

CSE140 HW5 Solution(100 Points)

June 4, 2018

1 Subtractor (10pts)

1.1 Design the subtractor with minimal SOP expressions. (5pts for completion)

A full subtractor inputs (x, y, b_{in}) and outputs (b_{out}, d) . The truth table for the subtractor is,

x	y	b_{in}	b_{out}	d
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

By using K-maps, the expressions for output bits b_{out} and d can be simplified as

$$\begin{aligned}b_{out} &= x'y + (x' + y)b_{in} \\ d &= x \oplus y \oplus b_{in}\end{aligned}$$

1.2 Design a four-bit borrow look ahead subtractor. (5pts for completion)

For each column of input bits, it will generate a borrow out if $x_i < y_i \Rightarrow x'_i = y_i = 1$. The **Generate** (G_i) for subtractor is

$$G_i = x'_i y_i$$

For each column of input bits, it will propagate a borrow in to borrow out if $x_i \leq y_i \Rightarrow x'_i \text{ OR } y_i = 1$. The **Propagate** (P_i) for subtractor is

$$P_i = x'_i + y_i$$

The borrow out of a column i is

$$b_{i+1} = G_i + P_i b_i$$

The difference of a column i is

$$d_i = x_i \oplus y_i \oplus b_i$$

Notice when $i = 0$, the borrow in is b_{-1} .

With (G_i, P_i) for each column, we can design a borrow look ahead subtractor.

