

CSE140 Midterm2 Solution

Total 100 points (pts)

Question 1. Universal Set of Gates (15pts)

Check if the set in the following list is universal and explain your decision in less than 10 words.
Assuming constants 0 and 1 are available as inputs (15 points).

Correct True and False

- 3pts no matter whether one sentence explanation exists, or not

Incorrect True and False answer, but correct statement

- 1pts

Otherwise, 0pt

1. {AND, OR}

False.

The given set cannot implement NOT gate.

2. {XOR, NOT}.

False.

Either AND or OR gate cannot be built with only NOT and XOR gates.

3. {NOR}.

True.

NOR gate is a universal gate.

4. { $f(x,y)$ }, where $f(x,y) = xy + x'y'$.

FALSE.

$f(x,y) = xy + x'y' = x \text{ XNOR } y$, and XNOR is not a universal gate.

5. { $f(a,b,c)$ }, where $f(a,b,c) = (a' + b + c)(a + b' + c)(a + b + c')$.

True.

NOT: $f(a,0,0) = (a' + 0 + 0)(a + 1 + 0)(a + 0 + 1) = a' \cdot 1 \cdot 1 = a' = \text{NOT } a$.

OR: $f(a,b,1) = (a' + b + 1)(a + b' + 1)(a + b + 0) = 1 \cdot 1 \cdot (a + b) = a + b = a \text{ OR } b$.

AND: AND gate can be built with NOT and OR gates.

Question 2. Other Types of Gates (10pts)

Consider the function $f(x, y)$ where $\oplus$ is an Exclusive OR operator:

$$f(x, y) = (xy) \oplus (x'y') \oplus x \oplus x' \oplus (x + y')$$

Simplify the function into a minimal sum of products expression using switching function techniques, e.g., Shannon's expansion. Show your derivation. (10 points).

If one uses Shannon's expansion,

3 points for correctness of $f(0, y)$ or $f(x, 0)$

3 points for correctness of $f(1, y)$ or $f(x, 1)$

2 points for correctly applying Shannon's expansion

2 points for the final answer

If one does not use Shannon's expansion,

Check the final answer with four cases. (-2pts/each)

Check if minimum expression or not. (-2pts)

x	y	f(x,y)
0	0	1
0	1	1
1	0	0
1	1	1

$$\begin{aligned} f(1, y) &= (1 \cdot y) \oplus (1' \cdot y') \oplus 1 \oplus 0 \oplus (1 + y') \\ &= y \oplus 0 \oplus 1 \oplus 0 \oplus 1 \\ &= y \oplus 0 \\ &= y \end{aligned}$$

// The same number of 0 and 1.

// $0 \oplus A = A$

$$\begin{aligned} f(0, y) &= (0 \cdot y) \oplus (0' \cdot y') \oplus 0 \oplus 1 \oplus (0 + y') \\ &= 0 \oplus y' \oplus 0 \oplus 1 \oplus y' \\ &= 0 \oplus 0 \oplus 1 \oplus y' \oplus y' \\ &= 0 \oplus 0 \oplus 1 \oplus 0 \\ &= 1 \end{aligned}$$

