

Name _____

CSE140 Midterm 1 with Solution, Tuesday May 2, 2017

Please read the following instructions carefully: If you are unclear about any of the questions on the exam, make the most plausible assumption to answer the question. Instructors and proctors will not answer questions on the exam material. Please write your name on the top of each page. Write the names of the students to your left and right in the space provided.

Name of student to your left: _____

Name of student to your right: _____

	Total Points	Points Awarded
Problem 1	24	
Problem 2	16	
Problem 3	20	
Problem 4	20	
Problem 5	20	
Total	100	

All the Best!

1 True or False

Circle your choice of true or false. Use one sentence to explain your choice. (24 pts)

1. The duality of Boolean algebra only applies to the statement that can be proven by the laws/theorems of Boolean algebra. (4pts)

True False

4 points total, 2 points for wrong choice, 2 points for no or wrong explanation.

The definition of the Boolean algebra.

2. For an OR gate, a "0" at input dominates the output (blocks the message of other inputs). (4pts)

True False

4 points total, 2 points for wrong choice, 2 points for no or wrong explanation.

For an OR gate, a "0" passes the input, so it does not dominate the output.

3. Shannon's expansion can be proven by the laws and theorems of Boolean algebra. (4pts)

True False

4 points total, 2 points for wrong choice, 2 points for no or wrong explanation. If anyone chose True but mentioned switch function in the explanation, deduct 2 points only

Shannon's expansion can be proved by the laws and theorems of switch function because it only used two variable 0 and 1. Boolean algebra is too general.

4. {NAND} is a universal set of gates. (4pts)

True False

4 points total, 2 points for wrong choice, 2 points for no or wrong explanation.

a NAND 1 = a' gives a "NOT".

"NOT" and NAND combination can give an "OR" or "AND".

5. $\{f(a,b,c) = a'b' + b'c' + a'c'\}$ is a universal set of gates. (4pts)

True False

4 points total, 2 points for wrong choice, 2 points for no or wrong explanation.

$f(a,0,1) = a' \cdot 1 + 1 \cdot 0 + a' \cdot 0 = a'$ NOT
 $f(a',b',1) = ab + b \cdot 0 + a \cdot 0 = ab$ AND
NOT and AND gives an OR

6. Let $f(a,b,c)$ be a majority voting machine, i.e. f is true when more than half of the input bits are 1, then $\{f(a,b,c)\}$ is a universal set of gates. **(4pts)**

True **False**

4 points total, 2 points for wrong choice, 2 points for no or wrong explanation.

$f(a,0,0) = \text{false}$, $f(a,1,1) = \text{true}$.
 $f(a,0,1)$ or $f(a,1,0) = a$, cannot make a "NOT" in this case.

2 Consensus Theorem

Prove the following equation using **Boolean algebra** (using the laws and theorems of Boolean algebra but not the techniques of switching function system). Write down explicitly the laws used in the derivation(16pts).

8 points total. Deduct 8 points if the student use Shannon's expansion.

1. $abd + cd'e + abce = abd + cd'e.$

LHS = $abd + cd'e + abce$. 1 (identity) 0.5 point for equation, 0.5 point for law
 $abd + cd'e + abce(d + d')$ (complement) 0.5 point for equation, 0.5 point for law; if the student did not write previous one, it is fine, then mark this line: 1 point for equation, 1 point for law
 $= abd + cd'e + abced + abced'$ (distributive) 0.5 point for equation, 0.5 point for law
 $= abd + abcde + cd'e + abcde$ (commutative) 0.5 point for equation, 0.5 point for law, if the student combine this line and previous line with distributive and commutative, then 1 point for equation, 1 point for two laws, deduct 0.5 for missing one law
 $= abd(1 + ce) + ced'(1 + abc)$ (distributive) 1 point for equation, 1 point for two laws
 $= abd + ced' = RHS$ (absorbency or identity) 1 point for equation, 1 point for two laws

8 points total. Deduct 8 points if the student use Shannon's expansion.