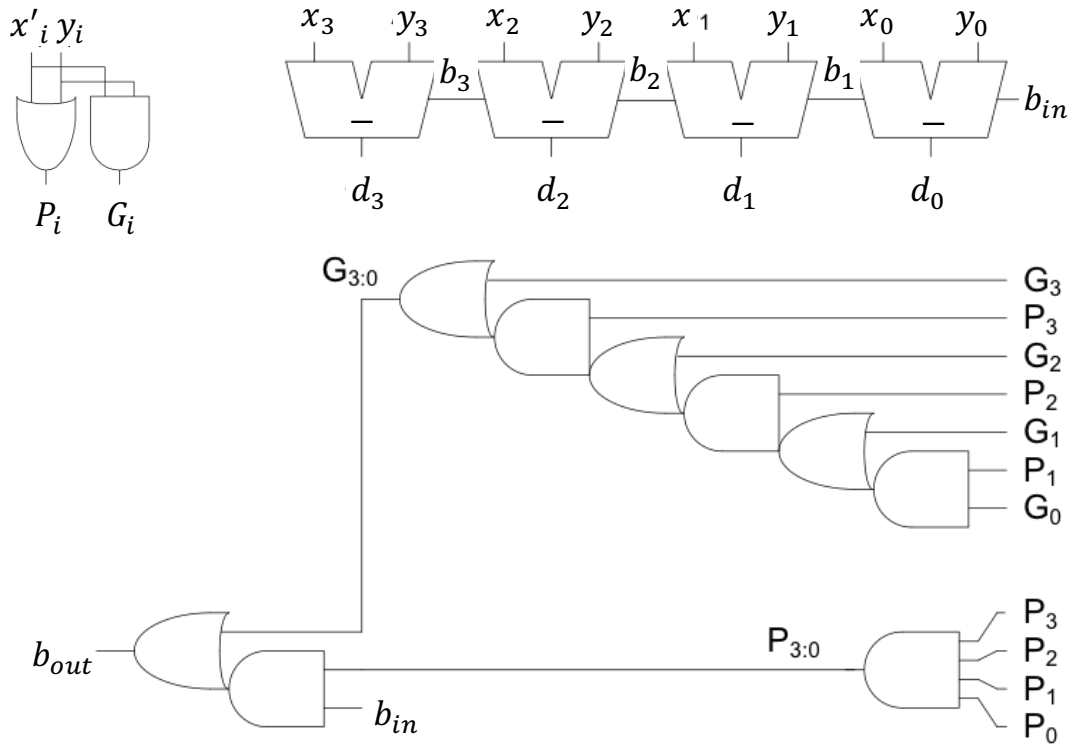
$$b_1 = G_0 + P_0 b_{in}$$

$$b_2 = G_1 + P_1 b_1$$

$$b_3 = G_2 + P_2 b_2$$

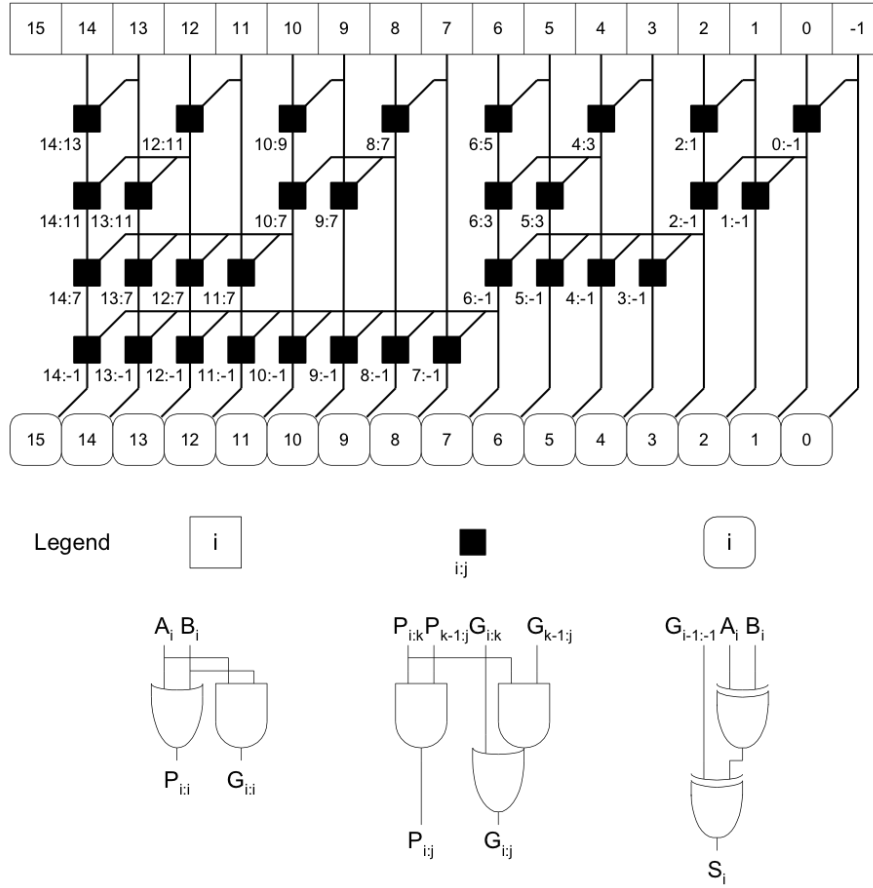
$$b_{out} = G_3 + P_3 b_3$$

We can reuse the structure the carry look ahead adder from slide 22 of lecture 14 for the borrow look ahead subtractor.



2 Prefix Adder (10pts)

Simulate the addition of two numbers with the prefix adder from slide 27 of lecture 14.



2.1 Simulate the additon of 289 and 756. (5pts for completion)

First we compute the pair $(G_{i:i}, P_{i:i})$ for each column. Since we don't have a carry in bit, so $(G_{-1}, P_{-1}) = (0, 0)$.

column i	15 ~ 10	9	8	7	6	5	4	3	2	1	0	-1
289	0	0	1	0	0	1	0	0	0	0	1	-
756	0	1	0	1	1	1	1	0	1	0	0	-
$(G_{i:i}, P_{i:i})$	(0,0)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)	(0,1)	(0,0)	(0,1)	(0,0)	(0,1)	(0,0)

Then compute the value (G, P) for each black box. For the first stage

$$\begin{aligned}
 G_{0:-1} &= G_{0:0} + P_{0:0}G_{-1:-1} = 0 & P_{0:-1} &= P_{0:0}P_{-1:-1} = 0 \\
 G_{2:1} &= G_{2:2} + P_{2:2}G_{1:1} = 0 & P_{2:1} &= P_{2:2}P_{1:1} = 0 \\
 G_{4:3} &= G_{4:4} + P_{4:4}G_{3:3} = 0 & P_{4:3} &= P_{4:4}P_{3:3} = 0 \\
 G_{6:5} &= G_{6:6} + P_{6:6}G_{5:5} = 1 & P_{6:5} &= P_{6:6}P_{5:5} = 1 \\
 G_{8:7} &= G_{8:8} + P_{8:8}G_{7:7} = 0 & P_{8:7} &= P_{8:8}P_{7:7} = 1 \\
 G_{10:9} &= G_{10:10} + P_{10:10}G_{9:9} = 0 & P_{10:9} &= P_{10:10}P_{9:9} = 0 \\
 G_{12:11} &= G_{12:12} + P_{12:12}G_{11:11} = 0 & P_{12:11} &= P_{12:12}P_{11:11} = 0 \\
 G_{14:13} &= G_{14:14} + P_{14:14}G_{13:13} = 0 & P_{14:13} &= P_{14:14}P_{13:13} = 0
 \end{aligned}$$