// Commutative

// $A \oplus A = 0$

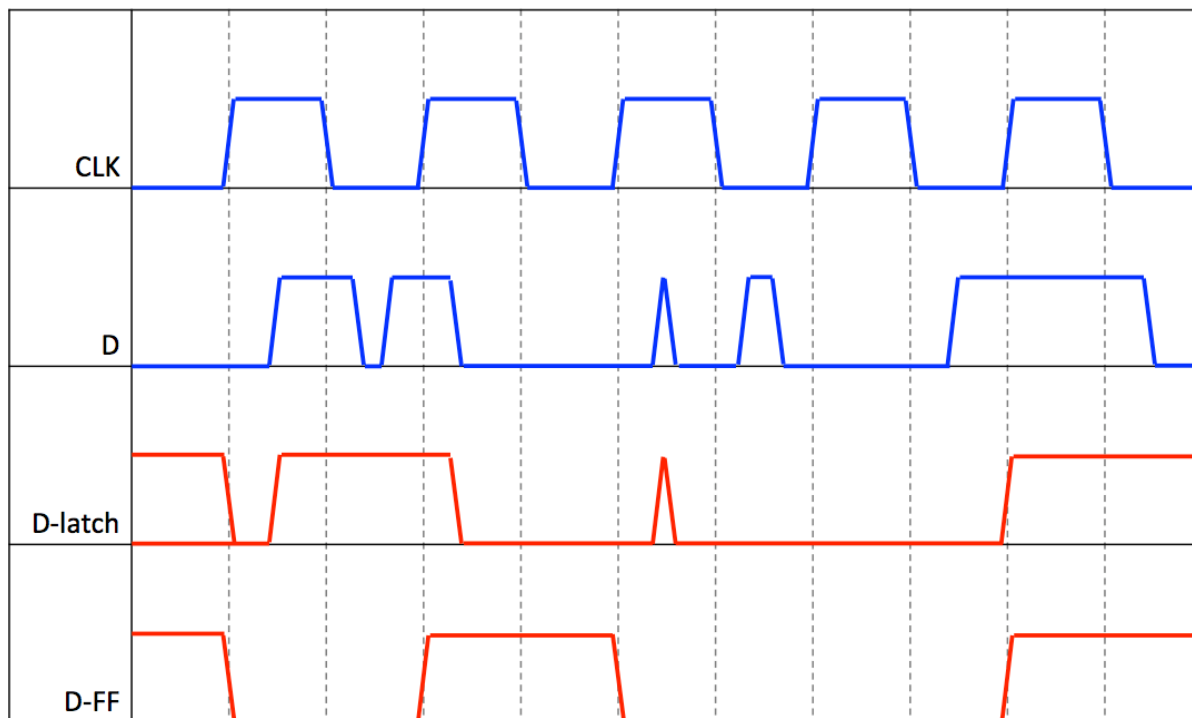
$$\begin{aligned} f(x, y) &= x \cdot (f(1, y)) + x' \cdot (f(0, y)) \\ &= xy + x' \\ &= x' + y \end{aligned}$$

// Consensus Theorem

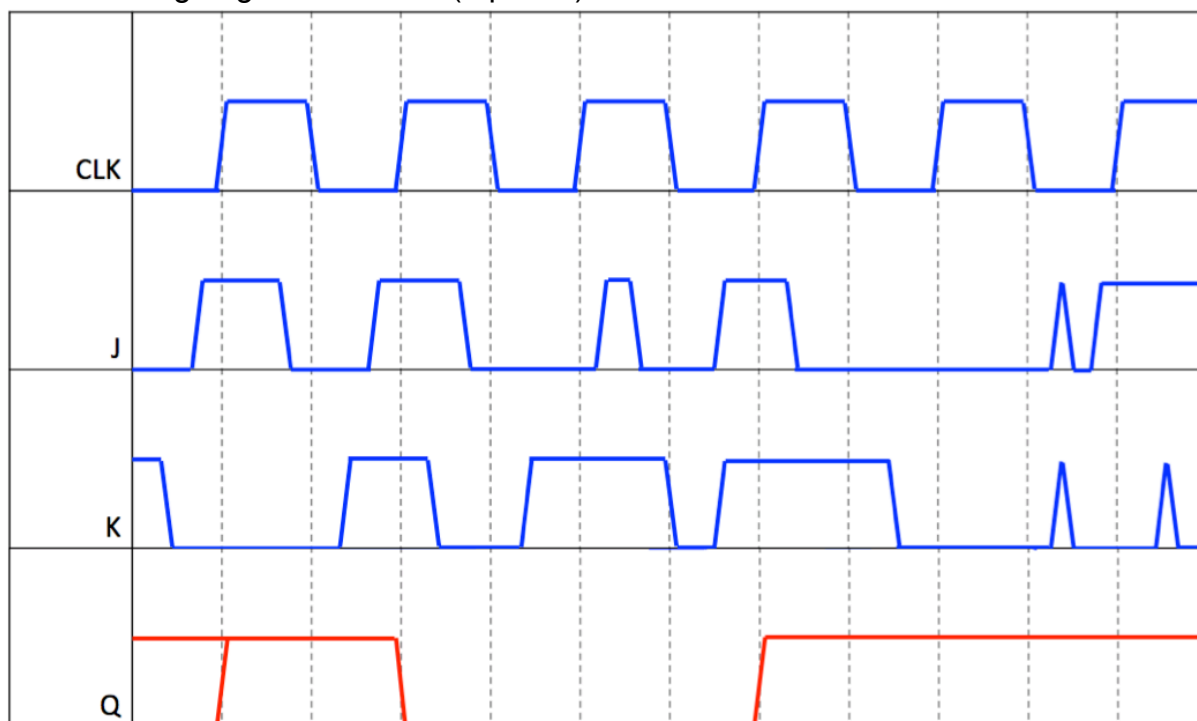
Question 3. Timing Diagram of Latch and Flip-Flop (15pts)

Deduct 1pt per each clock edge and its corresponding output. No more deduction than 5 points.