2. $(a + d + e)(b' + c + d')(a + b' + c + e) = (a + d + e)(b' + c + d')$

LHS = $(a + d + e)(b' + c + d')(a + b' + c + e + 1')$ (identify) 0.5 point for equation, 0.5 point for law
 $= (a + d + e)(b' + c + d')(a + b' + c + e + dd')$ (complement) 0.5 point for equation, 0.5 point for law; if the student did not write previous one, it is fine, then mark this line: 1 point for equation, 1 point for law
 $= (a + d + e)(b' + c + d')(a + b' + c + e + d)(a + b' + c + e + d')$ (distributive) 0.5 point for equation, 0.5 point for law
 $= (a + d + e)(a + b' + c + e + d)(b' + c + d')(a + b' + c + e + d')$ (commutative) 0.5 point for equation, 0.5 point for law, if the student combine this line and previous line with distributive and commutative, then 1 point for equation, 1 point for two laws, deduct 0.5 for missing one law

$= (a + d + e + 0)(a + b' + c + e + d)(b' + c + d' + 0)(a + b' + c + e + d')$ (identity) 0.5 point for equation, 0.5 point for law,
 $= (a + d + e) + 0(c + b')(b' + c + d) + 0(a + e)$ (distributive) 0.5 point for equation, 0.5 point for law,
 $= (a + d + e)(b' + c + d) = RHS$ (absorbency or identity) 1 point for equation, 1 point for two laws

3 Karnaugh Map: Sum of Products Expressions

Use Karnaugh map to simplify function

$$f(a,b,c,d) = \sum m(0,2,3,5,8,11,13) + \sum d(4,7,9,10,15).$$

List all possible minimal sum of products expressions. Write the switching function. No need for the logic diagram (20pts).

20 points total.

K-map

cd\ab	~~ab=00~~	01	11	~~10~~
cd=00	0 1	4 X	12 0	8 1
cd=01	1 0	5 1	13 1	9 X
cd=11	3 1	7 X	15 X	11 1
cd=10	2 1	6 0	14 0	10 X

K-map

ab\cd	~~cd=00~~	01	11	~~10~~
ab=00	0 1	1 0	3 1	2 1
ab=01	4 X	5 1	7 X	6 0
ab=11	12 0	13 1	15 X	14 0
ab=10	8 1	9 X	11 1	10 X

if any students just write answers equation without any processes, only give 2 points for each right equation (2 equations total), put a question mark there.

5 points for just drawing the k-map

(Essential Prime) $\sum m(0,2,8,10) b'd'$, $\sum m(5,7,13,15) bd$,

$\sum m(3,7,11,15) cd$, $\sum m(2,3,10,11) b'c$, 5 points for listed all the primes above or more if the student listed none of the equations is right.

There are 2 equations for this question, deduct 5 points for missing or equations.

K-map

cd\ab	~~ab=00~~	01	11	~~10~~
cd=00	0 1	4 X	12 0	8 1
cd=01	1 0	5 1	13 1	9 X
cd=11	3 1	7 X	15 X	11 1
cd=10	2 1	6 0	14 0	10 X

K-map				
ab\cd	~~cd=00~~	01	11	10
ab=00	0 1	1 0	3 1	2 1
ab=01	4 X	5 1	7 X	6 0
ab=11	12 0	13 1	15 X	14 0
ab=10	8 1	9 X	11 1	10 X

$$f(a,b,c,d) = \sum m(0,2,8,10) + \sum m(5,7,13,15) + \sum m(3,7,11,15)$$

$$= b'd' + bd + cd$$

deduct 5 points if missing this equation or the listing equation is wrong, only min equations with numbers is fine.//

K-map				
cd\ab	~~ab=00~~	01	11	10
cd=00	0 1	4 X	12 0	8 1
cd=01	1 0	5 1	13 1	9 X
cd=11	3 1	7 X	15 X	11 1
cd=10	2 1	6 0	14 0	10 X

K-map				
ab\cd	~~cd=00~~	01	11	10
ab=00	0 1	1 0	3 1	2 1
ab=01	4 X	5 1	7 X	6 0
ab=11	12 0	13 1	15 X	14 0
ab=10	8 1	9 X	11 1	10 X

$$f(a,b,c,d) = \sum m(0,2,8,10) + \sum m(5,7,13,15) + \sum m(2,3,10,11)$$

$$= b'd' + bd + b'c$$

deduct 5 points if missing this equation or the listing equation is wrong, only min equations with numbers is fine.//

4 Karnaugh Map: Product of Sums Expressions

Use Karnaugh map to simplify function

$$f(a,b,c) = \sum m(0,3,7) + \sum d(5).$$

List all possible minimal product of sums expressions. Write the switching function. No need for the logic diagram (20pts).