columns $i : j$	14 : 13	12 : 11	10 : 9	8 : 7	6 : 5	4 : 3	2 : 1	0 : -1
$(G_{i:j}, P_{i:j})$	(0,0)	(0,0)	(0,0)	(0,1)	(1,1)	(0,0)	(0,0)	(0,0)

For the second stage

$$\begin{aligned}
G_{1:-1} &= G_{1:1} + P_{1:1}G_{0:-1} = 0 & P_{1:-1} &= P_{1:1}P_{0:-1} = 0 \\
G_{2:-1} &= G_{2:1} + P_{2:1}G_{0:-1} = 0 & P_{2:-1} &= P_{2:1}P_{0:-1} = 0 \\
G_{5:3} &= G_{5:5} + P_{5:5}G_{4:3} = 1 & P_{5:3} &= P_{5:5}P_{4:3} = 0 \\
G_{6:3} &= G_{6:5} + P_{6:5}G_{4:3} = 1 & P_{6:3} &= P_{6:5}P_{4:3} = 0 \\
G_{9:7} &= G_{9:9} + P_{9:9}G_{8:7} = 0 & P_{9:7} &= P_{9:9}P_{8:7} = 1 \\
G_{10:7} &= G_{10:9} + P_{10:9}G_{8:7} = 0 & P_{10:7} &= P_{10:9}P_{8:7} = 0 \\
G_{13:11} &= G_{13:13} + P_{13:13}G_{12:11} = 0 & P_{13:11} &= P_{13:13}P_{12:11} = 0 \\
G_{14:11} &= G_{14:13} + P_{14:13}G_{12:11} = 0 & P_{14:11} &= P_{14:13}P_{12:11} = 0
\end{aligned}$$

columns $i : j$	14 : 11	13 : 11	10 : 7	9 : 7	6 : 3	5 : 3	2 : -1	1 : -1
$(G_{i:j}, P_{i:j})$	(0,0)	(0,0)	(0,0)	(0,1)	(1,0)	(1,0)	(0,0)	(0,0)

For the third stage

$$\begin{aligned}
G_{3:-1} &= G_{3:3} + P_{3:3}G_{2:-1} = 0 & P_{3:-1} &= P_{3:3}P_{2:-1} = 0 \\
G_{4:-1} &= G_{4:3} + P_{4:3}G_{2:-1} = 0 & P_{4:-1} &= P_{4:3}P_{2:-1} = 0 \\
G_{5:-1} &= G_{5:3} + P_{5:3}G_{2:-1} = 1 & P_{5:-1} &= P_{5:3}P_{2:-1} = 0 \\
G_{6:-1} &= G_{6:3} + P_{6:3}G_{2:-1} = 1 & P_{6:-1} &= P_{6:3}P_{2:-1} = 0 \\
G_{11:7} &= G_{11:11} + P_{11:11}G_{10:7} = 0 & P_{11:7} &= P_{11:11}P_{10:7} = 0 \\
G_{12:7} &= G_{12:11} + P_{12:11}G_{10:7} = 0 & P_{12:7} &= P_{12:11}P_{10:7} = 0 \\
G_{13:7} &= G_{13:11} + P_{13:11}G_{10:7} = 0 & P_{13:7} &= P_{13:11}P_{10:7} = 0 \\
G_{14:7} &= G_{14:11} + P_{14:11}G_{10:7} = 0 & P_{14:7} &= P_{14:11}P_{10:7} = 0
\end{aligned}$$

columns $i : j$	14 : 7	13 : 7	12 : 7	11 : 7	6 : -1	5 : -1	4 : -1	3 : -1
$(G_{i:j}, P_{i:j})$	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)	(1,0)	(0,0)	(0,0)

