

CSE140 HW2 Preparation

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Q1: Boolean Algebra

- Boolean Algebra $\neq$ Switching Algebra
- What makes a Boolean algebra?

A multiple valued logic $\{a_0, a_1, \dots, a_n\}$ defined over two operations $\{\textcolor{red}{op}_1, \textcolor{blue}{op}_2\}$ that satisfies the following properties:

✓ Associative

- for any elements (x, y, z) from $\{a_0, a_1, \dots, a_n\}$

$$\textcolor{red}{(x \text{ } op_1 \text{ } y) op_1 \text{ } z = x \text{ } op_1 \text{ } (y \text{ } op_1 \text{ } z)}$$

$$\textcolor{blue}{(x \text{ } op_2 \text{ } y) op_2 \text{ } z = x \text{ } op_2 \text{ } (y \text{ } op_2 \text{ } z)}$$

Q1: Boolean Algebra

✓ Commutative

- for each pair of elements (x, y) from $\{a_0, a_1, \dots, a_n\}$

$$x \text{ op}_1 y = y \text{ op}_1 x$$

$$x \text{ op}_2 y = y \text{ op}_2 x$$

✓ Distributive

- for any elements (x, y, z) from $\{a_0, a_1, \dots, a_n\}$

$$x \text{ op}_1 (y \text{ op}_2 z) = (x \text{ op}_1 y) \text{ op}_2 (x \text{ op}_1 z)$$

$$x \text{ op}_2 (y \text{ op}_1 z) = (x \text{ op}_2 y) \text{ op}_1 (x \text{ op}_2 z)$$

Q1: Boolean Algebra

✓ Commutative

- for each pair of elements (x, y) from $\{a_0$

$$x \text{ op}_1 y = y \text{ op}_1 x$$

$$x \text{ op}_2 y = y \text{ op}_2 x$$

✓ Distributive

- for any elements (x, y, z) from $\{a_0, a_1, \dots, a_n\}$

$$x \text{ op}_1 (y \text{ op}_2 z) = (x \text{ op}_1 y) \text{ op}_2 (x \text{ op}_1 z)$$

$$x \text{ op}_2 (y \text{ op}_1 z) = (x \text{ op}_2 y) \text{ op}_1 (x \text{ op}_2 z)$$

Example

(a)					(b)				
&	1	a	b	c	#	1	a	b	c
1	1	a	b	c	1	1	1	1	1
a	a	a	c	c	a	1	a	1	a
b	b	c	b	c	b	1	1	b	b
c	c	c	c	c	c	1	a	b	c

Table 1: 2-input operators # and &.

Choose elements
(a, b) on operator &
 $a \& b = c = b \& a$

Q1: Boolean Algebra

✓ Commutative

- for each pair of elements (x, y) from $\{a_i$

$$x \text{ op}_1 y = y \text{ op}_1 x$$

$$x \text{ op}_2 y = y \text{ op}_2 x$$

✓ Distributive

- for any elements (x, y, z) from $\{a_0, a_1, \dots, a_n\}$

$$x \text{ op}_1 (y \text{ op}_2 z) = (x \text{ op}_1 y) \text{ op}_2 (x \text{ op}_1 z)$$

$$x \text{ op}_2 (y \text{ op}_1 z) = (x \text{ op}_2 y) \text{ op}_1 (x \text{ op}_2 z)$$

(a)					(b)				
&	1	a	b	c	#	1	a	b	c
1	1	a	b	c	1	1	1	1	1
a	a	a	c	c	a	1	a	1	a
b	b	c	b	c	b	1	1	b	b
c	c	c	c	c	c	1	a	b	c

Table 1: 2-input operators # and &.

For the homework problem, you will have $4 \times 4 \times 4 = 64$ (x, y, z) , but **No Need** to list them all!

Q1: Boolean Algebra

✓ Identity

- there exist element x such that: $x \text{ } op_1 \text{ } a_i = a_i$, for any $a_i \in \{a_0, a_1, \dots, a_n\}$

We call x the identity element of op_1

- there exist element y such that: $y \text{ } op_2 \text{ } a_i = a_i$, for any $a_i \in \{a_0, a_1, \dots, a_n\}$

We call y the identity element of op_2

✓ Complement

- each element x has a unique complement element y from $\{a_0, a_1, \dots, a_n\}$

$x \text{ } op_1 \text{ } y = \text{the identity element of } op_2$

$x \text{ } op_2 \text{ } y = \text{the identity element of } op_1$

- Boolean algebra has even number of values.

Q2: Multiplication Machine

- What is *Multiplication* between binary inputs?

✓ Given 2 **n-bits** input, how many bits for the output?

