

Solutions for Midterm1A are provided.

1 True or False

Grading: For each question, 4 points for correct answer; 2 points for wrong answer (wrong choice but reasonable explanation); 0 for making no attempts / wrong choice without explanation

- (1) For Boolean algebra, given an equation $a + b = a$, we can deduce that $b = 0$.
False. $b = a + X$ is a valid choice.
- (2) In Boolean algebra, given the theorem that $a + 1 = 1$, we can derive from duality that $a' \cdot 0 = 0$ but not $a \cdot 0 = 0$.
False. To perform duality, we swap $(+, \cdot)$ operators, and interchange all $(1, 0)$, and eventually arrive at $a \cdot 0 = 0$.
- (3) For a four input switching function, i.e. $f(a, b, c, d)$ and its corresponding K-map, the number of essential prime implicants may not be larger than 8.
True. There are 16 slots on the K-map, and at most 8 interleaves.
- (4) Shannon's expansion has $f(x, y, z) = ((x' + f(0, y, z)) \cdot (x + f(1, y, z)))$.
False. The correct form of Shannon's expansion is $f(x, y, z) = ((x' + f(1, y, z)) \cdot (x + f(0, y, z)))$.
- (5) $\{\text{AND}, \text{OR}\}$ is a universal set of gates.
False. A universal set should be able to implement $\{\text{AND}, \text{OR}, \text{NOT}\}$; however, Not is unable to be constructed by $\{\text{AND}, \text{OR}\}$.
- (6) $\{f(a, b) = a' + b'\}$ is a universal set of gates.
True. $f(a, 1)$ is NOT; $f(a', b') = a + b$ is OR; and $\{\text{OR}, \text{NOT}\}$ is a universal set.

2 Consensus Theorem Application

Grading:

- no points taken for writing out 0/1 in 2.1/2.2
- 0.5 point taken for any additional wrong term(s)

2.1

- (1) 2 points for each term
- (2) 2 points for each term
- (3) 2 points for each term
- (4) 0.5 point for each term, no points for blank

2.2

- (1) 2 points for each term
- (2) 2 points for each term, no points for blank
- (3) 1 point for each term, no points for blank
- (4) 0.5 point for each term, no points for blank

2.1

- (1) $a'b' + a'c + bc$, Consensus(es): $a'c$

- (2) $ab'cd + abde' + a'c'e$, Consensus(es): $acde'$
- (3) $abcd + a'd + efg$, Consensus(es): bcd
- (4) $ab'cd + a'de + bde'$, Consensus(es): $b'cde, acde', a'bd$

2.2

- (1) $(a' + b')(a' + c')(b + c')$, Consensus(es): $(a' + c')$
- (2) $(a + b' + c')(a + c + e)(a' + c + d)$, Consensus(es): $(a + b' + e), (c + d + e)$
- (3) $(a + b' + c + d)(a + c + d + e)(a' + c + d)$, Consensus(es): $(b' + c + d), (c + d + e)$
- (4) $(a + b + c)(a + d')(a + b' + e)(a' + c + d)$, Consensus(es): $(a + c + e), (b + c + d), (b' + c + d + e)$

3 Karnaugh Map: Sum of Products Expressions

Grading:

- 20 points for correct answers
- 14-19 points for partial correct switching functions, 1 point taken for each incorrect/missing/extra switching function
- 12 points for wrong K-map, and wrong switching functions (correct derivation)
- 10 points for correct K-map, but no switching functions
- 10 points for correct switching functions without K-map
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

$$f(a, b, c, d) = \Sigma m(0, 1, 3, 5, 7, 8, 12, 14, 15) + \Sigma d(6, 10, 11).$$

The Karnaugh map:

First, we find the essential prime implicants: $\Sigma m(1, 3, 5, 7), \Sigma m(8, 10, 12, 14)$. Then, for the 1s left, we find all possible prime implicants.

For $m(0)$, we have

- $\Sigma m(0, 1)$
- $\Sigma m(0, 8)$

For $m(15)$, we have

- $\Sigma m(3, 7, 11, 15)$
- $\Sigma m(6, 7, 14, 15)$
- $\Sigma m(10, 11, 14, 15)$

Thus, we arrive at $3 \times 2 = 6$ possible minimal sum of products expressions.

- (1) $f = a'd + ad' + a'b'c' + cd$
- (2) $f = a'd + ad' + b'c'd' + cd$
- (3) $f = a'd + ad' + a'b'c' + bc$
- (4) $f = a'd + ad' + b'c'd' + bc$
- (5) $f = a'd + ad' + a'b'c' + ac$
- (6) $f = a'd + ad' + b'c'd' + ac$