For the forth stage

$$\begin{aligned}
G_{7:-1} &= G_{7:7} + P_{7:7}G_{6:-1} = 1 & P_{7:-1} &= P_{7:7}P_{6:-1} = 0 \\
G_{8:-1} &= G_{8:7} + P_{8:7}G_{6:-1} = 1 & P_{8:-1} &= P_{8:7}P_{6:-1} = 0 \\
G_{9:-1} &= G_{9:7} + P_{9:7}G_{6:-1} = 1 & P_{9:-1} &= P_{9:7}P_{6:-1} = 0 \\
G_{10:-1} &= G_{10:7} + P_{10:7}G_{6:-1} = 0 & P_{10:-1} &= P_{10:7}P_{6:-1} = 0 \\
G_{11:-1} &= G_{11:7} + P_{11:7}G_{6:-1} = 0 & P_{11:-1} &= P_{11:7}P_{6:-1} = 0 \\
G_{12:-1} &= G_{12:7} + P_{12:7}G_{6:-1} = 0 & P_{12:-1} &= P_{12:7}P_{6:-1} = 0 \\
G_{13:-1} &= G_{13:7} + P_{13:7}G_{6:-1} = 0 & P_{13:-1} &= P_{13:7}P_{6:-1} = 0 \\
G_{14:-1} &= G_{14:7} + P_{14:7}G_{6:-1} = 0 & P_{14:-1} &= P_{14:7}P_{6:-1} = 0
\end{aligned}$$

columns $i : j$	14 : -1	13 : -1	12 : -1	11 : -1	10 : -1	9 : -1	8 : -1	7 : -1
$(G_{i:j}, P_{i:j})$	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)	(1,0)	(1,0)

So we have all the prefixes $G_{i:-1}$, $0 \leq i < 15$, for each column the sum equals

$$S_i = a_i \oplus b_i \oplus G_{i-1:-1}$$

column i	15 ~ 11	10	9	8	7	6	5	4	3	2	1	0
289	0	0	0	1	0	0	1	0	0	0	0	1
756	0	0	1	0	1	1	1	1	0	1	0	0
$G_{i-1:-1}$	0	1	1	1	1	1	0	0	0	0	0	0
Sum	0	1	0	0	0	0	0	1	0	1	0	1

2.2 Simulate the additon of 321 and 456. (5pts for completion)

First we compute the pair $(G_{i:i}, P_{i:i})$ for each column. Since we don't have a carry in bit, so $(G_{-1}, P_{-1}) = (0, 0)$.

column i	15 ~ 9	8	7	6	5	4	3	2	1	0	-1
321	0	1	0	1	0	0	0	0	0	1	-
456	0	1	1	1	0	0	1	0	0	0	-
$(G_{i:i}, P_{i:i})$	(0,0)	(1,1)	(0,1)	(1,1)	(0,0)	(0,0)	(0,1)	(0,0)	(0,0)	(0,1)	(0,0)

Then compute the value (G, P) for each black box. For the first stage

$$\begin{aligned}
G_{0:-1} &= G_{0:0} + P_{0:0}G_{-1:-1} = 0 & P_{0:-1} &= P_{0:0}P_{-1:-1} = 0 \\
G_{2:1} &= G_{2:2} + P_{2:2}G_{1:1} = 0 & P_{2:1} &= P_{2:2}P_{1:1} = 0 \\
G_{4:3} &= G_{4:4} + P_{4:4}G_{3:3} = 0 & P_{4:3} &= P_{4:4}P_{3:3} = 0 \\
G_{6:5} &= G_{6:6} + P_{6:6}G_{5:5} = 1 & P_{6:5} &= P_{6:6}P_{5:5} = 0 \\
G_{8:7} &= G_{8:8} + P_{8:8}G_{7:7} = 1 & P_{8:7} &= P_{8:8}P_{7:7} = 1 \\
G_{10:9} &= G_{10:10} + P_{10:10}G_{9:9} = 0 & P_{10:9} &= P_{10:10}P_{9:9} = 0 \\
G_{12:11} &= G_{12:12} + P_{12:12}G_{11:11} = 0 & P_{12:11} &= P_{12:12}P_{11:11} = 0 \\
G_{14:13} &= G_{14:14} + P_{14:14}G_{13:13} = 0 & P_{14:13} &= P_{14:14}P_{13:13} = 0
\end{aligned}$$

columns $i : j$	14 : 13	12 : 11	10 : 9	8 : 7	6 : 5	4 : 3	2 : 1	0 : -1
$(G_{i:j}, P_{i:j})$	(0,0)	(0,0)	(0,0)	(1,1)	(1,0)	(0,0)	(0,0)	(0,0)

