

CSE140 Homework 2, Due 11:59PM, Mon 4/23/18

Homework 2 covers the Boolean algebra, combinational logic specification, and implementation. The first problem reviews the definition of Boolean algebra using a multiple-element system to distinguish from a switching function system. For the next two problems, we practice more on the specification, in particular, when the number of input bits is not small. For the last two problems, we practice design minimization using Karnaugh maps in sum of products and product of sums formats.

1 Boolean Algebra

Given a mathematical system $M = (\{1, a, b, c\}, \#, \&)$ where the two operators $\#$ and $\&$ are defined in the following two subtables.

(a)					(b)				
$\&$	1	a	b	c	$\#$	1	a	b	c
1	1	a	b	c	1	1	1	1	1
a	a	a	c	c	a	1	a	1	a
b	b	c	b	c	b	1	1	b	b
c	c	c	c	c	c	1	a	b	c

Table 1: 2-input operators $\#$ and $\&$.

1. Verify whether the system is a Boolean algebra.
2. List the complements of elements 1, a, b, and c if the system is a Boolean algebra.

2 Multiplication Machine

A multiplication machine reads two binary input vectors (a_1, a_0) and (b_1, b_0) and generates a binary number (m_3, m_2, m_1, m_0) that is the product of the two binary inputs. For example, when $(a_1, a_0) = (1, 0)$ and $(b_1, b_0) = (1, 1)$, we have output $(m_3, m_2, m_1, m_0) = (0, 1, 1, 0)$. When $(a_1, a_0) = (1, 1)$ and $(b_1, b_0) = (1, 1)$, we have output $(m_3, m_2, m_1, m_0) = (1, 0, 0, 1)$.

Derive the canonical sum of **minterms** expression of the four output bits.

3 Priority Encoder

A priority encoder reads a vector of eight binary bits $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0)$ and produces a binary number (d_2, d_1, d_0) that presents the highest index of the input bit which is true. For example, when $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) = (0, 0, 1, 0, 1, 0, 0, 0)$, the output $(d_2, d_1, d_0) = (1, 0, 1)$; $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) = (0, 1, 0, 0, 0, 0, 0, 0)$, the output $(d_2, d_1, d_0) = (1, 1, 0)$; and when $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) = (0, 0, 0, 0, 0, 0, 0, 1)$, the output $(d_2, d_1, d_0) = (0, 0, 0)$. However, when none of the input bits asserts a true value, i.e. $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) = (0, 0, 0, 0, 0, 0, 0, 0)$, the output remains to be $(d_2, d_1, d_0) = (0, 0, 0)$.

1. Derive the **sum of products** expression of the three output bits.
2. fill in the following Bluespec code that implements the priority encoder.

```
function Bit#(3) priorityEncoder(Bit#(8) in);
  Integer highestIndex = 0;
  for ( Integer i = 0; i < 8; i = i + 1 ) begin

    if ( in[i] == _____ ) begin

      _____;

    end

  end
  Bit#(3) ret = fromInteger(highestIndex);
  return ret;
endfunction
```

4 Minimal Sum of Products Expression

Implementation from truth table to sum of products expressions.

1. Use Karnaugh map to simplify function

$$f(a, b, c) = \sum m(0, 2, 3, 7) + \sum d(4, 5).$$

List **all** possible minimal two-level sum of products expressions. Show the switching functions. No need for the schematic diagram.

2. Use Karnaugh map to simplify function

$$f(a, b, c, d) = \sum m(0, 4, 5, 7, 8, 10, 13) + \sum d(9, 14, 15).$$

List all possible minimal two-level sum of products expressions. Show the switching functions. No need for the schematic diagram.

3. Use Karnaugh map to simplify function

$$f(a, b, c, d, e) = \sum m(0, 2, 5, 8, 13, 14, 16, 18, 19, 22, 24, 27, 29, 31) + \sum d(6, 9, 11, 15, 25, 30)$$

Write **one** minimal two-level sum of products expression. Show the switching functions. No need for the schematic diagram.

5 Minimal Product of Sums Expression

Implementation from truth table to product of sums expressions.

1. Use Karnaugh map to simplify function

$$f(a, b, c) = \sum m(1, 6) + \sum d(2, 4).$$

List all possible minimal two-level product of sums expressions. Show the switching functions. No need for the schematic diagram.

2. Use Karnaugh map to simplify function

$$f(a, b, c, d) = \sum m(1, 3, 6, 7, 9, 11) + \sum d(8, 12, 14).$$

List all possible minimal two-level product of sums expressions. Show the switching functions. No need for the schematic diagram.

3. Use Karnaugh map to simplify function

$$f(a, b, c, d, e) = \sum m(1, 3, 4, 6, 11, 12, 14, 15, 17, 19, 20, 22, 27, 28, 30) + \sum d(5, 7, 8, 10, 24).$$

Write **one** minimal two-level product of sums expression. Show the switching functions. No need for the schematic diagram.