

Solutions for Midterm1B are provided.

1 True or False

Grading: For each question, 4 points for correct answer; 2 points for wrong answer (wrong choice but reasonable explanation); 0 for making no attempts / wrong choice without explanation

- (1) For Boolean algebra, given an equation $xyz = xz$, we can deduce that $y = 1$.
False. $y = x + z + W$ is a valid choice.
- (2) For Boolean algebra, the statement $a + a = a$ is true and can be proven by the laws of Boolean algebra.
True. $a + a = a$ can be proven true by distributive law and identity law of Boolean algebra.
- (3) For a switching function of three inputs, i.e. $f(a, b, c)$ and its corresponding K-map, the number of prime implicants can be 5 or more.
True. An example is $\Sigma m(0, 1, 2, 5, 6, 7)$, in which we can find 6 prime implicants.
- (4) {AND,NOT} is a universal set of gates.
True. One can use DeMorgan's law to implement OR from {AND,NOT}, so the {AND,NOT} is a universal set.
- (5) {NOT,XOR} is a universal set of gates.
False. A universal set should be able to implement {AND,OR,NOT}; however, AND is unable to be constructed by {NOT,XOR}.
- (6) $\{f(a, b, c) = a'b' + b'c' + a'c'\}$ is a universal set of gates.
True. f can implement NOT with $f(a, 0, 1) = a'$, and OR with $f(f(a, b, 1), 0, 1)$; given {OR,NOT} is a universal set, f is a universal set of gates.

2 Consensus Theorem Application

Grading:

- no points taken for writing out 0/1 in 2.1/2.2
- 0.5 point taken for any additional wrong term(s)

2.1

- (1) 2 points for each term
- (2) 1 point for each term, no points for blank
- (3) 2 points for each term
- (4) 0.5 point for each term, no points for blank

2.2

- (1) 2 points for each term
- (2) 1 point for each term, no points for blank
- (3) 1 point for each term, no points for blank
- (4) 0.4 point for each term, no points for blank

2.1

- (1) $a'b + ac' + bc'$, Consensus(es): bc'
- (2) $abc'd' + acd'e' + a'be$, Consensus(es): $abd'e', bc'd'e$
- (3) $abd + a'cd + bfg$, Consensus(es): bcd
- (4) $a'b'cd + a'd'e' + bde'$, Consensus(es): $a'b'ce', a'cde', a'be'$

2.2

- (1) $(a + b')(a' + c')(b' + c')$, Consensus(es): $(b' + c')$
- (2) $(a + c' + e)(a + c + e)(a' + c + d)$, Consensus(es): $(a + e), (c + d + e)$
- (3) $(a + b + c + d')(a + c' + d' + e)(a + c' + d)$, Consensus(es): $(a + b + d' + e), (a + c' + e)$
- (4) $(a + b + c)(a' + d)(a + b' + d)(a' + c + d')$, Consensus(es): $(b + c + d), (a + c + d), (b + c + d'), (b' + d), (a' + c)$

3 Karnaugh Map: Sum of Products Expressions

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Grading:

- 20 points for correct answers
- 16-19 points for partially correct switching functions, 1 point taken for each incorrect/missing/extra switching function
- 12 points for wrong K-map, and wrong switching functions (correct derivation)
- 10 points for correct K-map, but no switching functions
- 10 points for correct switching functions without K-map
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

$$f(a, b, c, d) = \Sigma m(0, 1, 2, 5, 7, 9, 12, 14) + \Sigma d(6, 4, 11).$$

The Karnaugh map:

First, we find the essential prime implicants: $\Sigma m(4, 5, 6, 7)$, $\Sigma m(0, 2, 4, 6)$, $\Sigma m(4, 6, 12, 14)$ Then, for the 1s left, we find all possible prime implicants.

For $m(9)$, we have

- $\Sigma m(9, 11)$
- $\Sigma m(1, 9)$

For $m(1)$, we have

- $\Sigma m(0, 1, 4, 5)$

To construct the minimal sum of products expression, $\Sigma m(1, 9)$ is the only choice.