Output will have
 $(n + n) = 2n$ bits!

$$\begin{array}{r} \\ \\ \times \\ \hline \\ \\ \\ \\ \hline 1 1 1 0 \end{array}$$

← 3-bits

← 3-bits

← 6-bits

- Truth table: with 2 n -bits input elements and m -bits output
- Minterms & Maxterms

Q2: Multiplication Machine

- What is Multiplication between binary inputs?
- Truth table: with 2 n -bits input elements and $m = 2n$ bits output
- Minterms & Maxterms

Input 1							Input 2			Output	
id	a_{n-1}	...	a_0	b_{n-1}	...	b_0	p_{m-1}	...	p_0		
0	0	...	0	0	...	0					
1	0	...	0	0	...	1					
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$					
$2^{2n} - 1$	1	...	1	1	...	1					

$$= \begin{cases} 1, & \text{minterm } m_0 = a'_{n-1} a'_{n-2} \cdots b'_0 \\ 0, & \text{maxterm } M_0 = a_{n-1} + \cdots + b_0 \end{cases}$$



$$p_0 = \sum m(p, q, \dots) \text{ or } \prod_8 M(r, s, \dots)$$

Q3: Priority Encoder

- Truth table: **2^8 lines ! Are all the lines necessary?**
- don't care terms

Example: when $(a_7, a_6, a_5, a_4, a_3, a_2, a_1, a_0) =$

$$\begin{cases} (0,0,0,0,0,1,0,0) \\ (0,0,0,0,0,1,0,1) \\ (0,0,0,0,0,1,1,0) \\ (0,0,0,0,0,1,1,1) \end{cases} \text{ the output } (d_2, d_1, d_0) = (0,1,0)$$

So we could simplify the 4 inputs as $(0,0,0,0,0,1, X, X)$.

Q4&5: Logic Minimization

- **Adjacency**: two minterms (maxterms) are adjacent, if they differ only in one variable

We can merge each Adjacent pair into one term !

- K-maps
 - ✓ searching for adjacent pairs are the basis of logic minimization.
 - ✓ present a visual way to efficiently look for this pairs!
 - ✓ all neighbor cells in a K-maps are adjacent. Don't forget the edges!!

Q4: Minimize SOP

- implicant, **prime** implicant and **essential** prime implicant
- K-map with 5 variables $f(a, b, c, d, e) \Rightarrow$ 5 neighbors in K-map!!

$a = 0$

$de \backslash bc$	00	01	11	10
00	1 ₀	0 ₄	0 ₁₂	1 ₈
01	1 ₁	1 ₅	1 ₁₃	1 ₉
11	0 ₃	1 ₇	X ₁₅	0 ₁₁
10	0 ₂	1 ₆	0 ₁₄	0 ₁₀

Is $\sum m(5,7,13) + d(15)$ a prime implicant?

$a = 1$

$de \backslash bc$	00	01	11	10
00	1 ₁₆	0 ₂₀	0 ₂₈	1 ₂₄
01	1 ₁₇	1 ₂₁	X ₂₉	1 ₂₅
11	0 ₁₉	1 ₂₃	X ₃₁	0 ₂₇
10	0 ₁₈	0 ₂₂	0 ₃₀	0 ₂₆

Is $\sum m(6,7)$ an essential prime implicant?

Q4: Minimize SOP

- implicant, **prime** implicant and **essential** prime implicant
- K-map with 5 variables $f(a, b, c, d, e) \Rightarrow$ 5 neighbors in K-map!!

$a = 0$

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00	1 ₀	0 ₄	0 ₁₂	1 ₈
01	1 ₁	1 ₅	1 ₁₃	1 ₉
11	0 ₃	1 ₇	X ₁₅	0 ₁₁
10	0 ₂	1 ₆	0 ₁₄	0 ₁₀

Is $\sum m(5,7,13,15)$ a prime implicant?

NO! It's covered by
 $\sum m(5,7,13,15,21,23,29,31)$

$a = 1$

$de \backslash bc$	00	01	11	10
00	1 ₁₆	0 ₂₀	0 ₂₈	1 ₂₄
01	1 ₁₇	1 ₂₁	X ₂₉	1 ₂₅
11	0 ₁₉	1 ₂₃	X ₃₁	0 ₂₇
10	0 ₁₈	0 ₂₂	0 ₃₀	0 ₂₆

Is $\sum m(6,7)$ an essential prime implicant?

Yes! $m(6)$ is not covered by any other prime implicant

Q4: Minimize SOP

- implicant, **prime** implicant and **essential** prime implicant
- K-map with 5 variables $f(a, b, c, d, e) \rightarrow$ 5 neighbors in K-map!!