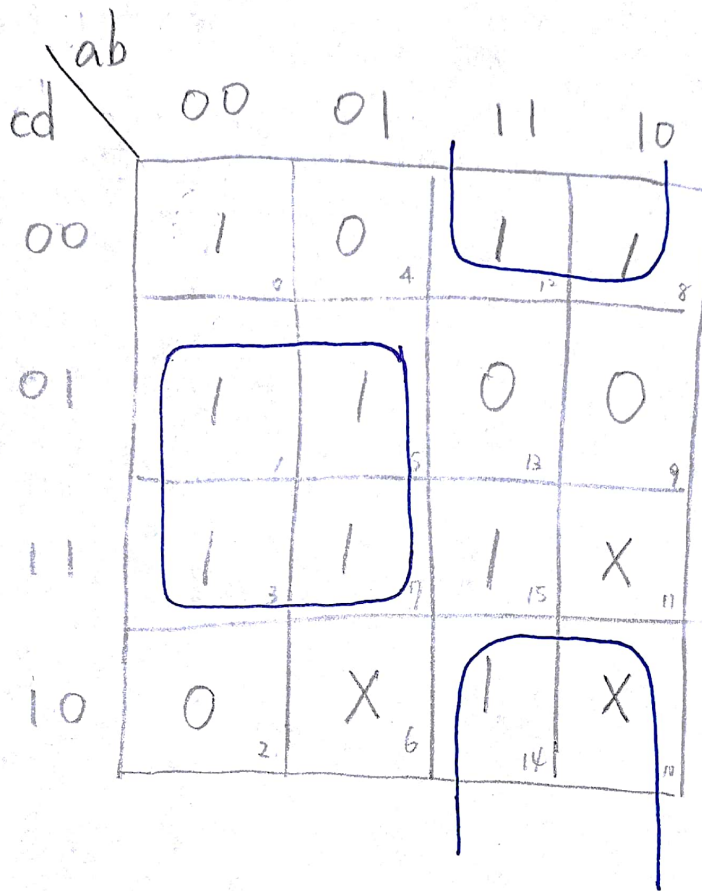


FIGURE 1. Karnaugh map

4 Karnaugh Map: Product of Sums Expressions

Grading:

- 20 points for correct answers
- 16,18 points for partial correct switching functions, 2 points taken for each wrong switching function (half of the points deducted for correct answer in wrong format)
- 15 points for correct K-map, essential/ prime implicants (correct/wrong), but no switching functions
- 12 points for wrong K-map, and wrong switching functions (correct derivations)
- 10 points for correct K-map, but no switching functions
- 5-9 points for partially correct K-map without switching functions, 1 point taken for each wrong slot,

- 10 points for correct switching functions without K-map
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

$$f(a, b, c) = \Sigma m(0, 2, 5) + \Sigma d(3, 6)$$

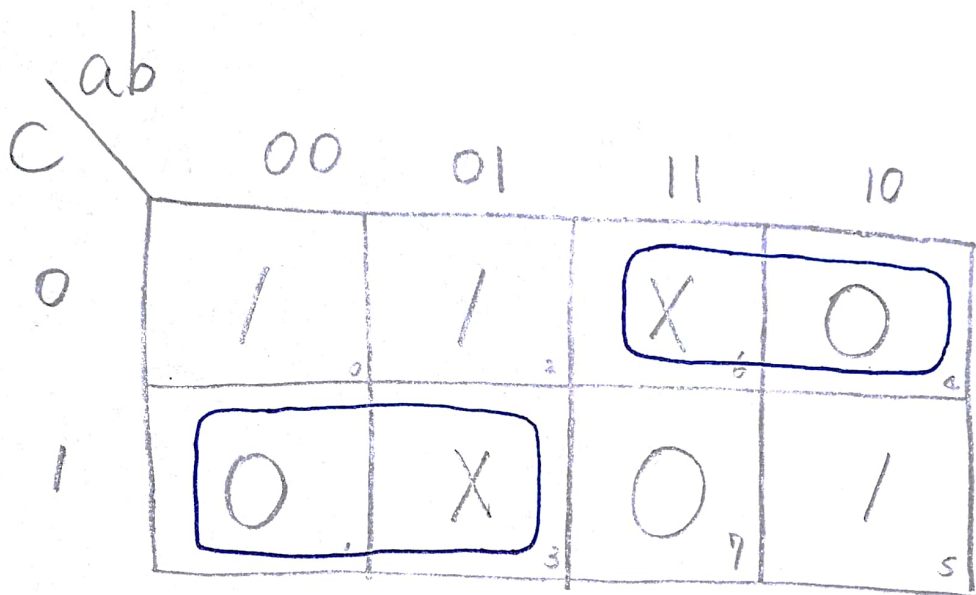


FIGURE 2. Karnaugh map

First, we find the essential prime implicants: $\Pi M(1, 3), \Pi M(4, 6)$. Then, for the 0s left, we find all possible prime implicants.

For $M(7)$, we have

- $\Pi M(3, 7)$
- $\Pi M(6, 7)$

Thus, we arrive at 2 possible minimal product of sums expressions.

- (1) $f = (a' + c)(a + c')(b' + c')$
- (2) $f = (a' + c)(a + c')(a' + b')$

5 Other Types of Gates

Grading:

- 20 points for correct answers
- 3 points taken out for wrong or unsimplified answer; for using Shannon's expansion, 3 points for each correct term in (3) by applying correct methodology, and 3 points for simplifying each term correctly.
- if setting up incorrect Shannon's equation (i.e swapped 0 and 1), 5 points are deducted
- 10 points for correct answer, but no derivations / process
- 5 points for writing out the correct methodologies without any implementation
- 2 points for writing out the wrong methodologies without any implementation
- 0 point for making no attempt

$$f(x, y) = x'y \oplus xy' \oplus (x + y) \oplus (x' + y') \oplus y'$$

By Shannon's expansion, we arrive at

$$\begin{aligned} (1) \quad f(x, y) &= xf(1, y) + x'f(0, y) \\ (2) \quad &= x(0y \oplus 1y' \oplus (1 + y) \oplus (0 + y') \oplus y') + x'(1y \oplus 0y' \oplus (0 + y) \oplus (1 + y') \oplus y') \\ (3) \quad &= x(0 \oplus y' \oplus 1 \oplus y' \oplus y') + x'(y \oplus 0 \oplus y \oplus 1 \oplus y') \end{aligned}$$

The properties of XOR that we can use are:

- Associative
- Commutative
- $x \oplus x = 0$
- $x \oplus x' = 1$
- $x \oplus 0 = x$
- $x \oplus 1 = x'$

Eventually, we have

$$\begin{aligned} (4) \quad f(x, y) &= x(y) + x'(y) \\ (5) \quad &= y \end{aligned}$$