For the second stage

$$\begin{aligned}
G_{1:-1} &= G_{1:1} + P_{1:1}G_{0:-1} = 0 & P_{1:-1} &= P_{1:1}P_{0:-1} = 0 \\
G_{2:-1} &= G_{2:1} + P_{2:1}G_{0:-1} = 0 & P_{2:-1} &= P_{2:1}P_{0:-1} = 0 \\
G_{5:3} &= G_{5:5} + P_{5:5}G_{4:3} = 0 & P_{5:3} &= P_{5:5}P_{4:3} = 0 \\
G_{6:3} &= G_{6:5} + P_{6:5}G_{4:3} = 1 & P_{6:3} &= P_{6:5}P_{4:3} = 0 \\
G_{9:7} &= G_{9:9} + P_{9:9}G_{8:7} = 0 & P_{9:7} &= P_{9:9}P_{8:7} = 0 \\
G_{10:7} &= G_{10:9} + P_{10:9}G_{8:7} = 0 & P_{10:7} &= P_{10:9}P_{8:7} = 0 \\
G_{13:11} &= G_{13:13} + P_{13:13}G_{12:11} = 0 & P_{13:11} &= P_{13:13}P_{12:11} = 0 \\
G_{14:11} &= G_{14:13} + P_{14:13}G_{12:11} = 0 & P_{14:11} &= P_{14:13}P_{12:11} = 0
\end{aligned}$$

columns $i : j$	14 : 11	13 : 11	10 : 7	9 : 7	6 : 3	5 : 3	2 : -1	1 : -1
$(G_{i:j}, P_{i:j})$	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)	(0,0)	(0,0)	(0,0)

For the third stage

$$\begin{aligned}
G_{3:-1} &= G_{3:3} + P_{3:3}G_{2:-1} = 0 & P_{3:-1} &= P_{3:3}P_{2:-1} = 0 \\
G_{4:-1} &= G_{4:3} + P_{4:3}G_{2:-1} = 0 & P_{4:-1} &= P_{4:3}P_{2:-1} = 0 \\
G_{5:-1} &= G_{5:3} + P_{5:3}G_{2:-1} = 0 & P_{5:-1} &= P_{5:3}P_{2:-1} = 0 \\
G_{6:-1} &= G_{6:3} + P_{6:3}G_{2:-1} = 1 & P_{6:-1} &= P_{6:3}P_{2:-1} = 0 \\
G_{11:7} &= G_{11:11} + P_{11:11}G_{10:7} = 0 & P_{11:7} &= P_{11:11}P_{10:7} = 0 \\
G_{12:7} &= G_{12:11} + P_{12:11}G_{10:7} = 0 & P_{12:7} &= P_{12:11}P_{10:7} = 0 \\
G_{13:7} &= G_{13:11} + P_{13:11}G_{10:7} = 0 & P_{13:7} &= P_{13:11}P_{10:7} = 0 \\
G_{14:7} &= G_{14:11} + P_{14:11}G_{10:7} = 0 & P_{14:7} &= P_{14:11}P_{10:7} = 0
\end{aligned}$$

columns $i : j$	14 : 7	13 : 7	12 : 7	11 : 7	6 : -1	5 : -1	4 : -1	3 : -1
$(G_{i:j}, P_{i:j})$	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)	(0,0)	(0,0)	(0,0)