$a = 0$

$de \backslash bc$	00	01	11	10
00	1 0	0 4	0 12	1 8
01	1 1	1 5	1 13	1 9
11	0 3	1 7	X 15	0 11
10	0 2	1 6	0 14	0 10

prime implicants:

Essential prime implicants:

$a = 1$

$de \backslash bc$	00	01	11	10
00	1 16	0 20	0 28	1 24
01	1 17	1 21	X 29	1 25
11	0 19	1 23	X 31	0 27
10	0 18	0 22	0 30	0 26

Minimal SOP: $f(a, b, c, d, e) = ?$

Q4: Minimize SOP

- implicant, **prime** implicant and **essential** prime implicant
- K-map with 5 variables $f(a, b, c, d, e) \rightarrow$ 5 neighbors in K-map!!

$a = 0$

$de \backslash bc$	00	01	11	10
00	1 ₀	0 ₄	0 ₁₂	1 ₈
01	1 ₁	1 ₅	1 ₁₃	1 ₉
11	0 ₃	1 ₇	X ₁₅	0 ₁₁
10	0 ₂	1 ₆	0 ₁₄	0 ₁₀

$a = 1$

$de \backslash bc$	00	01	11	10
00	1 ₁₆	0 ₂₀	0 ₂₈	1 ₂₄
01	1 ₁₇	1 ₂₁	X ₂₉	1 ₂₅
11	0 ₁₉	1 ₂₃	X ₃₁	0 ₂₇
10	0 ₁₈	0 ₂₂	0 ₃₀	0 ₂₆

prime implicants:

$$\sum m(5, 7, 13, 15, 21, 23, 29, 31) = ce$$

$$\sum m(1, 5, 9, 13, 21, 23, 29, 31) = d'e$$

$$\sum m(0, 1, 8, 9, 16, 17, 24, 25) = c'd$$

$$\sum m(6, 7) = a'b'cd$$

Essential prime implicants:

$$\sum m(0, 1, 8, 9, 16, 17, 24, 25) = c'd$$

$$\sum m(6, 7) = a'b'cd$$

Minimal SOP: $f(a, b, c, d, e) = c'd + a'b'cd + ce$
 or $c'd + a'b'cd + d'e$

Q5: Minimize POS

- implicate, **prime** implicate and **essential** prime implicate
- K-map with 5 variables $f(a, b, c, d, e)$ 5 neighbors in K-map!!

$a = 0$

$de \backslash bc$	00	01	11	10
00	0 ₀	0 ₄	0 ₁₂	0 ₈
01	1 ₁	1 ₅	0 ₁₃	0 ₉
10	1 ₃	0 ₇	X ₁₅	1 ₁₁
11	1 ₂	1 ₆	0 ₁₄	1 ₁₀

$a = 1$

$de \backslash bc$	00	01	11	10
00	X ₁₆	0 ₂₀	0 ₂₈	0 ₂₄
01	0 ₁₇	1 ₂₁	X ₂₉	0 ₂₅
11	0 ₁₉	1 ₂₃	X ₃₁	1 ₂₇
10	0 ₁₈	1 ₂₂	1 ₃₀	1 ₂₆

Q5: Minimize POS

- implicate, **prime** implicate and **essential** prime implicate
- K-map with 5 variables $f(a, b, c, d, e)$ 5 neighbors in K-map!!

$a = 0$

$de \backslash bc$	00	01	11	10
00	0 ₀	0 ₄	0 ₁₂	0 ₈
01	1 ₁	1 ₅	0 ₁₃	0 ₉
10	1 ₃	0 ₇	X ₁₅	1 ₁₁
11	1 ₂	1 ₆	0 ₁₄	1 ₁₀

$a = 1$

$de \backslash bc$	00	01	11	10
00	X ₁₆	0 ₂₀	0 ₂₈	0 ₂₄
01	0 ₁₇	1 ₂₁	X ₂₉	0 ₂₅
11	0 ₁₉	1 ₂₃	X ₃₁	1 ₂₇
10	0 ₁₈	1 ₂₂	1 ₃₀	1 ₂₆

prime implicants:

$$\prod M(8,9,12,13,24,25,28,29) = b' + d$$

$$\prod M(0,4,8,12,16,20,24,28) = d + e$$

$$\prod M(12,13,14,15) = a + b' + c'$$

$$\prod M(16,17,18,19) = a' + b + c$$

$$\prod M(7,15) = a + c' + d' + e$$

Essential prime implicants: All the listed prime implicants

Minimal POS: $f(a, b, c, d, e) = (b' + d)(d + e)(a + b' + c')(a' + b + c)(a + c' + d' + e)$