

CSE140 Midterm1 Solution

Total 100 points (pts)

Question 1. True or False (20pts)

Circle your choice of true or false. Use one sentence to explain your choice.

Correct True and False

- 4pts no matter whether one sentence explanation exists, or not

Incorrect True and False answer, but correct statement

- 2pts

Otherwise, 0pt

1. There is no difference between a switching function system and a Boolean algebra system. (4pts)

False. A switching function system considers two-value logic system, but a Boolean function system considers multi-value logic system.

2. In a Boolean algebra system, the complement is unique (i.e. one and only one). (4pts)

True. By uniqueness of complement theorem.

3. For every postulate (e.g., distributive law) in a Boolean algebra system, if we transform the equation by swapping the identity elements 0 and 1 and exchanging the operators OR and AND, the equality remains to be true. (4pts)

True. By duality theorem.

4. An essential prime implicant can have its every on-set element covered by some other prime implicants. (4pts)

False. An essential prime implicant has at least one on-set element not covered by any other prime implicant.

5. For a K-map of n variables, each cell has exactly n neighbors. (4pts)

True. If a Boolean system has exactly n variables, then each cell has exactly n neighbors.

Question 2. Consensus Theorem (15pts)

Prove the following equation using Boolean algebra (by using the laws and theorems of Boolean algebra but not the techniques of switching function system) (15pts).

$$abd + a'bc + bcd = abd + a'bc$$

15pts for correctness to derive RHS

Each incorrect step (including final answer)

- Deduct 5pts
- Keep following student's procedure and grade next steps based on his or her previous step.

If one uses Shannon's expansion

- 10pts if the final answer is correct.
- Otherwise, 0pt

$$\begin{aligned} LHS &= abd + a'bc + bcd \\ &= abd + a'bc + (a + a')bcd && // \text{ Identity} \\ &= abd + a'bc + abcd + a'bcd && // \text{ Distributive} \\ &= abd + abcd + a'bc + a'bcd && // \text{ Commutative} \\ &= abd(1 + c) + a'bc(1 + d) && // \text{ Distributive} \\ &= abd + a'bc && // \text{ Identity} \\ &= RHS \end{aligned}$$

Question 3. Shannon's Expansion (15pts)

Prove the equality of the following switching functions using Shannon's expansion (15pts).

$$(a + b + d)(a' + b + c)(b + c + d + e + f) = (a + b + d)(a' + b + c)$$

15pts for correctness to derive RHS

5pts for correct simplification of $F(0, b, c, d, e, f)$

5pts for correct simplification of $F(1, b, c, d, e, f)$

5pts for correct application of Shannon's expansion and correct final answer, e.g.,

$$F(a, b, c, d, e, f) = (a + F(0, b, c, d, e, f)) \cdot (a' + F(1, b, c, d, e, f))$$

When we apply Shannon's expansion to a ,

$$F(a, b, c, d, e, f) = (a + b + d)(a' + b + c)(b + c + d + e + f)$$

$$F(0, b, c, d, e, f) = (b + d) \cdot 1 \cdot (b + c + d + e + f) = (b + d)(1 + c + e + f) = b + d$$

$$F(1, b, c, d, e, f) = 1 \cdot (b + c) \cdot (b + c + d + e + f) = (b + c)(1 + d + e + f) = b + c$$

$$\begin{aligned} F(a, b, c, d, e, f) &= (a + F(0, b, c, d, e, f)) \cdot (a' + F(1, b, c, d, e, f)) \\ &= (a + b + d)(a' + b + c) \end{aligned}$$

Question 4. Karnaugh Map: Sum of Products Expressions (25pts)

Use Karnaugh map to simplify function

$$f(a, b, c, d) = \sum m(0, 4, 5, 10, 11, 13, 15) + \sum d(2, 8, 9, 12).$$

List all possible minimal sum of products expressions. Show the Boolean expressions. No need for the logic diagram (25pts).

10pts for correctness of K-map

- Deduct 5pts in case of one or two errors
- Deduct 10pts in case of three or more errors
- Do not grade for defining prime and essential prime implicants

15pts for correctness of expression

- Deduct 5pts in case of one or more extra expression(s)
- Deduct 5pts each missing/extra term (up to 10pts deduction in case of extra terms)
- If one has many expressions, grade the closest expression to the solution

ab \ cd	00	01	11	10
00	1	1	×	×
01	0	1	1	×
11	0	0	1	1
10	×	0	0	1

Red circles are essential prime implicants.

$$f(a, b, c, d) = ad + bc' + b'd'$$

Question 5. Karnaugh Map: Product of Sums Expressions (25pts)

Use Karnaugh map to simplify function

$$f(a,b,c) = \sum m(1,6) + \sum d(2,7).$$

List all possible minimal product of sums expressions. Show the Boolean expressions. No need for the logic diagram (25pts).

7pts for correctness of K-map

- Deduct 3pts in case of one or two errors
- Do not grade for defining prime and essential prime implicants

18pts for correctness of expression

- Deduct 3pts in case of one or more extra expression(s)
- Deduct 3pts for each missing expression
- Deduct 1pt each missing/extra term (up to 2pts deduction in case of extra terms)

ab \ c	00	01	11	10
0	0	×	1	0
1	1	0	×	0

There are no essential prime implicants.

$$f(a,b,c) = (a + c)(a + b')(a' + b)$$

$$f(a,b,c) = (a + c)(b' + c')(a' + b)$$

$$f(a,b,c) = (b + c)(a + b')(a' + c')$$

$$f(a,b,c) = (b + c)(a + b')(a' + b)$$

$$f(a,b,c) = (b + c)(b' + c')(a' + c')$$

$$f(a,b,c) = (b + c)(b' + c')(a' + b)$$