For the forth stage

$$\begin{aligned}
G_{7:-1} &= G_{7:7} + P_{7:7}G_{6:-1} = 1 & P_{7:-1} &= P_{7:7}P_{6:-1} = 0 \\
G_{8:-1} &= G_{8:7} + P_{8:7}G_{6:-1} = 1 & P_{8:-1} &= P_{8:7}P_{6:-1} = 0 \\
G_{9:-1} &= G_{9:7} + P_{9:7}G_{6:-1} = 0 & P_{9:-1} &= P_{9:7}P_{6:-1} = 0 \\
G_{10:-1} &= G_{10:7} + P_{10:7}G_{6:-1} = 0 & P_{10:-1} &= P_{10:7}P_{6:-1} = 0 \\
G_{11:-1} &= G_{11:7} + P_{11:7}G_{6:-1} = 0 & P_{11:-1} &= P_{11:7}P_{6:-1} = 0 \\
G_{12:-1} &= G_{12:7} + P_{12:7}G_{6:-1} = 0 & P_{12:-1} &= P_{12:7}P_{6:-1} = 0 \\
G_{13:-1} &= G_{13:7} + P_{13:7}G_{6:-1} = 0 & P_{13:-1} &= P_{13:7}P_{6:-1} = 0 \\
G_{14:-1} &= G_{14:7} + P_{14:7}G_{6:-1} = 0 & P_{14:-1} &= P_{14:7}P_{6:-1} = 0
\end{aligned}$$

columns $i : j$	14 : -1	13 : -1	12 : -1	11 : -1	10 : -1	9 : -1	8 : -1	7 : -1
$(G_{i:j}, P_{i:j})$	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)	(1,0)

So we have all the prefixes $G_{i:-1}$, $0 \leq i < 15$, for each column the sum equals

$$S_i = a_i \oplus b_i \oplus G_{i-1:-1}$$

column i	15 ~ 10	9	8	7	6	5	4	3	2	1	0
321	0	0	1	0	1	0	0	0	0	0	1
456	0	0	1	1	1	0	0	1	0	0	0
$G_{i-1:-1}$	0	1	1	1	0	0	0	0	0	0	0
Sum	0	1	1	0	0	0	0	1	0	0	1

3 Carry Save Adder (40pts)

3.1 Simulate the additon of 28, 19, 11 and 14. (30pts)

- 2pts for each wrong pair (2bits) of fill-ins in the rows A, B, C, Sum.

Weight	64	32	16	8	4	2	1
n1=28	0	0	1	1	1	0	0
n2=19	0	0	1	0	0	1	1
n3=11	0	0	0	1	0	1	1
n4=14	0	0	0	1	1	1	0

n1	00	00	10	10	10	00	00
n2	0	0	1	0	0	1	1
A=n1+n2	00	01	00	10	10	10	10
n3	0	0	0	1	0	1	1
B=A+n3	00	10	01	00	11	01	00
n4	0	0	0	1	1	1	0
C=B+n4	00	10	10	11	11	00	00
2 bits of C	0	0	0	1	1	0	0
	0	1	1	1	1	0	0
Sum	1	0	0	1	0	0	0

3.2 Simulate the additon of 17, 18, 31 and 15. (5pts for completion)

Weight	64	32	16	8	4	2	1
n1=17	0	0	1	0	0	0	1
n2=18	0	0	1	0	0	1	0
n3=31	0	0	1	1	1	1	1
n4=15	0	0	0	1	1	1	1

n1	00	00	10	00	00	00	10
n2	0	0	1	0	0	1	0
A=n1+n2	00	01	00	00	00	10	10
n3	0	0	1	1	1	1	1
B=A+n3	00	10	10	10	11	01	00
n4	0	0	0	1	1	1	1
C=B+n4	00	10	11	01	11	00	10
2 bits of C	0	0	1	1	1	0	0
	0	1	1	0	1	0	1
Sum	1	0	1	0	0	0	1

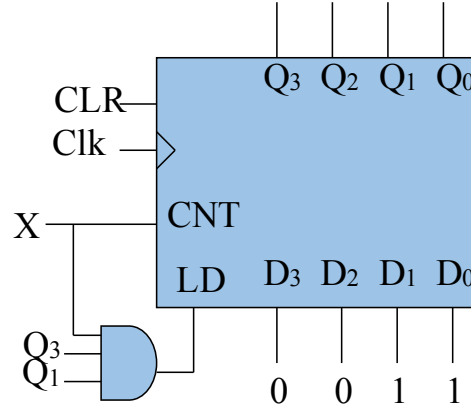
3.3 (5pts for completion)

In a carry save adder, every column computes the sum of three input bits and creates a carry out to the next column. Since the sum of three binary bits is at most 11 which consists at most 2 digits, we only need one digit to advance the carry out.

