	⁰ 1	⁴ 0	¹² 0	⁸ 1
	01	¹ 1	⁵ 0	¹³ 0	⁹ X
	11	³ 0	⁷ 0	¹⁵ 0	¹¹ 0
	10	² 1	⁶ 0	¹⁴ 1	¹⁰ X

PI Q: Do the E-primes cover the entire on set?

- A. Yes
B. No



Reducing Canonical Expressions

1. Prime implicants: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
2. Essential Primes: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
3. Min exp: $\Sigma(\text{Essential Primes}) = \Sigma m(0, 1, 8, 9) + \Sigma m(0, 2, 8, 10) + \Sigma m(10, 14)$

$$f(a,b,c,d) = ?$$

ab		00	01	11	10
cd					
00	0 1	4 0	12 0	8 1	
01	1 1	5 0	13 0	9 X	
11	3 0	7 0	15 0	11 0	
10	2 1	6 0	14 1	10 X	

PI Q: Do the E-primes cover the entire on set?

A. Yes

B. No



Reducing Canonical Expressions

1. Prime implicants: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
2. Essential Primes: $\Sigma m(0, 1, 8, 9)$, $\Sigma m(0, 2, 8, 10)$, $\Sigma m(10, 14)$
3. Min exp: $\Sigma(\text{Essential Primes}) = \Sigma m(0, 1, 8, 9) + \Sigma m(0, 2, 8, 10) + \Sigma m(10, 14)$

$$f(a,b,c,d) = b'c' + b'd' + acd'$$

ab		00	01	11	10
cd	00	0 1	4 0	12 0	8 1
	01	1 1	5 0	13 0	9 X
	11	3 0	7 0	15 0	11 0
	10	2 1	6 0	14 1	10 X

PI Q: Do the E-primes cover the entire on set?

A. Yes

B. No

Another example

Given $F(a,b,c,d) = \sum m(0, 3, 4, 14, 15)$

$D(a,b,c,d) = \sum m(1, 11, 13)$

1. Draw the K-Map

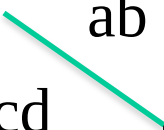


ab		00	01	11	10
cd	00				
	01				
	11				
	10				

Another example

Given $F(a,b,c,d) = \sum m(0, 3, 4, 14, 15)$

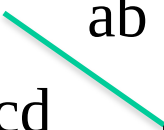
$D(a,b,c,d) = \sum m(1, 11, 13)$



		ab			
		00	01	11	10
cd	00	0 1	4 1	12 0	8 0
	01	1 X	5 0	13 X	9 0
	11	3 1	7 0	15 1	11 X
	10	2 0	6 0	14 1	10 0

Reducing Canonical Expressions

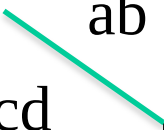
1. Prime implicants: $\Sigma m(0, 4), \Sigma m(0, 1), \Sigma m(1, 3), \Sigma m(3, 11), \Sigma m(14, 15), \Sigma m(11, 15), \Sigma m(13, 15)$
2. Essential Primes: $\Sigma m(0, 4), \Sigma m(14, 15)$



		ab			
		00	01	11	10
cd	00	0 1	4 1	12 0	8 0
	01	1 X	5 0	13 X	9 0
	11	3 1	7 0	15 1	11 X
	10	2 0	6 0	14 1	10 0

Reducing Canonical Expressions

1. Prime implicants: $\Sigma m(0, 4), \Sigma m(0, 1), \Sigma m(1, 3), \Sigma m(3, 11), \Sigma m(14, 15), \Sigma m(11, 15), \Sigma m(13, 15)$
2. Essential Primes: $\Sigma m(0, 4), \Sigma m(14, 15)$
3. Min exp: $\Sigma m(0, 4), \Sigma m(14, 15), (\Sigma m(3, 11) \text{ or } \Sigma m(1, 3))$
4. $f(a,b,c,d) = a'c'd' + abc + b'cd \text{ (or } a'b'd)$



		ab			
		00	01	11	10
cd	00	0 1	4 1	12 0	8 0
	01	1 X	5 0	13 X	9 0
	11	3 1	7 0	15 1	11 X
	10	2 0	6 0	14 1	10 0 ₂₉