1. Given the input waveforms shown below, sketch the output, Q, of a D latch and D flip-flop (10 points).



2. Given the input waveforms shown below, sketch the output, Q, of a JK flip-flop. The flip-flop is triggered at the rising edge of the clock (5 points).



Question 4. Finite State Machine: State Diagram (30pts)

A pattern recognizer produces output $y = 1$ whenever its input $x(t-4, t) = 01010$, otherwise output $y = 0$.

1. Implement the system with a Mealy machine using a minimal number of states.

(i) Use a state diagram to demonstrate the design. Label the initial state (the state when the machine is turned on and before any input is read) as S0 (10 points).

Deduct 2 points if not minimal.

Deduct 6 points if not Mealy machine.

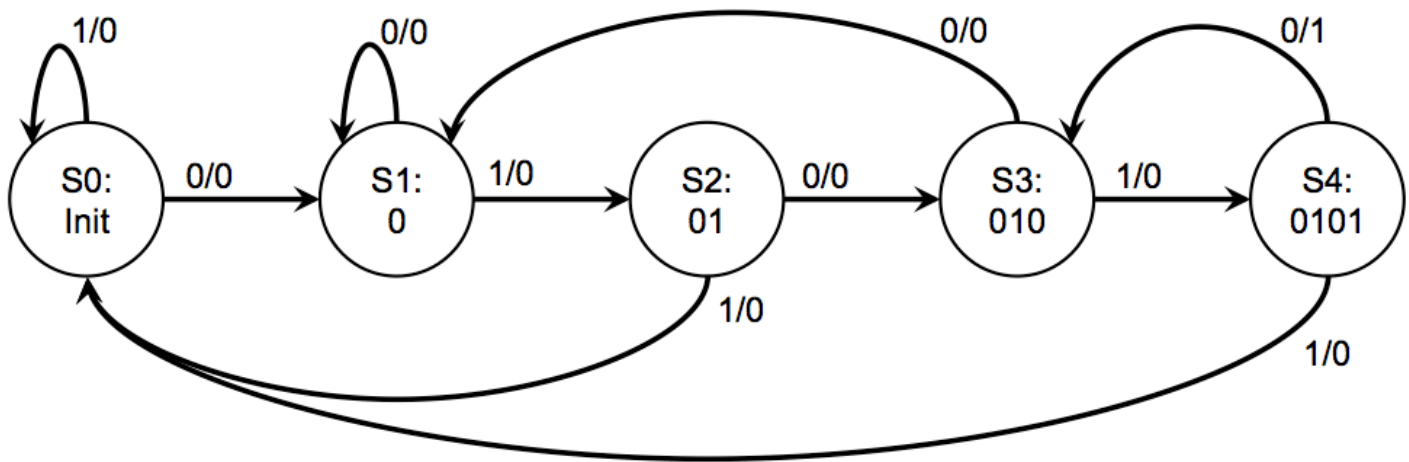
2 Points for FSM's structural correctness.

Apply three patterns, and deduct 2 points for incorrectness of each test pattern.

Input: 0101011 Output: 0000100

Input: 0101000 Output: 0000100

Input: 0101010 Output: 0000101



(ii) Fill the following table as the machine starts at state S0 and process through the given input x from CLK (Clock) sequence = 1 to 10 (5 points).

Correctness of each $y = 0.5$ points

CLK	1	2	3	4	5	6	7	8	9	10
x	0	0	1	0	1	0	1	0	1	0
state	S0	S1	S1	S2	S3	S4	S3	S4	S3	S4
y	0	0	0	0	0	1	0	1	0	1

2. Implement the system with a Moore machine with a minimal number of states.

(i) Use a state diagram to demonstrate the design. Again, label the initial state as S0 (10 points).