$$(1) f = bd' + a'd' + a'b + b'c'd$$

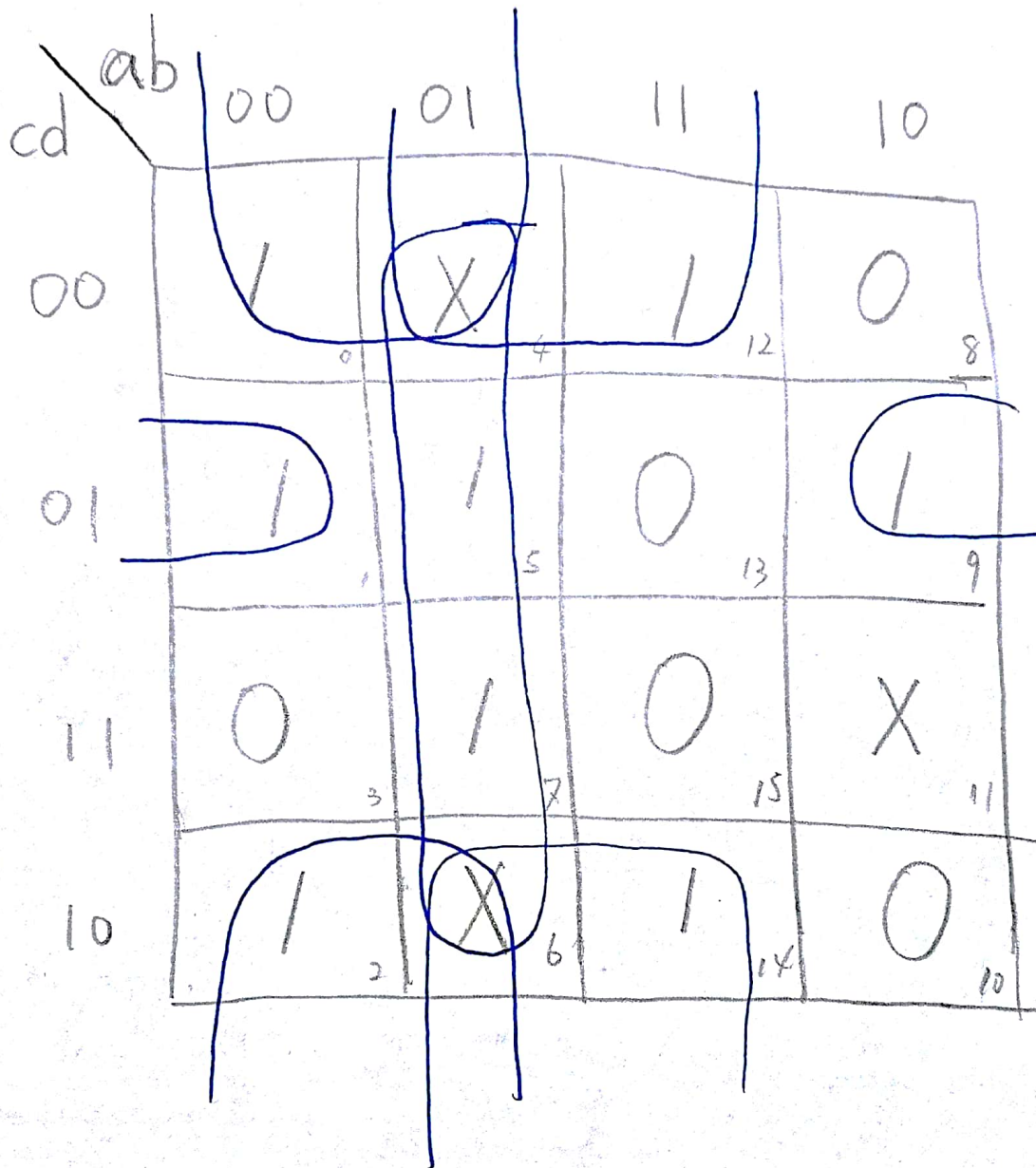


FIGURE 1. Karnaugh map

4 Karnaugh Map: Product of Sums Expressions

Grading:

- 20 points for correct answers
- 16,18 points for partial correct switching functions, 2 points taken for each wrong switching function (half of the points deducted for correct answer in wrong format)
- 15 points for correct K-map, essential/ prime implicants (correct/wrong), but no switching functions

- 12 points for wrong K-map, and wrong switching functions (correct derivations)
- 10 points for correct K-map, but no switching functions
- 5-9 points for partially correct K-map without switching functions, 1 point taken for each wrong slot,
- 10 points for correct switching functions without K-map
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