20 points total.

K-map				
ab\c	00	01	11	10
c=0	0 1	2 0	6 0	4 0
c=1	1 0	3 1	7 1	5 X

if any students just write answers equation without any processes, only give 2 points for each right equation (2 equations total), put a question mark there.

5 points for just drawing the k-map

(Essential Prime) $\prod M(2,6)$ $b'+c$, $\prod M(1,5)$ $b+c'$,

$\prod M(4,5)$ $a'+b$, $\prod M(4,6)$ $a'+c$, 5 points for listed all the primes above or more if the student listed none of the equations is right.

There are 2 equations for this question, deduct 5 points for missing or equations.

K-map				
ab\c	00	01	11	10
c=0	0 1	2 0	6 0	4 0
c=1	1 0	3 1	7 1	5 X

$$f(a,b,c,d) = \prod M(2,6) \prod M(1,5) \prod M(4,5)$$

$$=(b'+c)(b+c')(a'+b)$$

deduct 5 points if missing this equation or the listing equation is wrong, only min equations with numbers is fine.//

K-map				
ab\c	00	01	11	10
c=0	0 1	2 0	6 0	4 0
c=1	1 0	3 1	7 1	5 X

$$f(a,b,c,d) = \prod M(2,6) \prod M(1,5) \prod M(4,6)$$

$$=(b'+c)(b+c')(a'+c)$$

deduct 5 points if missing this equation or the listing equation is wrong, only min equations with numbers is fine.//

5 Other Types of Gates

Consider the function $f(x,y)$ where $\oplus$ is an Exclusive OR operator:

$$f(x,y) = (x+y) \oplus (x'+y') \oplus xy \oplus x'y' \oplus (x+y').$$

Simplify the function into a minimal sum of products expression using switching function techniques e.g. Shannon's expansion. Show your derivation (20pts).

20 points total. Deduct 15 points if the student only list the equation without any process.

5 points for just listing the idea to attempt to solve the question by shannon's equation: the full equation of $f(x,y) = \dots$, $f(1,y) = \dots$ or $f(x,1) = \dots$, $f(0,y) = \dots$ or $f(x,0) = \dots$

If the student tries to use the other methods but did not get the right answer, ask TA about the partial credit.

$$f(x,y) = xf(1,y) + x'f(0,y) \text{ or } f(x,y) = (x+f(0,y))(x'+f(1,y))$$

The two equations below can be combined in one equation.

$$\begin{aligned} f(1,y) &= (1+y) \oplus (0+y') \oplus 1y \oplus 0y' \oplus (1+y') \\ &= 1 \oplus y' \oplus y \oplus 0 \oplus 1 \\ &= y \oplus y \oplus 0 \oplus 1 \\ &= 0 \oplus 0 \oplus 1 \\ &= 0 \oplus 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} f(0,y) &= (0+y) \oplus (1+y') \oplus 0y \oplus 1y' \oplus (0+y') \\ &= y \oplus 1 \oplus 0 \oplus y' \oplus y' \\ &= y' \oplus 0 \oplus y' \oplus y' \\ &= y' \oplus y' \oplus y' \\ &= 0 \oplus y' \\ &= y' \end{aligned}$$

$$\begin{aligned} f(x,y) &= xf(1,y) + x'f(0,y) \\ &= x + x'y' \\ &= (x+x')(x+y') \\ &= 1(x+y') = x+y' \end{aligned}$$

$$\begin{aligned} f(x,y) &= (x+f(0,y))(x'+f(1,y)) \\ &= (x+y')(x'+1) \\ &= x+y' \end{aligned}$$