Deduct 2 points if not minimal.

Deduct 6 points if not Moore machine.

Deduct 6 points if there are no outputs.

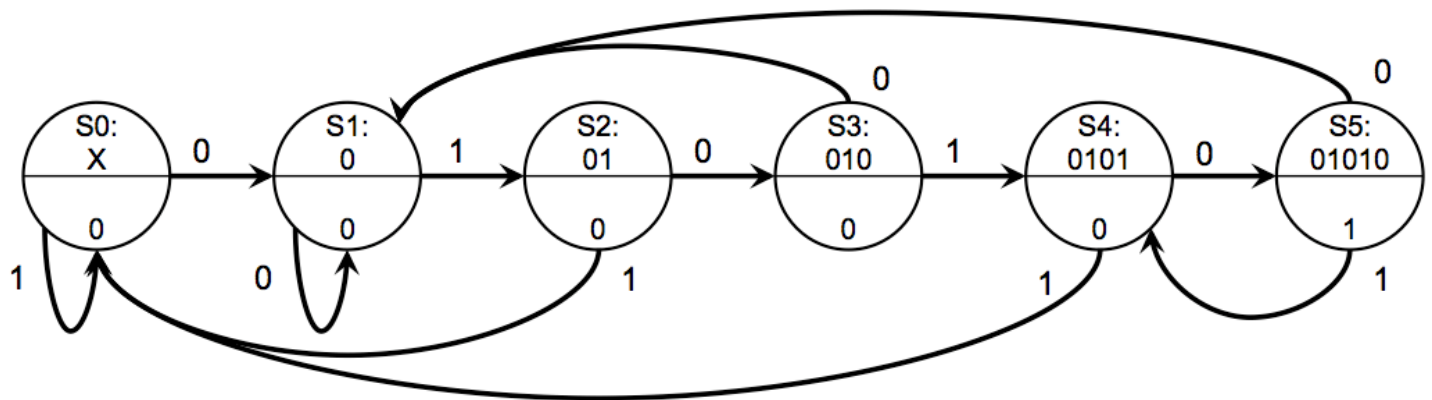
2 Points for FSM's structural correctness.

Apply three patterns, and deduct 2 points for incorrectness of each test pattern.

Input: 0101011 Output: 00000100

Input: 0101000 Output: 00000100

Input: 0101010 Output: 00000101



(ii) Fill the following table as the machine starts at state S0 and process through the given input x from CLK (Clock) sequence = 1 to 10 (5 points).

Correctness of each $y = 0.5$ points

CLK	1	2	3	4	5	6	7	8	9	10
x	1	0	1	0	1	0	1	0	1	1
state	S0	S0	S1	S2	S3	S4	S5	S4	S5	S4
y	0	0	0	0	0	0	1	0	1	0

Question 5. Implementation (30pts)

A state machine has one input $x(t)$ and two-bit state $(Q_1(t), Q_0(t))$. The machine is described by the following state equations.

$$Q_1(t+1) = x'(t)Q_1'(t)Q_0(t),$$

$$Q_0(t+1) = Q_1(t)Q_0(t).$$

Use two **T flip-flops** and a minimal sum-of-products network to implement the machine.

(i). Fill the state table of the system (15 points).

Deduct 1 point each error (up to 15 points)

ID	$Q_1(t)$	$Q_0(t)$	$x(t)$	$Q_1(t+1)$	$Q_0(t+1)$	T_1	T_0
0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0
2	0	1	0	1	0	1	1
3	0	1	1	0	0	0	1
4	1	0	0	0	0	1	0
5	1	0	1	0	0	1	0
6	1	1	0	0	1	1	0
7	1	1	1	0	1	1	0

(ii). Describe the design using a logic diagram. Show your derivation (K maps) (15 points).

5 pts for correctness of K-map (T1)

5 pts for correctness of K-map (T0)

5 pts for correctness of logic diagram

$$T_1 = Q_1(t) + Q_0(t)x'(t)$$

$Q_1(t)Q_0(t)$	00	01	11	10
$x(t)$				
0	0	1	1	1
1	0	0	1	1

$$T_0 = Q_1'(t)Q_0(t)$$

$Q_1(t)Q_0(t)$	00	01	11	10
$x(t)$				
0	0	1	0	0
1	0	1	0	0