$$f(a, b, c) = \sum m(1, 4, 6) + \sum d(3, 5)$$

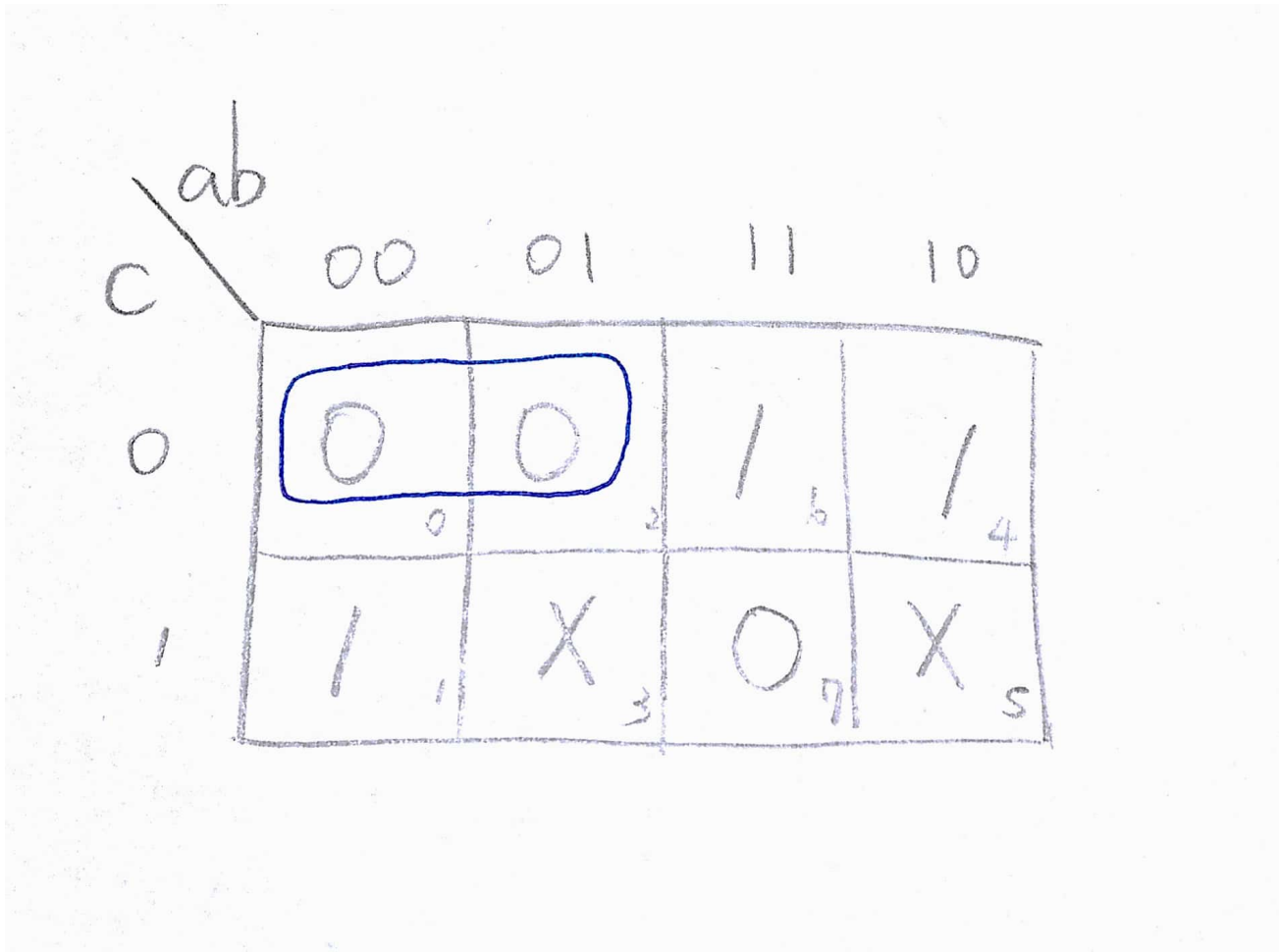


FIGURE 2. Karnaugh map

First, we find the essential prime implicants: $\Pi M(0, 2)$. Then, for the 0s left, we find all possible prime implicants.

For $M(7)$, we have

- $\Pi M(3, 7)$
- $\Pi M(5, 7)$

Thus, we arrive at 2 possible minimal product of sums expressions.

(1) $f = (a + c)(b' + c')$

(2) $f = (a + c)(a' + c')$

5 Other Types of Gates

Grading:

- 20 points for correct answers
- 3 points taken out for wrong or unsimplified answer; for using Shannon's expansion, 3 points for each correct term in (3) by applying correct methodology, and 3 points for simplifying each term correctly.
- if setting up incorrect Shannon's equation (i.e swapped 0 and 1), 5 points are deducted
- 10 points for correct answer, but no derivations / process
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

$$f(x, y) = (x' + y) \oplus (x + y') \oplus xy \oplus x'y' \oplus x'$$

By Shannon's expansion, we arrive at

$$\begin{aligned}(1) \quad f(x, y) &= xf(1, y) + x'f(0, y) \\(2) \quad &= x((0 + y) \oplus (1 + y') \oplus 1y \oplus 0y' \oplus 0) + x'((1 + y) \oplus (0 + y') \oplus 0y \oplus 1y' \oplus 1) \\(3) \quad &= x(y \oplus 1 \oplus y \oplus 0 \oplus 0) + x'(1 \oplus y' \oplus 0 \oplus y' \oplus 1)\end{aligned}$$

The properties of XOR that we can use are:

- Associative
- Commutative
- $x \oplus x = 0$
- $x \oplus x' = 1$
- $x \oplus 0 = x$
- $x \oplus 1 = x'$

Eventually, we have

$$(4) \quad f(x, y) = x(1) + x'(0)$$

$$(5) \quad = x$$