4 Counter (40pts)

4.1 Given a modulo 16 counter, construct a 3 to 10 counter. (5pts for completion)

We set signal $LD = 1$ when $CNT = 1$ and $(Q_3Q_2Q_1Q_0) = (1010)$, at the same time set $(D_3D_2D_1D_0) = (0011)$.



4.2 Given a modulo 16 counter, construct a counter that counts in sequence 1, 2, 5, 6, 7, 0, 4 with minimal networks of AND, OR gates. (30pts)

- -1pt for each missing or wrong label of $CNT, LD, D_3 \sim D_0$, no more than 5pts
- 5pts for each correct input of $LD, D_3 \sim D_0$; -1pt for any wrong input in each minimal expression.

We need to load new numbers from D to Q

- When $Q = 2$, load $D = 5$
- When $Q = 7$, load $D = 0$
- When $Q = 0$, load $D = 4$
- When $Q = 4$, load $D = 1$

The truth table for the design

ID	$Q_3Q_2Q_1Q_0$	LD	D_3	D_2	D_1	D_0
0	0000	1	0	1	0	0
1	0001	0	-	-	-	-
2	0010	1	0	1	0	1
3	0011	-	-	-	-	-
4	0000	1	0	0	0	1
5	0001	0	-	-	-	-
6	0010	0	-	-	-	-
7	0011	1	0	0	0	0
8~15	1000~1111	-	-	-	-	-

Through K-maps, we derive the minimal expressions of LD, D_3, D_2, D_1 .

We have $LD = Q'_1Q'_0 + Q_1Q_0 + Q'_2Q'_0$.

LD

$Q_1Q_0 \setminus Q_3Q_2$	00	01	11	10
00	1 0	1 4	X 12	X 8
01	0 1	0 5	X 13	X 9
11	X 3	1 7	X 15	X 11
10	1 2	0 6	X 14	X 10

Obviously we can tell from the truth table that $D_3 = 0, D_1 = 0$. We have $D_2 = Q'_2$ and $D_0 = Q_2Q'_1(or \quad Q_2Q'_0) + Q_1Q'_0(or \quad Q'_2Q_1)+$ with the following two K-maps.

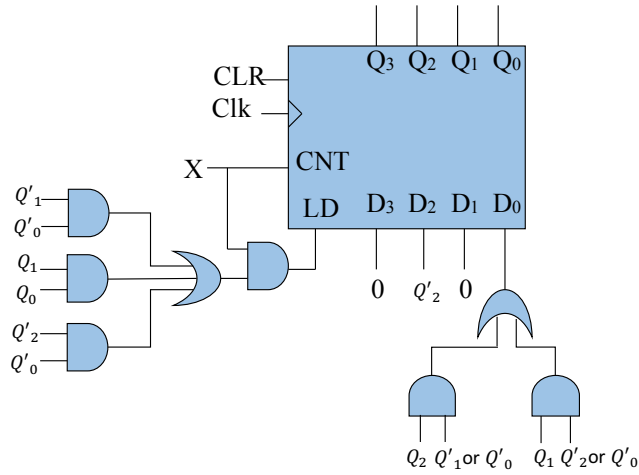
D_2

$Q_1Q_0 \setminus Q_3Q_2$	00	01	11	10
00	1 0	0 4	X 12	X 8
01	X 1	X 5	X 13	X 9
11	X 3	0 7	X 15	X 11
10	1 2	X 6	X 14	X 10

D_0

$Q_1Q_0 \setminus Q_3Q_2$	00	01	11	10
00	0 0	1 4	X 12	X 8
01	X 1	X 5	X 13	X 9
11	X 3	0 7	X 15	X 11
10	1 2	X 6	X 14	X 10

The minimal network



4.3 Construct a modulo 64 counter using modulo 8 counter. (5pts for completion)

Use the signal $TC = 1$ when the modulo m counter has $Q(t) = m - 1$ and $CNT = 1$. The cascade counter with modulo 8 counters will do $Q_5(t+1)Q_4(t+1)Q_3(t+1) = Q_5(t)Q_4(t)Q_3(t) + 1 \bmod 8$ when $T_{c0} = 1$.

